Winkler Coefficient for Beams on Elastic Foundation

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ABSTRACT

Using the elastic continuum approach for the analysis of beam on elastic foundation, a method is developed to calculate an equivalent coefficient of subgrade reaction k to use in the Winkler model. It is shown that dividing the contact pressure produced from elastic continuum model by the corresponding settlement along the beam length generates length dependent coefficient of subgrade reaction that accurately describes the interaction between the beam and the supporting soils. Results from the developed equivalent k parameter are compared with those from elastic continuum model and by using different k values predicted by other methods that are currently in use. For uniformly loaded beam, graphs and regression equations are provided from which an equivalent k value at different positions along the beam length (e.g., at the center and the edge of the beam) can be calculated as a function of the elastic soil properties and the rigidity of the beam.

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1. INTRODUCTION

The response of the supporting soil is considered the most important parameter in the analysis of soil-structure interaction problems such as beam on elastic foundation (Hain and Lee, 1974; Horvath, 1993; Ulrich, 1995; El-Garhy, 2001). There are two approaches that can be used to model the response of the supporting soils. The first approach is the Winkler approach or the subgrade reaction approach, in which the soil is represented by a series of independent linear springs, each spring having stiffness called the coefficient of subgrade reaction. Winkler suggested that the contact pressure, P, at a point is proportional to the displacement, w, at that point, Eq. (1).

$$P = kw \tag{1}$$

Where

k = the coefficient of subgrade reaction (F/L^3).

The main drawback in the Winkler model is that, it cannot consider the continuity of the soil. Recently attempts were made to overcome this drawback by assuming some kind of interaction between soil springs and added a second parameter to Eq. (1) (Horvath, 1989; Shirima and Giger, 1992).

The second approach is the elastic continuum approach, in which the soil is assumed to be a semi-infinite elastic continuum. The elastic continuum approach is shown to be the most realistic approach representing the performance of the subgrade (Hain and Lee, 1974; El-Garhy, 2001). However, the effective use of this approach requires a careful determination of the elastic soil properties to establish values appropriate to the field stress state (Hain and Lee, 1974).

Mathematically, the elastic continuum approach is considered much more complex than the Winkler approach. Therefore, the Winkler approach has been used by most commercial computer codes to solve many soil-foundation interaction problems. The user of these codes has to determine the coefficient of subgrade reaction, k, to represent the soil. There is no easy way to determine the k value because its value is not unique for a given type of soil (Daloglu and Vallabhan, 2000).

In the analysis of soil-foundations interaction, using a constant k value to model the supporting soil leads to wrong design (Ulrich, 1995). If the analysis is performed for uniformly loaded beam or raft foundations using a constant k value, the values of differential settlements, bending moments, and shear forces in the foundations will be equal to zero, which doubtless is not correct. To overcome this problem Ulrich (1991) suggested the use of the discrete area method, in which varying coefficient of subgrade reaction is considered and ACI Committee 336 (1991) and Bowles (1996) have suggested to double the value of k at the edges of the foundations. The actual subgrade responses beneath beam and raft foundations lead to deflections and contact pressures that result in non-uniform coefficient of subgrade reaction (Daloglu and Vallabhan, 2000).

Because of the extensive use of Winkler approach with constant k value as a subgrade model by most of commercial computer codes and the variety of methods suggested over the years for calculating the value of the constant k, an evaluation of relative accuracy of these methods is of some interest. The most important task is developing a rational and simple method to calculate the actual distribution of the coefficient of subgrade reaction beneath the foundations. There are a number of factors that govern the value of the coefficient of subgrade reaction, these factors include elastic soil properties, foundation rigidity, and the applied load pattern.

In this paper the different methods for calculating the constant coefficient of subgrade reaction are presented and evaluated. Using the elastic continuum model, more-accurate method is developed to calculate equivalent values of the coefficient of subgrade reaction along the beam length to use in the Winkler model. Comparison between the results of the elastic continuum model with those of Winkler model using the equivalent k values and five different values of k as suggested in the literature are presented and discussed. For uniformly loaded beam, graphs and regression equations are provided from which an equivalent k value at different positions along the beam length (e.g., at the center and the edge of the beam) can be determined as a function of the relative stiffness and L/B ratio.

2. PREVIOUS METHODS TO CALCULATE k VALUE

The available methods used to calculate the constant value of the coefficient of subgrade reaction can be classified into two categories: table or chart methods, and those based on the theory of elasticity. The most widely referenced table/chart methods are based on plate load tests data (Terzaghi, 1955; Scot, 1981; Bazaraa and Howeedy, 1975; Ismael, 1987). This paper will focus on the methods based on the theory of elasticity. These methods are presented and discussed in the next section.

2.1. Biot Method

Biot (1937) solved the problem of an infinite beam with a concentrated load resting on two and three-dimensional elastic soil continuum. Biot found a correlation of the continuum elastic theory and the Winkler model where the maximum moments in the beam are equated.

In the case of infinite beam resting on two-dimensional elastic foundation, Biot developed the following equation for k value.

$$k = 0.71 E_s \left[\frac{E_s b^4}{EI} \right]^{1/3} \tag{2}$$

Where

E_s = modulus of elasticity of the soil

EI = bending rigidity of the beam

b = half beam width

In the case of infinite beam resting on three-dimensional elastic foundation, Biot developed the following equation for k value.

$$k = \frac{1.23E_s}{BC(1-\nu^2)}\left[\frac{1}{C(1-\nu^2)}\frac{E_s b^4}{EI}\right]^{0.11} \tag{3}$$

where

C = 1.0 if distribution of pressure across beam width is uniform
= 1.13 if deflection across the beam width is uniform

The value of C can be taken 1.10 for practical purposes (Vesic, 1961).

