THE EFFECT OF ANNEALING ON THE STRENGTH OF GLASS AND THE CONDITIONS OF CRACKING

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ABSTRACT

The scope of this paper is to investigate the effect of annealing operations of glass on its strength and cracking conditions. This annealing operation reduces the residual stresses and strains, caused by uneven cooling conditions during manufacturing.

Firstly, the equations of fracture mechanics of ordinary glass were demonstrated ; after that the annealing factor in the equations, was introduced.

The importance of this research comes from the fact that improving the strength of glass will be reflected immediately in vast applications, specially in sheet glass industry.

It was found , when the residual stresses were eliminated,that the crack extension force increased by 68% mor than the ordinary glass.

KEYWORDS

Glass annealing, Fracture mechanies, theory of elasticity, elastic fracture, brittle fracture, cracking conditions.

NOMENCLATURE

E	Modulus of elasticity
G	Crack extension force.
G_t	Crack extension force of annealed glass.
K_I	Stress intensity factor.
K_t	Stress intensity factor in annealed glass.
r, θ	Polar coordinates.
x,y,z	Rectangular coordinates.
u,v,w	Components of displacements.
α	distance.
σ_i	Internal stress in glass.
$\sigma_x, \sigma_y, \sigma_z$	Normal components of the stress parallel to x,y, and z axes, respectively.
ϵ	Unit elongation.
$\epsilon_x, \epsilon_y, \epsilon_z$	Unit elongation in X,y and z directions, respectively.
μ	modulus of rigidity.
ν	Poisson's ratio.

INTRODUCTION

Glass is a very important engineering material, and every body can visualize its great applications everywhere. Indead, glass has useful properties such as cheapness, easy manufacturing, and many favorable physical properties such as transparency, insulation, hardness, ... ect.

In the last decade, much more interest was paid to the theoretical analysis and study in the field of fracture mechanics and glass technology.

The strength of glass seems to be the major handicaping property, and it is also affected to a great extent by the thermal and surface conditions of glass. This is the main problem that will be discussed in this paper. Afterthe analysis of the glass fracture mechanics and its cracking condition, the annealing factor was introduced to investigate its effect on the strength and cracking.

PART (I)

THE ELASTIC CRACK TIP STRESS FIELD

Using Westergaard's complex stress function, Latzko*1 demonstrated that the elastic stress field near the tip of the crack in a sheet of brittle material, could be described by:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad \text{(1-a)}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \quad \text{(1-b)}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2} \quad \text{(1-c)}$$

Where the radius, r, and the angle, θ, are defined as in Fig (1)*1.

The corresponding displacements for plane strain ($\epsilon_z = 0$) become*1:

$$u = \frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}} \cos\frac{\theta}{2}\left(1 - 2\nu + \sin^2\frac{\theta}{2}\right) \quad \text{(2-a)}$$

$$v = \frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}} \sin\frac{\theta}{2}\left(2 - 2\nu - \cos^2\frac{\theta}{2}\right) \quad \text{(2-b)}$$

$$w = 0 \quad \text{(2-c)}$$

By using $\theta = 0$ in equation (1-b), and $\theta = \pi$ in equation (2-b), and $r = (\alpha - x)$*2:

$$\therefore \sigma_y = \frac{K_I}{\sqrt{2\pi x}} \quad \text{and} \quad v = \frac{4(1-\nu^2)}{E} K_I \sqrt{\frac{\alpha - x}{2\pi}}$$

$$\text{But } G = -\lim \frac{1}{2\alpha} \int_0^{\alpha} \sigma_y \cdot 2v \cdot dx \quad \text{(3-a)}$$

$$\therefore G = (1 - \nu^2)\frac{K_1^2}{E} \quad \text{(3-b)}$$

PART (11)

THE EFFECT OF ANNEALING ON CRACK EXTENSION FORCE IN GLASS

In order to find the effect of annealing on crack extension force in glass, an attempt will be made to adapt the equations of the crack extension force in brittle materials.

