Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Science Dep. Post Graduate Examination, 2015-2016 Date of Exam : 30/ 05 / 2018	Minoufia University	Subject: Introduction to Ordinary Differential Equations Code: BES 506 Time Allowed : 3hrs Total Marks: 100 Marks

Answer all the following questions الامتحان في صفحتان

Question 1 [25 Marks]

(A) Find the general solution of the following first order first degree ordinary differential equations:

(1) $\left(x^3 y\right)\frac{dy}{dx} = (1+x)\sec 3y$

(2) $\left(y\, e^{\frac{y}{x}} + x\right) dx - x\, e^{\frac{y}{x}} = 0$

(3) $x\frac{dy}{dx} + 3\,y = \frac{\sin 2x}{x}$

(B) Find the general solution of the first order first degree ordinary differential equation:

$$x\,\frac{dy}{dx} - y^2 \ln x + y = 0$$

(C) Find the general solution of the first order first degree ordinary differential equation:

$$(x + y^2 \sin x - y^3)\, dx = (3\,x\,y^2 + 2\,y\cos x)\, dy$$

Question 2 [25 Marks]

(A) Explain all cases of the integrating factor to reduce the first order first degree ordinary differential equation to an exact equation. Solve this equation as an example

$$(y + x\,y^2)dx - x\,dy = 0$$

(B) Find the general solution of the first order but not of first degree ordinary differential equations:

(1) $\left(\frac{dy}{dx}\right)^2 + 2\,y\cot x\frac{dy}{dx} = y^2$

(2) $\left(\frac{dy}{dx}\right)^2 - 2x\frac{dy}{dx} + y = 0$

(C) Find the general solution of the second order first degree ordinary differential equations:

(1) $x^2\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = 0$

(2) $y\frac{d^2y}{dx^2}+1=\left(\frac{dy}{dx}\right)^2$

Question 3 [25 Marks]

(A) Prove that if $y_1=\cos 4x, y_2=\sin 4x$, and $y_3=e^{3x}$ are linearly independent functions. Discus completely all the difference between the general solution and particular solution of an ordinary differential equation.Find the homogeneous differential equation which the complement solution is :

$y_c=c_1y_1+c_2y_2+c_3y_3$ *where* $c_1, c_2,$ *and* c_3 *are constants* .

(B) Find the general solution of the non-homogenous system of differential equations:

$$\frac{d^2x}{dt^2}-y=e^{2t} \quad \text{and} \quad \frac{dy}{dt}-x=sin(2t)$$

(C) Find the total solution of the following non-homogenous differential equation by the undetermined coefficients method.

$$\frac{d^4y}{dx^4}-y=4\sinh 3x+x^2$$

Question 4 [25 Marks]

(A) Find the total solution of the following non-homogenous differential equation by the undetermined coefficients method.

$$\frac{d^2y}{dx^2}+16y=sin(4x)$$

(B) Find the total solution of the following non-homogenous differential equation by the undetermined coefficients method.

$$[(D)(D-1)(D-2)]\,x=t^3+e^{2x}\ ,\ D=\frac{d}{dt}$$

(C) Determine the power series solution of $\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+y=0$, using the Leibniz-Maclaurin method

given the boundary conditions that at $x=0$, $y=0$ and $\frac{dy}{dx}=1$.

With my best wishes *Dr. Mohamady Bassioni*