2.2. Vesic Method

Vesic (1961) extended Biot's solution of infinite beam resting on three-dimensional elastic foundation to the case of loading by a couple and introduced a criterion for using Winkler model with beams of finite length. On similar manner of Biot, Vesic developed the following equation for k value

$$k = \frac{0.65E_s}{B(1-\nu^2)}\sqrt[12]{\frac{E_s B^4}{EI}} \tag{4}$$

where

B = $2b$
= beam width

All the terms in Eq. (4) are the same as in Eq. (3).

2.3. Horvath Method

Horvath (1983a) developed a new subgrade model for raft-soil interaction. Horvath model was based on the theory of elasticity and was called the Reissner Simplified Continuum (RSC). Using RSC, Horvath (1983b) developed equations to calculate the coefficient of subgrade reaction as a function of the soil modulus of elasticity, E_s, and the depth of elastic soil layer, H, taking into account the change of E_s with the depth of elastic soil layer.

For constant soil modulus of elasticity with depth, Horvath developed the following equation:

$$k = \frac{E_s}{H} \tag{5}$$

For soil modulus of elasticity change with depth according to Eq. (6), Horvath developed Eq. (7) to calculate k value:

$$E_s = E_o + Az \tag{6}$$

where

E_o = soil modulus of elasticity directly beneath the loaded area
A = the rate of change of soil modulus of elasticity with depth

$$k = \frac{A}{\ln(E_o + AH) - \ln(E_o)} \tag{7}$$

For soil modulus of elasticity change with depth according to Eq. (8), Horvath developed Eq. (9) to calculate k value:

$$E_s = E_o + Az^{0.5} \tag{8}$$

where all the terms in Eq. (8) are the same as in Eq. (6).

$$k = \frac{A^2}{2\left[(E_o + AH^{0.5}) - E_o \ln(E_o + AH^{0.5}) - E_o + E_o \ln(E_o)\right]} \tag{9}$$

To calculate the k value by Horvath method, two parameters should be known: the layer thickness, H, and the soil modulus of elasticity. Horvath (1983b) pointed out that H should be selected as the depth to the rigid layer. If the width of the loaded area is small with respect to the depth to the actual rigid layer, Horvath suggested that the value of H could be taken equal to a depth less than the depth to the actual rigid layer, where vertical settlement can be assumed to be negligible. Horvath (1983b) suggested the following equation to calculate the value of H :

$$H = I_k B \tag{10}$$

where

I_k	=	an influence factor to be determined
B	=	the width of the loaded area (beam width)

I_k was taken as 2 as suggested by Schmertmann in settlement calculation (Horvath, 1983b).

2.4. Bowles Method

Bowles (1996) developed the following equation, Eq. (11), to calculate the k value. This equation produced from the immediate settlement equation based on the theory of elasticity.

$$k = \frac{E_s}{BI_s I_F (1 - \nu_s^2)} \tag{11}$$

where

I_s	=	an influence factor that depends on L/B and H/B
I_F	=	an influence factor that depends on L/B and D/B
B	=	the width of the loaded area (beam width)
L	=	the length of the loaded area (beam length)
H	=	thickness of soil layer
	=	$5B$ as recommended by Bowles, 1996
D	=	foundation depth

The influence factors I_s and I_F can be determined from tables prepared by Bowles (1996). Equivalent

3. NEW METHOD TO CALCULATE EQUIVALENT k VALUES

The elastic continuum model is used in an attempt to determine equivalent values for the coefficient of subgrade reaction along the beam length. After performing the analysis of the beam using elastic continuum model, the equivalent k values are computed by dividing the contact pressure by the corresponding settlement at each node along the beam length. Using the equivalent k values, the same beam is analyzed by the Winkler model. The resulting bending moments and settlements compared to that produced form elastic continuum model. It is shown that using the equivalent k values with the Winkler model describes accurately the interaction between the beam and the supporting soil like the elastic continuum model.

The authors developed a computer program called WCBECM (Winkler Coefficient for Beam on Elastic Continuum Model) using the elastic continuum model and the Finite Element Method to analyze the interaction between the beam and the supporting soils. WCBECM is able to calculate the values of settlement, contact pressure, and equivalent k value at each node in the finite element mesh. WCBECM is able to analyze the beam when subjected to different load patterns. WCBECM is coded in FORTRAN and suitable for microcomputers.

3.1. Numerical Examples to Check the New Method

Numerical examples are selected to compare the results of the elastic continuum model with those of Winkler model using the equivalent k values determined by WCBECM and five different values of constant k as determined from the previous equations. The first example is a beam subjected to two symmetrical concentrated loads of 500 kN and resting on elastic half space as shown in Fig. (1a). The beam is of 10-m length, 1-m width, and 0.5-m thickness. The modulus of elasticity of the beam is $2.11\times10^7\,kN/m^2$. The modulus of elasticity and Poisson's ratio of the subgrade are $1.0\times10^4\,kN/m^2$ and 0.2 respectively.