Taking the annealing effect into account, the coordinate system at the crack tip may be as shown in Fig. (3). Subsequently, the equations of stress and displacements become:

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \frac{k_t}{\sqrt{2\pi r}} \cos\frac{\theta_1}{2}\left(1-\sin\frac{\theta_1}{2}\sin\frac{3\theta_1}{2}\right) \quad \text{... (a-1)}$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) + \frac{k_t}{\sqrt{2\pi r}} \cos\frac{\theta_1}{2}\left(1+\sin\frac{\theta_1}{2}\sin\frac{3\theta_1}{2}\right) \quad \text{... (a-2)}$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \left(\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right) + \frac{K_t}{\sqrt{2\pi r}} \left(\sin\frac{\theta_1}{2}\cos\frac{\theta_1}{2}\cos\frac{3\theta_1}{2}\right) \quad \text{......... (a-3)}$$

$$u = \frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}} \cos\frac{\theta}{2}\left(1-2\nu+\sin^2\frac{\theta}{2}\right) + \frac{K_t}{\mu}\sqrt{\frac{r}{2\pi}} \cos\frac{\theta_1}{2}\left(1-2\nu+\sin^2\frac{\theta_1}{2}\right) \quad \text{...... (b-1)}$$

$$v = \frac{K_I}{\mu}\sqrt{\frac{r}{2\pi}} \sin\frac{\theta}{2}\left(2-2\nu-\cos^2\frac{\theta}{2}\right) + \frac{K_t}{\mu}\sqrt{\frac{r}{2\pi}} \sin\frac{\theta_1}{2}\left(2-2\nu-\cos^2\frac{\theta_1}{2}\right) \quad \text{..... (b-2)}$$

$$w = 0 \quad \text{.. (b-3)}$$

To obtain the value of θ_1 at the minimum internal stress :

$$\sigma_i = \frac{K_t}{\sqrt{2\pi r}} \cos\frac{\theta_1}{2}\left(1+\sin\frac{\theta_1}{2}\sin\frac{3\theta_1}{2}\right)$$

$$\frac{d\sigma_i}{d\theta_1} = \frac{K_t}{\sqrt{2\pi r}}\left(3\sin\frac{\theta_1}{2}\cos\frac{3\theta_1}{2}+\sin\frac{3\theta_1}{2}\cos\frac{\theta_1}{2}\right)$$

$$+\left(1+\sin\frac{\theta_1}{2}\sin\frac{3\theta_1}{2}\right)\left(\frac{-K_t}{2\sqrt{2\pi r}}\sin\frac{\theta_1}{2}\right)$$

$$= \frac{K_t}{2\sqrt{2\pi r}}\left[\left(3\sin\frac{\theta_1}{2}\cos\frac{3\theta_1}{2}+\sin\frac{3\theta_1}{2}\cos\frac{\theta_1}{2}\right)\right.$$

$$\left.-\sin\frac{\theta_1}{2}\left(1+\sin\frac{\theta_1}{2}\sin\frac{3\theta_1}{2}\right)\right]$$

When

$$\frac{d\sigma_i}{d\theta_1} = 0$$

$$\therefore \frac{K_t}{2\sqrt{2\pi r}}\left[\left(3\sin\frac{\theta_1}{2}\cos\frac{3\theta_1}{2}+\sin\frac{3\theta_1}{2}\cos\frac{\theta_1}{2}\right)-\sin\frac{\theta_1}{2}\left(1+\sin\frac{\theta_1}{2}\sin\frac{3\theta_1}{2}\right)\right]=0$$

$$f' = 3\sin\frac{\theta_1}{2}\cos\frac{3\theta_1}{2}+\sin\frac{3\theta_1}{2}\cos\frac{\theta_1}{2}-\sin\frac{\theta_1}{2}\left(1+\sin\frac{\theta_1}{2}\sin\frac{3\theta_1}{2}\right)=0$$

By drawing the function, Fig (4)it was found that $\theta_1 = 62^0$.

By using $\theta = 0$ and $\theta_1 = 62^0$ in equation (a-2) :

$$\therefore \sigma_y = \frac{K_I}{\sqrt{2\pi x}} + 1,3\frac{k_t}{\sqrt{2\pi x}}$$

E.B. Shand[*3] reported that the increase of strength as a result of annealing = 0,4 experimentally.

So it may be $0 < K_t \leqslant 0,4\,K_I$

$$\therefore \sigma_y = \frac{K_I}{\sqrt{2\pi x}}(1+0,52) = 1,52\frac{K_I}{\sqrt{2\pi x}} \qquad \dots\dots\dots\dots\dots\dots \quad (c)$$

By using $\theta = 180^0$ and $\theta_1 = 62^0$ in equation (b-2)

$$\therefore v = \frac{4(1-\nu^2)}{E}K_I\sqrt{\frac{\alpha-x}{2\pi}}+\frac{2(1+\nu)}{E}K_t\sqrt{\frac{\alpha-x}{2\pi}}\sin\frac{\theta_1}{2}\left(2-2\nu-\cos^2\frac{\theta_1}{2}\right)$$

$$= \frac{4(1-\nu^2)}{E}K_I\sqrt{\frac{\alpha-x}{2\pi}}+\frac{4(1-\nu^2)}{E}K_t\sqrt{\frac{\alpha-x}{2\pi}}\cdot\frac{0,2(2-2\nu-0,735)}{2(1-\nu)}$$

