Eng. Research Bull
Faculty of Eng. & Tech.
Menoufia University
Vol. VI Part 1,1984

pp. 141 - 157

A DIGITAL CONTROL SYSTEM FOR A SYNCHRONOUS GENERATING UNIT.

*S. M. OSHEBA** *A. M. SARHAN*** *M. A. ABO-SINA***

ABSTRACT

This paper presents a discrete proportional plus integral control algorithm and procedures for designing such controllers. The procedures have been applied and a controller is designed for a synchronous generating unit, connected to a large power system. Simulation results are presented which illustrate that the controller improves considerably system performance and is very effective in damping power system oscillations.

1. *INTRODUCTION*

Traditionally, power system controllers are designed using the well-established single-input/ single-output techniques. This, however, does not take account of interaction between various control loops in the system which are now becoming very pronounced because of the increasing size of interconnections.The increasing complexity of power systems have also renders power system engineers to search for new methods of operation and control.Modern control theory offers a good alternative as it takes account of interaction between various control loops and also the resulting controllers could be implemented by using on-line computers(1-7).

* *Dept. of Electrical Power and Machines.*

** *Dept. of Engineering Mathematics.*

Faculty of Eng. & Tech. Menoufia University, Shebin El-Kom, EGYPT.

To design a discrete optimal controller, a linear model of the system should be obtained in state-space form[3]. A cost function is then defined which when minimized as a multi-stage decision process and subject to the system model, leads to a constant feedback matrix. One basic problem may be faced when implementing such controllers is to be minimizing the offset in some state-variables when the system is subjected to unknown type of disturbance. To overcome this problem, a proportional plus integral control algorithm is introduced in this paper. The integral action is employed on some state-variables only which need to be controlled predisely in steady-state. A discrete-proportional plus integral control algorithm for a turbogenerator system is designed. Simulation results are presented to show the effectiveness of the proposed control algorithm and design technique.

2. CONTROL ALGORITHM

In a standard form, a linear time-invariant state-space model can be written as follows :

$$X = A\,X + B\,U \tag{1}$$

where X is nx1 state-vector, U is mx1 input vector, A is nxn system matrix, B is nxm input matrix, n and m are the system order and the number of inputs respectively.

The solution of Eqn.(1), yields

$$X(t) = e^{A(t-t_o)} \,.\, X(t_o) + \int_{t_o}^{t} e^{A(t-\tau)} \,.\, B.U(\tau).d\tau \tag{2}$$

In computer control systems, which employ digital techniques, the input is constant over intervals of T_s seconds. Therefore,

$$U(\tau) = U(KT_s), \quad KT_s \leq T_s \leq (K+1)\; T_s \tag{3}$$

For $t_o = KT_s$ and $t = (k+1)T_s$, Eqn.(2) gives the following standard discrete-time model.

$$X_{k+1} = \phi X_k + \Delta U_k \tag{4}$$

where ϕ is the state-transition matrix and Δ is the deriving input matrix.

Let Z_k be a vector contains the showen variables for integral control, where Z_k is related to X_k as follows :

$$Z_{k+1} = C X_k \tag{5}$$

where C is mxn matrix contains the necessary elements from a unit matrix. Eqn.(4) can then be augmented by Eqn.(5) to give the following augmented model :

$$\begin{bmatrix} X_{k+1} \\ Z_{k+1} \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} X_k \\ Z_k \end{bmatrix} + \begin{bmatrix} \Delta \\ 0 \end{bmatrix} U_k \tag{6}$$

in a compact form :

$$\hat{X}_{k+1} = \hat{\phi} \hat{X}_k + \hat{\Delta} U_k \tag{7}$$

using Eqn.(7), a digital controller can be developed by minimizing the quadratic performance index J :

$$J = \sum_{k=0}^{k} (X^t_{k+1} Q.X_{k+1} + U^t R U_k) \tag{8}$$

where Q and R are nxn and mxm weighting matrices. The minimization technique would lead to the following control law :

$$U_k = F . \hat{X}_k \tag{9}$$

partitioning the feedback matrix, F , yields

$$U_k = \begin{bmatrix} F_1 & F_2 \end{bmatrix} \begin{bmatrix} X_k \\ Z_k \end{bmatrix} \tag{10}$$

$$= F_1 X_k + F_2 Z_k \tag{11}$$

$$U_k = F_1 X_k + F_2 \sum_{k=1}^{k} TC X_k \qquad (12)$$

Eqn.(2) illustrates a proportional plus integral control law with integral control on selected variables which can be determined by elements in the C matrix.