The program WCBECM is used to determine the equivalent k values along the beam length for the beam shown in Fig. (1a). These values are drawn against the beam length as shown in Fig. (1b). The displacements and the bending moments along the beam length are presented in Figs. (1c, 1d). Referring to these figures it is observed that:

1. Excellent comparison between the results of Winkler model using equivalent k values and those from the elastic continuum model. The maximum bending moment at the center of the beam is calculated as 315.4 $kN.m$ using equivalent k values against 315.3 $kN.m$ using elastic continuum model.
2. The results of Winkler Model using Biot equation (2D), Eq. (2), are highly greater than those from the elastic continuum model. The

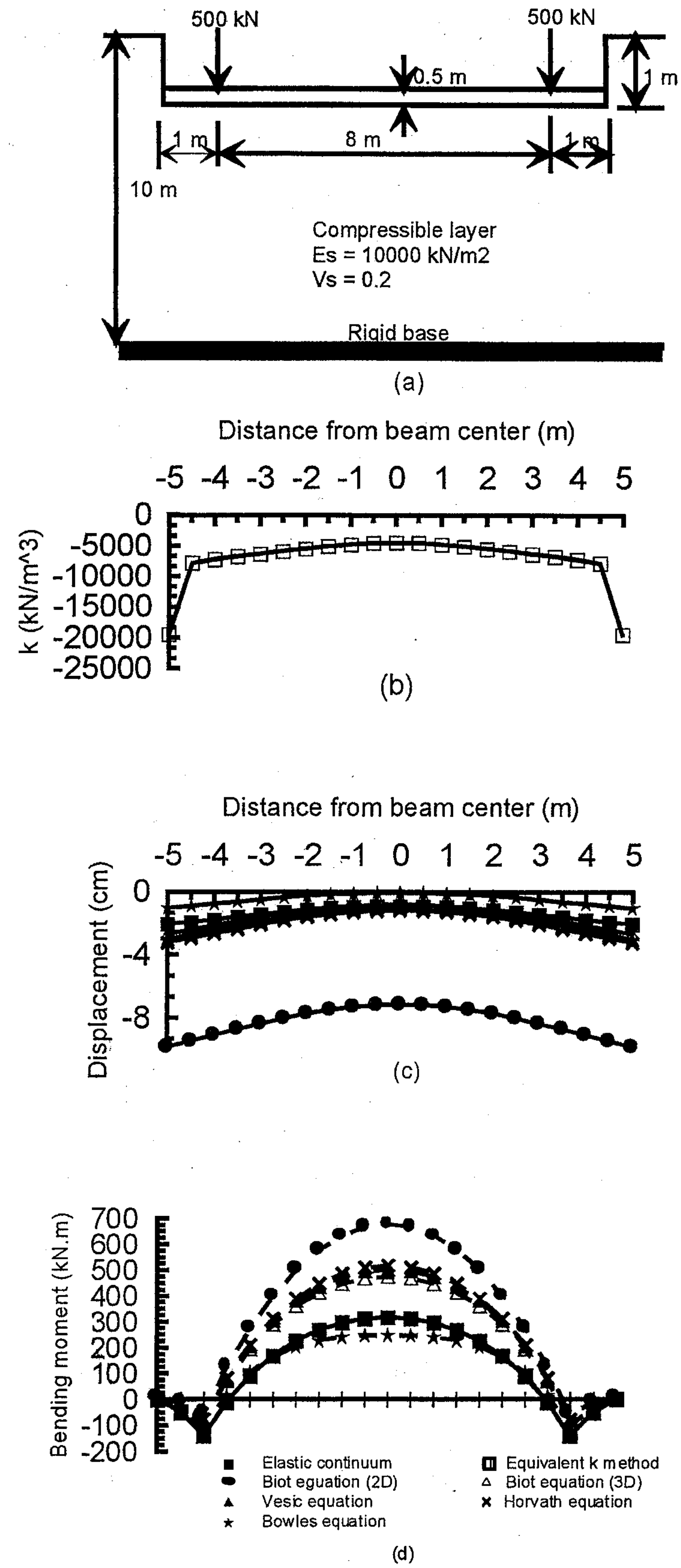


Fig. (1). Comparison of Results for Two Concentrated Loads: (a) Beam Geometry; (b) Equivalent k Values; (c) Displacement; (d) Moments maximum bending moment at the center of the beam is calculated as $675.9\,kN.m$ using Biot equation (2D) against $315.3\,kN.m$ using elastic continuum model.

3. The results of Winkler Model using Bowles equation, Eq. (11), are smaller than those from the elastic continuum model. The maximum bending moment at the center of the beam is calculated as $246.2\,kN.m$ using Bowles equation against $315.3\,kN.m$ using elastic continuum model.
4. The results of Winkler Model using Biot equation (3D), Eq. (3), Vesic equation, Eq. (4), and Horvath equation, Eqs. (5, 10), are greater than those from the elastic continuum model. The maximum bending moment at the center of the beam is calculated as $473.2\,kN.m$, $501.4\,kN.m$, and $516.8\,kN.m$ using Biot equation (3D), Vesic equation, and Horvath equation, respectively, against $315.3\,kN.m$ using elastic continuum model.