Poisson's ratio can be taken as 0.23 for calculations[*4]

$$\therefore v = 1,105\frac{4(1-\nu^2)}{E}K_I\sqrt{\frac{\alpha-x}{2\pi}}$$

By using equation (3-a):

$$\therefore G_t = 1,68\frac{(1-\nu^2)}{E}K_I^2$$

But $G = \frac{1-\nu^2}{E}K_I^2$ $\qquad \therefore G_t = 1,68\ G$

CONCLUSION

The theoretical result shows that the increase of the crack extension force in glass, at zero residual stress, may be 68 % ; comparing with the ordinary glass.

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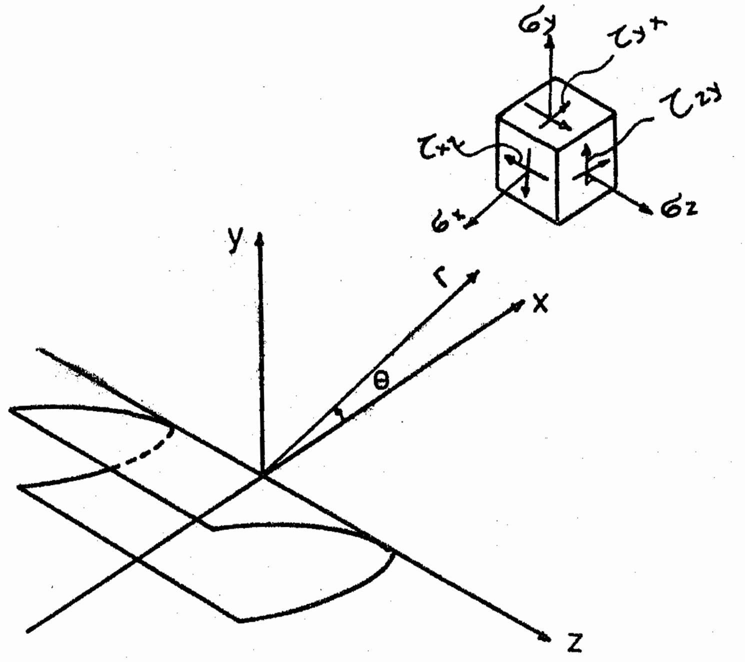
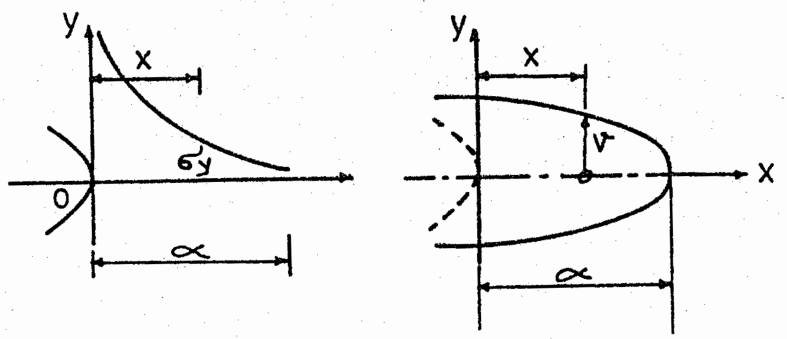


Fig.(1): Coordinate system and stress components at the crack tip.[*1]

Fig.(2): Schematic representation of the tip of a crack.[*2]