3. *DESIGN TECHNIQUE*

The design of the control system developed in this paper is based on adapting the eigengalues shift policy in Reference (7) to become suitable for this case. The eigenvalues shift is assumed to give its main effects on the movements of the real part of the eigenvalues towards the origin of the unit circle in the Z-plane. Let :

Q : is nxn weighting matrix contains the initial weighting elements.

λ_i : is the initial value of the <u>ith</u> eigenvalue.

$\Delta\lambda_i$: is the increment change in the <u>ith</u> eigenvalue.

Δ_q : is the change in the diagonal elements of the weighting matrix Q between two successive iterations.

The increment change for an eigenvalue which results from changes in the diagonal elements of the weighting matrix Q is given by[7] :

$$\Delta\lambda_i = \lambda^t_{i,q} \cdot \Delta q \qquad (13)$$

where $\lambda^t_{i,q} = \dfrac{\partial \lambda_i}{\partial q}$ = Sensitivity coefficients

Assuming the general case in which λ_i is complex conjugate. Then, separating Eqn. (13) into real and imaginary parts gives :

$$\Delta\lambda_i = \Delta E_i + J \cdot \Delta H_i \qquad (14)$$

Considering only the movements in the real parts, then :

$$\Delta E = S . \Delta q \qquad (15)$$

where ΔE = vector contains real parts of dominant eigenvalues,

$$S = \text{Real} (\lambda^t , q).$$

Now with the knowledge of S,how q can be determined to achieve specefic shift?.In order to determine Δq, let :

ΔE_{Tw} = Sum of weighted real shift for dominant m eigenvalues.

$$= \sum_{i=1}^{m} B_i \Delta E_i$$

Then from Eqn.(15), one may have

$$\Delta E_{Tw} = \psi^t . \Delta q \qquad (16)$$

where,

$$\psi^i = B_i S(1,i) + B_2 . S(2,i) + \ldots + B_m S(m,i)$$

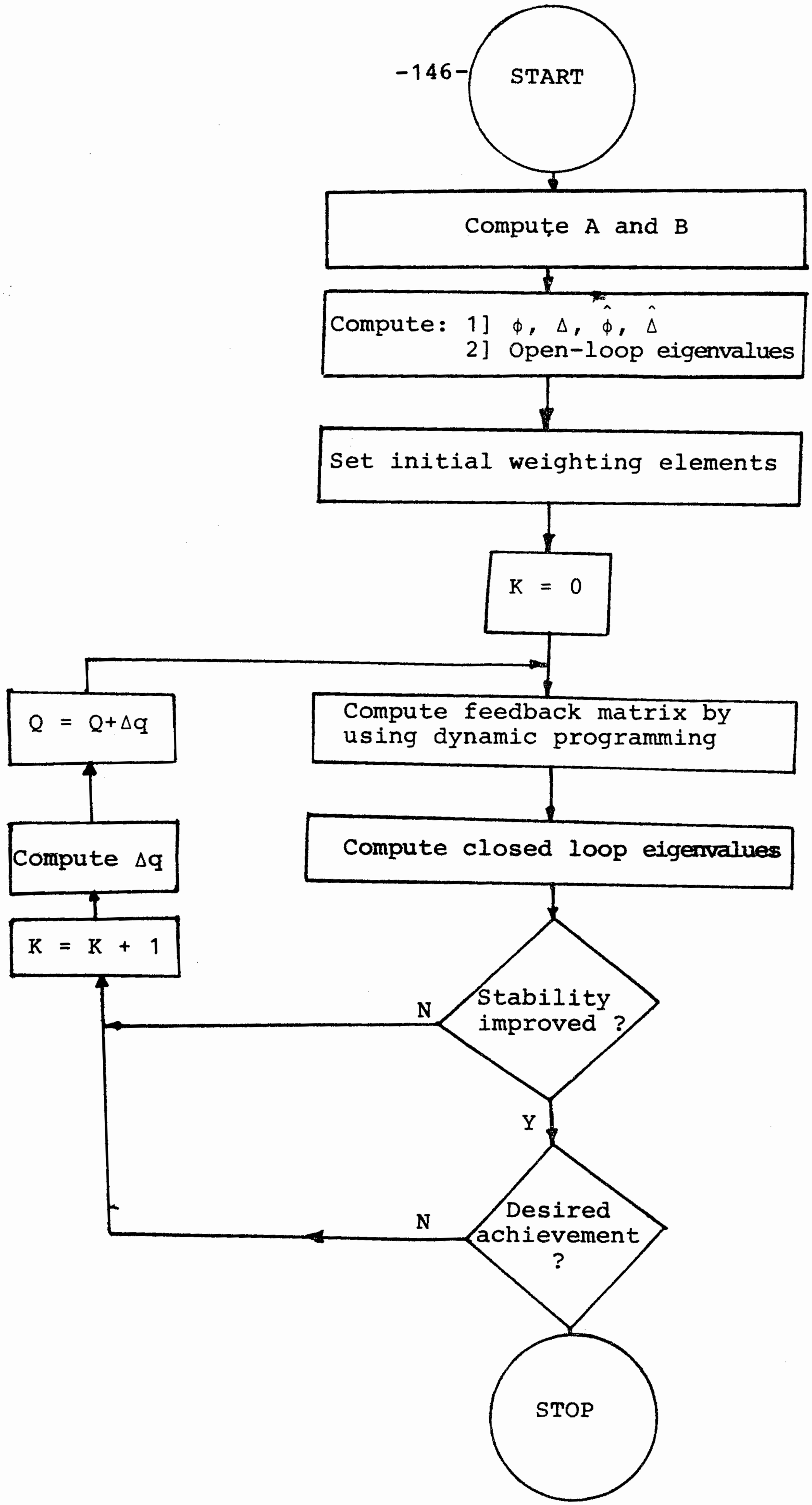
where, B's are positive weighting elements to distingush between the relative shifted eigenvalues. Since linear relationship is valid between the real shift and the increment change in Δq.[7], then