The second example is a beam subjected to uniformly distributed load of $100kN/m^2$ and resting on elastic half space. The beam is of 10-m length, 1.25-m width, and 0.5-m thickness. The modulus of elasticity of the beam is $2.11\times10^7\,kN/m^2$. The modulus of elasticity and Poisson's ratio of the subgrade are $1.2\times10^4\,kN/m^2$ and 0.3 respectively.

The equivalent k values are determined by the program WCBECM and drawn against the beam length as shown in Fig. (2a). The displacements and the bending moments along the beam length are presented in Figs. (2b, 2c). Refering to these figures it is observed that:

1. Excellent comparison between the results of Winkler model using equivalent k values and those from the elastic continuum model. The maximum bending moment at the center of the beam is calculated as 73.8 $kN.m$ using equivalent k values against $74.6\,kN.m$ using elastic continuum model.
2. The results of Winkler model using constant coefficient of subgrade reaction calculated by the previous equations produced constant displacements and consequently zero bending moments.

4. EQUIVALENT k VALUES FOR UNIFORMLY LOADED BEAM

WCBECM is used to determine the equivalent k values along the beam length for many problems of uniformly loaded beams resting on elastic continuum model. The data of the beam-soil system used in the study are given in Table 1.

For convenience, Poisson's ratio of the soil and the modulus of elasticity of the beam material are assumed constant and taken 0.3 and $2.2\times10^7\,kN/m^2$ respectively in this research

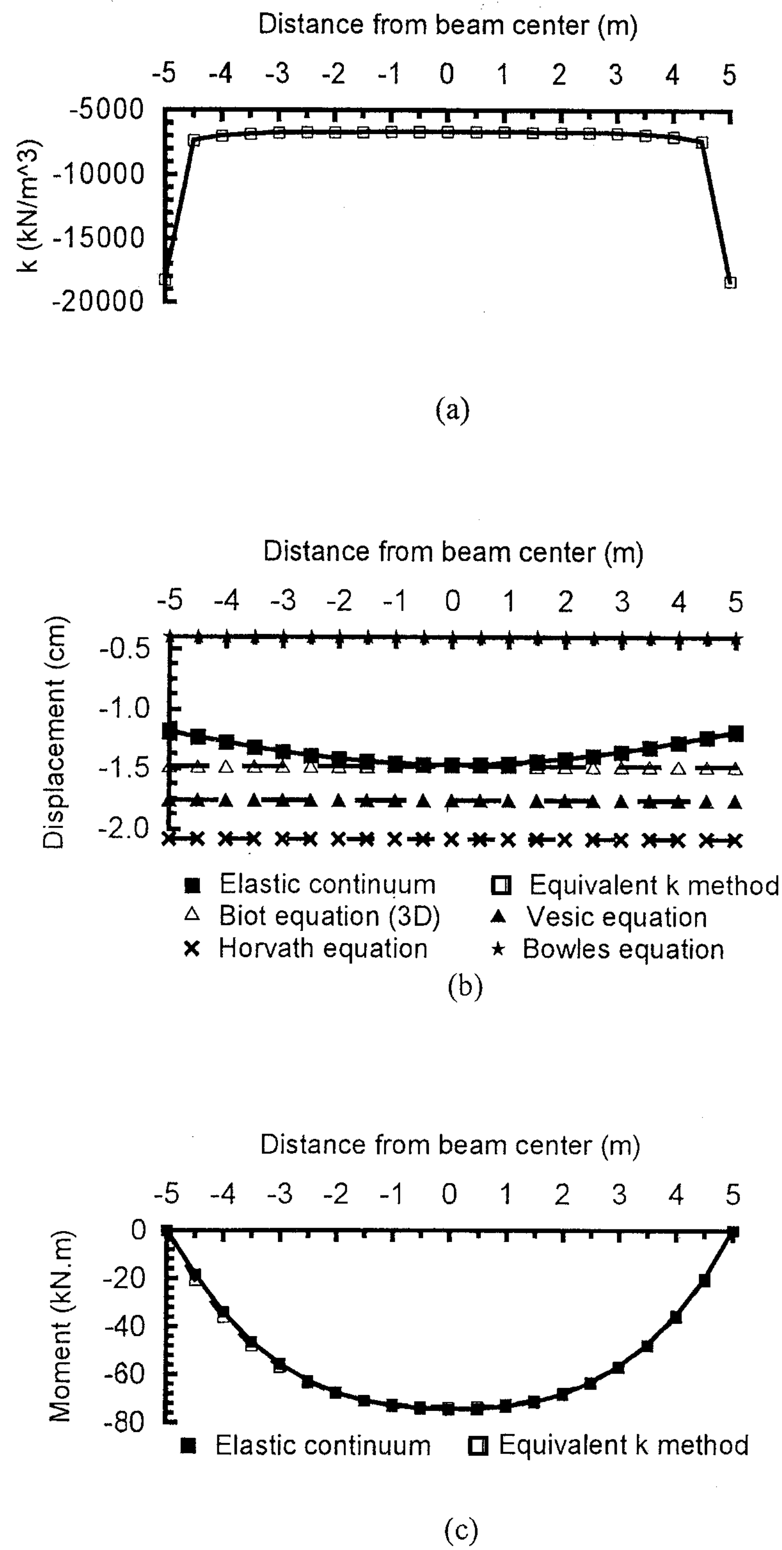


Fig. (2). Comparison of Results for Uniformly Distributed Load:
(a) Equivalent k Values; (b) Displacement; (c) Moment