before it advances a distance α

after advancing

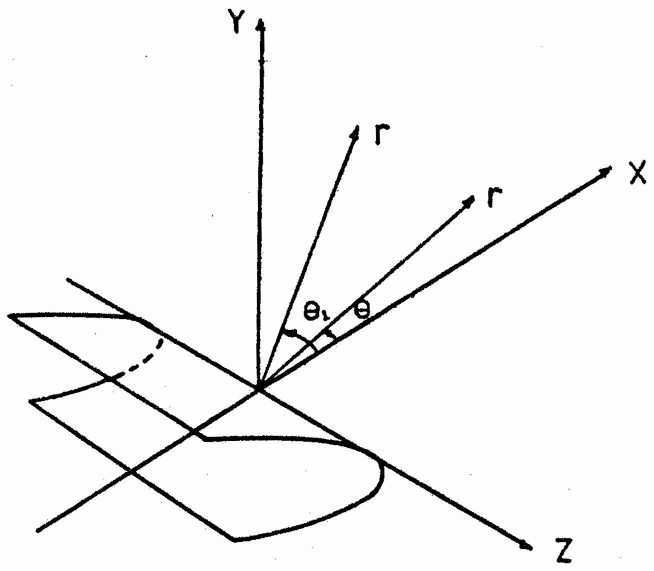
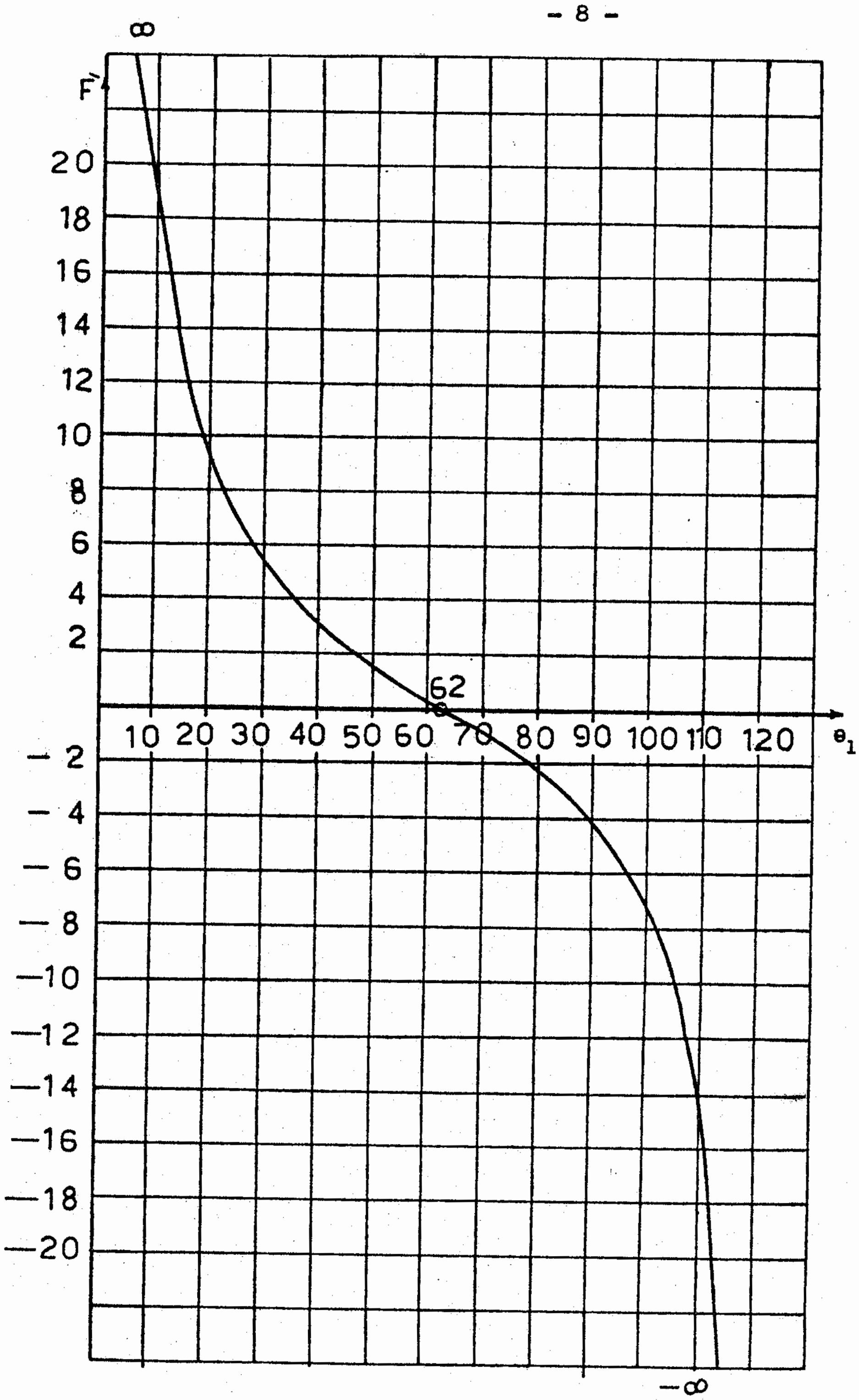


Fig. (3): Coordinate system at the crack tip after annealing.



Fig(4): Assessment of θ_1 value.