$$\Delta q = K \psi \quad , \quad K \epsilon E \qquad (17)$$

The controller may then be designed according to the algorithm explained by the flow chart shown in Fig.1.

4. STATE-SPACE MODELLING

The system considered in this study is a two-pole synchronous generating unit. It consists of a synchronous generator connected to a large power system via transformer and a douple circuit transmission line.The generator is deriven by a steam turbine and is excited


START
Compute A and B
Compute: 1] ϕ, Δ, ϕ̂, Δ̂
2] Open-loop eigenvalues
Set initial weighting elements
K = 0
Compute feedback matrix by using dynamic programming
Compute closed loop eigenvalues
Stability improved ?
N
Y
Desired achievement ?
N
K = K + 1
Compute Δq
Q = Q+Δq
STOP

via a thyristor exciter fast valving and fast acting excitation system are assumed. The system is shown in Fig.2 and is modelled based on park's non-linear equations[3].

The system non-linear equations are perturbed about an operating point and arranged into a set of state-variables, where $P\,\psi_d$ and $P\,\psi_q$ are neglected and the constants appear in the appendix(II).

$$\Delta X^t = (\Delta\delta \;,\; P\,\Delta\delta,\; \Delta\psi f_d \;,\; \Delta\, G_v \;,\; \Delta\, T_M) \qquad (18)$$

$$\Delta U^t = (\Delta\, U_e \;,\; \Delta\, U_g) \qquad (19)$$

$$\Delta X = A\,\Delta X + B\,.\,\Delta U \qquad (20)$$

where,

$$A = \begin{matrix} 0 & 1 & 0 & 0 & 0 \\ K_g & \frac{-K_d.W_o}{2H} & K_{10} & 0 & \frac{W_o}{2H} \\ K_{14} & 0 & K_{13} & 0 & 0 \\ 0 & 0 & 0 & K_{12} & 0 \\ 0 & 0 & 0 & K_{23} & K_{22} \end{matrix} \quad ; \quad B = \begin{matrix} 0 & 0 \\ 0 & 0 \\ K_{19} & 0 \\ 0 & K_{24} \\ 0 & 0 \end{matrix}$$

The constants appear in the appendix (II).