Table 1. The data of the beam-soil system used in the study

Beam properties			Beam thickness	E_s
L (m)	B (m)	L/B	(m)	(kN/m^2)
5	0.8	6.25	0.3	5000
10	1.0	10.0	0.5	10000
15	1.2	12.5	0.7	15000
20	1.35	14.8	0.9	20000
				30000
				40000

Non-dimensional terms used in the analysis are given as follows:

$$K_r = \frac{16(1-\nu_s)^2 EI}{\pi E_s L^4} \tag{12}$$

$$K_w = \frac{kBL^4}{EI} \tag{13}$$

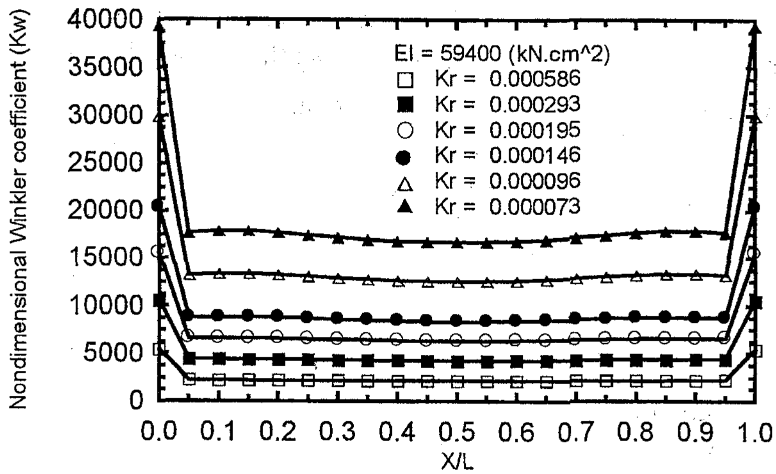
where

K_r = relative stiffness as defined by Poulos and Davis (1974)
E_s = modulus of elasticity of the soil
E = modulus of elasticity of the beam material
I = moment of inertia of the beam
L = beam length
B = beam width
K_w = non-dimensional coefficient of subgrade reaction
k = coefficient of subgrade reaction (F/L^3)

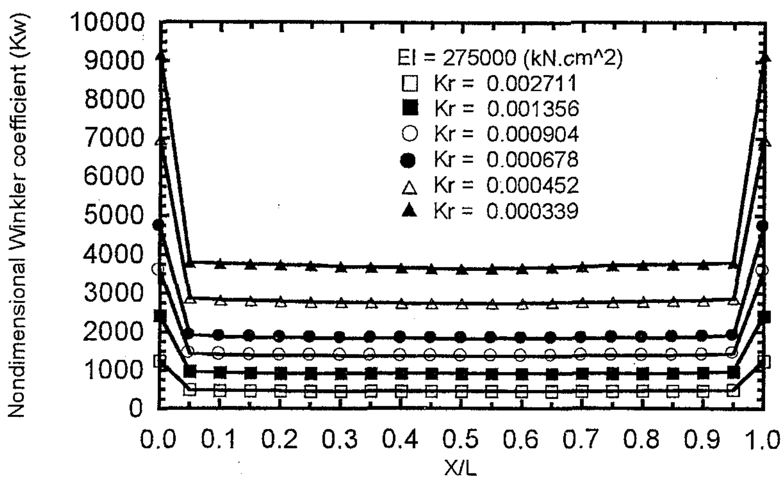
Samples from the results are presented in Fig. (3). Fig. (3) shows the variation of the non-dimensional coefficient of subgrade reaction, K_w, along the beam length at different values of relative stiffness, K_r, and beam rigidity, EI. Referring to Fig. (3) it is observed that:

1. The distribution of the coefficient of subgrade reaction is non-uniform along the beam length. However, over most of the middle beam length (approximately 90% of the beam length) the rate of change of K_w is small.
2. In all studied cases, the ratio between the value of K_w at the edge of the beam to the value of K_w at the center of the beam is not constant and grater than 2. This ratio increases as the beam rigidity increases and decreases as the relative stiffness decreases or as the modulus of elasticity of the soil increases.

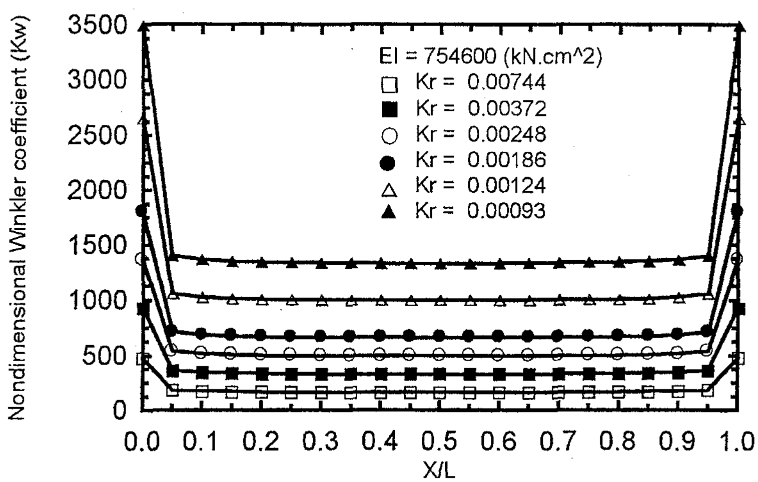
The non-dimensional coefficient of subgrade reaction, K_w, at different positions along the beam length and the relative stiffness, K_r, obtained in the



(a)



(b)



(c)

Fig. (3). Variation of Non-dimensional Coefficient of Subgrade Reaction along the Beam Length at Different Beam Rigidity and Relative Stiffness above analysis is presented in a graphical form using logarithmic scale in both axes as shown in Fig. (4). Referring to Fig. (4) the relationship between K_w and K_r form a straight line for different values of L/B ratio at different positions along the beam length. The following equation is developed to represent the relationship between K_w and K_r:

$$K_w = C_1(K_r)^{-C_2} \tag{14}$$

Values of the constants C_1 and C_2 of Eq. (14) at different positions along the beam length depend on the value of L/B ratio and can be determined from the following regression equations:

At the edge of the beam:

$$C_1 = 12.089 - 1.04154\left(\frac{L}{B}\right) + 0.0337841\left(\frac{L}{B}\right)^2 \tag{15}$$

$$C_2 = 1.02187 - 0.00501044\left(\frac{L}{B}\right) - 0.000014387\left(\frac{L}{B}\right)^2 \tag{16}$$

At 0.05L:

$$C_1 = 5.52562 - 0.479999\left(\frac{L}{B}\right) + 0.0122506\left(\frac{L}{B}\right)^2 \tag{17}$$

$$C_2 = 1.03551 - 0.00960827\left(\frac{L}{B}\right) + 0.000453882\left(\frac{L}{B}\right)^2 \tag{18}$$

At 0.3L:

$$C_1 = 5.47845 - 0.564369\left(\frac{L}{B}\right) + 0.017558\left(\frac{L}{B}\right)^2 \tag{19}$$

$$C_2 = 0.985124 + 0.00401982\left(\frac{L}{B}\right) - 0.000181195\left(\frac{L}{B}\right)^2 \tag{20}$$

At 0.5L (at the center of the beam):

$$C_1 = 5.38563 - 0.557404\left(\frac{L}{B}\right) + 0.0176956\left(\frac{L}{B}\right)^2 \tag{21}$$

$$C_2 = 0.987278 + 0.00451304\left(\frac{L}{B}\right) - 0.000256051\left(\frac{L}{B}\right)^2 \tag{22}$$

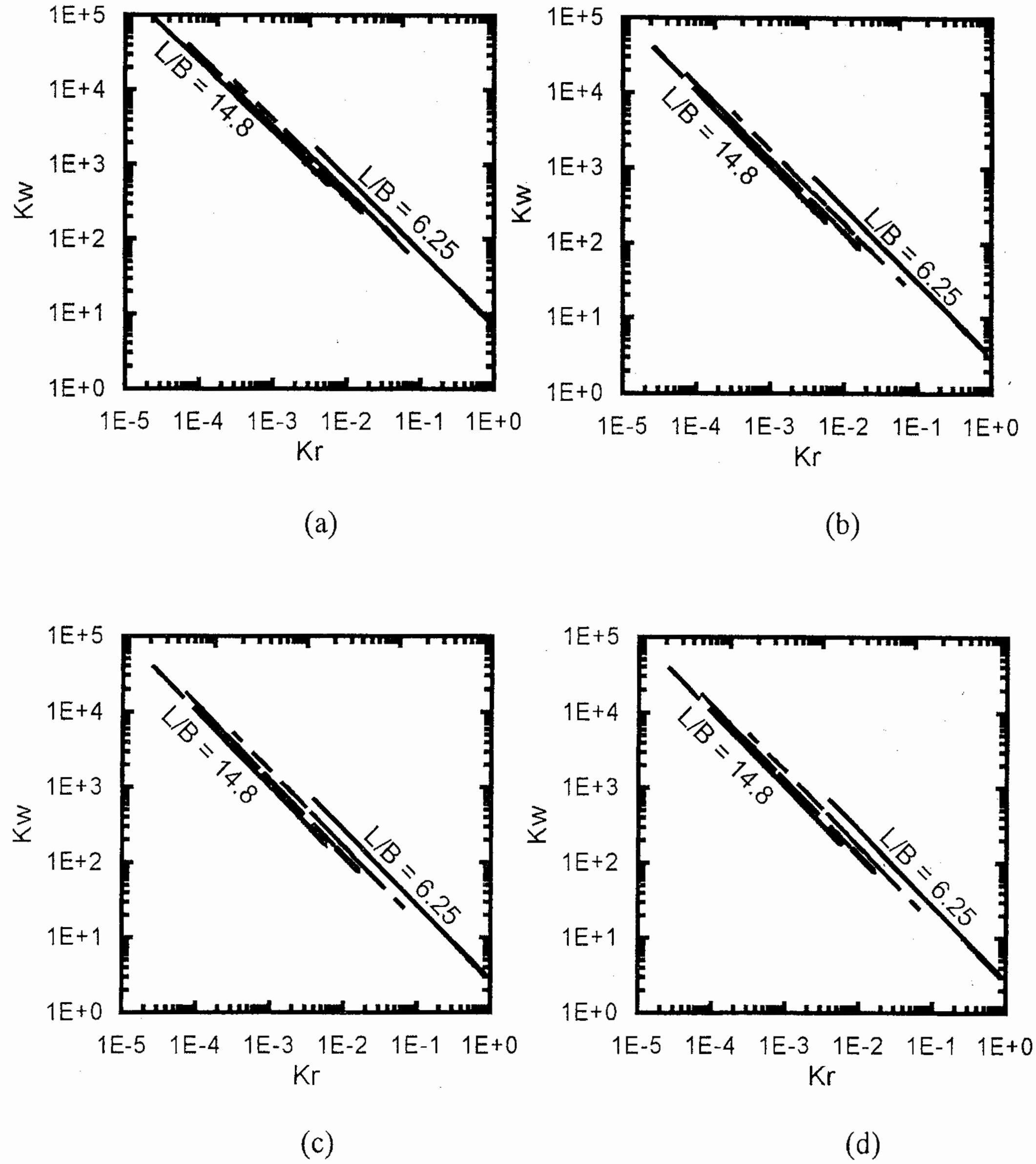


Fig. (4). Non-dimensional Coefficient of Subgrade Reaction as a Function of Relative Stiffness and L/B Ratio along the Beam Length: (a) At the Edge; (b) At 0.05 L; (c) At 0.3 L; At the Center

The values of the non-dimensional coefficient of subgrade reaction K_w at the remaining positions along the beam length can be determined by interpolation.

4.1. Numerical Examples to Check Eq. (14)

Using the above equations, the equivalent k values along the beam length are calculated and presented as a comparison with the values of k from the program WCBECM for two examples with different L/B ratio as shown in Figs. (5a, 6a). The first example is a beam subjected to uniformly distributed load of $100 kN/m^2$ and resting on elastic half space. The beam is of 10-m length, 1.25-m width, and 0.5-m thickness. The modulus of elasticity of the beam is $2.11 \times 10^7 kN/m^2$. The modulus of elasticity and Poisson's ratio of the subgrade are $1.2 \times 10^4 kN/m^2$ and 0.3 respectively. The second example is a beam subjected to uniformly distributed load of $100 kN/m^2$ and resting on elastic half space. The beam is of 20-m length, 1.0-m width, and 0.5-m thickness. The modulus of elasticity of the beam is $2.11 \times 10^7 kN/m^2$. The modulus of elasticity and Poisson's ratio of the subgrade are $1.75 \times 10^4 kN/m^2$ and 0.3 respectively.

The displacements and the bending moments along the beam length for the two examples are presented in Figs. (5b, 5c) and Figs. (6b, 6c). Refering to these figures it is observed that the developed equation, Eq. (14), gives values of k which match with the values of k from the program WCBECM along the beam length. The maximum error in the values of k is found to be in the range of 0.7% to 19%. The maximum error in the maximum displacements and bending moments due to this error found out to be in the range of 9.5% to 18%.

5. CONCLUSIONS

A simple rational method capable of providing improved estimates of the actual distribution of the coefficient of subgrade reaction to be used in the analysis of soil-foundation interaction is introduced. The following conclusions is reached from this study:

1. The coefficient of subgrade reaction is not a constant value but it is varying along the beam length. The variation of the coefficient of subgrade reaction is manly dependent on the elastic properties of the supporting soil and the beam rigidity in addition to the applied load pattern.
2. The results presented illustrate the significant shortcomings of different methods that are still widely used to calculate the constant k value.
3. For uniformly loaded beam, the ratio between the coefficient of subgrade reaction at the edge and the center of the beam is not constant and depends on the elastic properties of the supporting soil and the beam rigidity.
4. For uniformly loaded beam, graphs and regression equations are provided for non-dimensional coefficient of subgrade reaction as a function of relative stiffness and L/B ratio of the beam from which equivalent k values can be calculated.

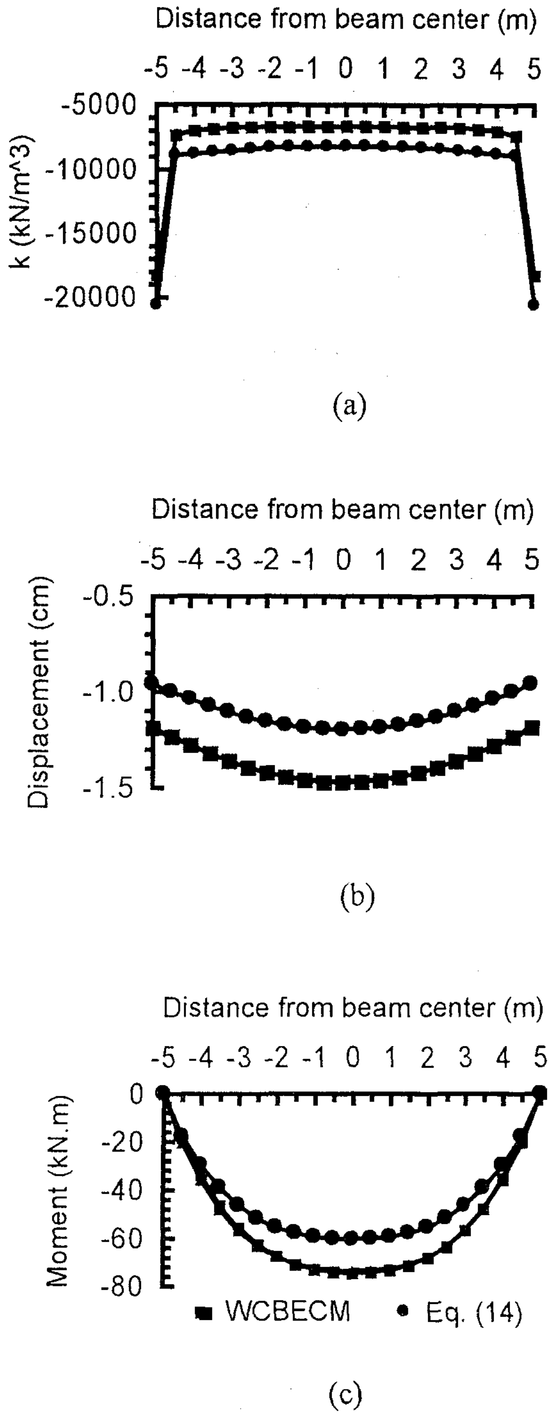


Fig. (5). Comparison of Results for Beam (1.25-m X 10-m) Subjected to Uniformly Distributed Load:(a) Equivalent *k* Values; (b) Displacement; (c) Moment

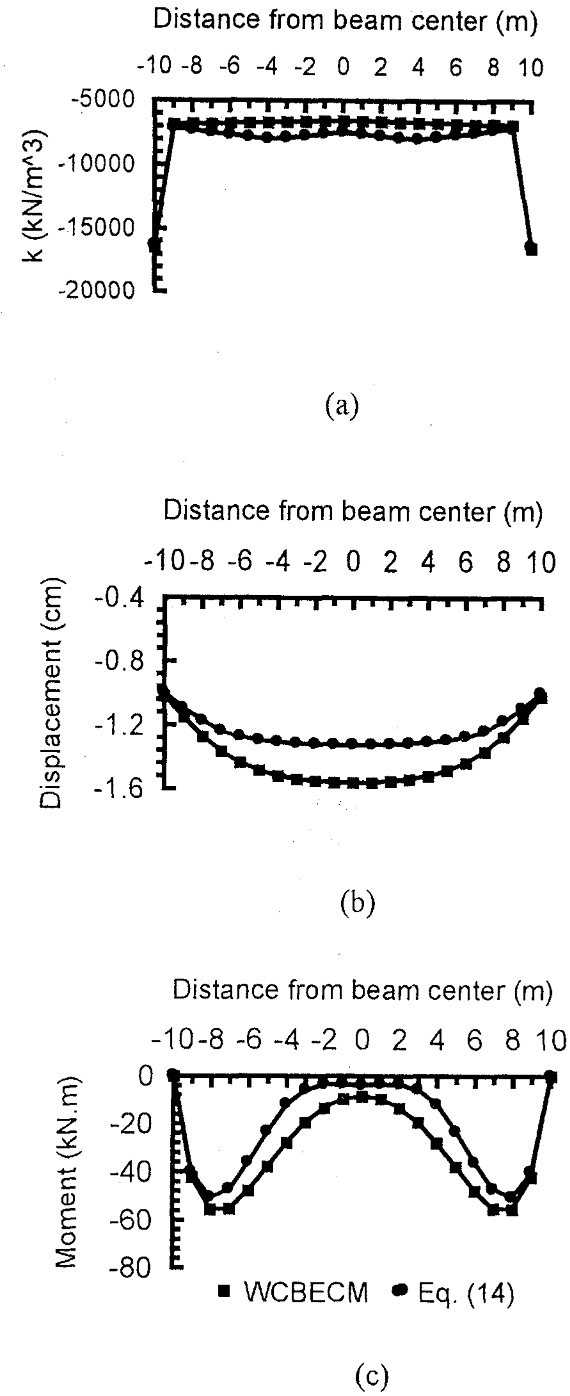


Fig. (6). Comparison of Results for Beam (1-m X 20-m) Subjected to Uniformly Distributed Load:(a) Equivalent *k* Values; (b) Displacement; (c) Moment

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معامل ونكلر للكمر المرتكز على التربة المرنة

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١ . مدرس _ قسم مدني _ كلية الهندسة بشبين الكوم _ جامعة المنوفية

٢ . مدرس _ قسم مدني _ كلية الهندسة _ جامعة طنطا

باستخدام النموذج المرن لتحليل الأساس الشريطي المرتكز على التربة المرنة تم اقتراح طريقة لحساب معامل رد فعل التربة يستخدم في نموذج ونكلر. اتضح أنة بقسمة ضغط التماس الناتج من النموذج المرن على الهبوط المناظر عند النقط المختلفة على طول القاعدة الشريطية ينتج عنة معامل رد فعل للتربة يستخدم مع نموذج ونكلر لحساب رد الفعل المتبادل بين الأساس و التربة بدقة عالية. تم مقارنة النتائج باستخدام قيم معامل رد فعل التربة المقترح و نموذج ونكلر مع نتائج النموذج المرن وكذلك مع النتائج باستخدام طرق أخرى لحساب معامل رد فعل التربة والشائعة الاستخدام مع نموذج ونكلر. بالنسبة للقاعدة الشريطية المعرضة لحمل منتظم تم اقتراح مخطط بياني ومعادلات تقريبية منهما يمكن حساب معامل رد فعل التربة عند النقط المختلفة على طول القاعدة الشريطية (مثل حافة و مركز القاعدة) كدالة في الخواص المرنة للتربة و جساءة القاعدة الشريطية.