5. *SIMULATION RESULTS*

The system shown in Fig.2. is operated at an operating point,defined by an active power P = 0.80 P.U and reactive power Q = 0.60 P.U and the controller is designed using the algorithm described in sections 3 and 4, as follows :

Weighting elements

Q_{diag} = Starting weighting elements

$$\begin{matrix} & \delta & P\delta & \psi fd & G_v & T_M \\ = (& 1000, & 1000, & 1000, & 1000, & 1000) \end{matrix}$$

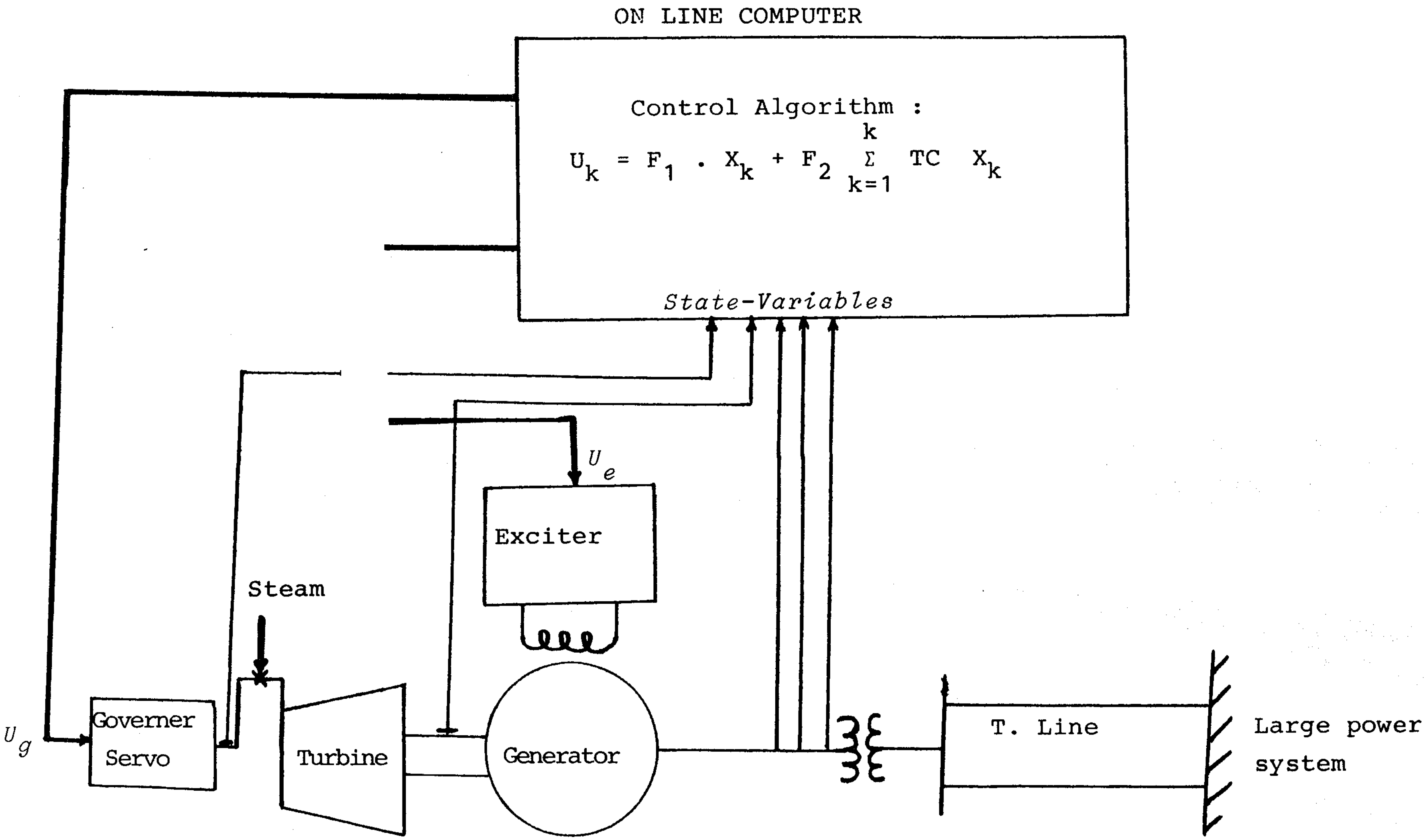


Fig.2. Schematic Diagram of a Digital Control System

$$R_{diag} = (\overset{U_e}{1}, \overset{U_g}{10})$$

Δ_q diag (After one iteration) =

(-6.687 , 6.073 , 0.009 , 0.003 , 0.044).

Eigenvalues

Open loop eigenvalues : $\lambda_{1,2} = 0.978 \pm J\ 0.147$
$\lambda_3 = 0.988$
$\lambda_4 = 0.818$
$\lambda_5 = 0.966$

Closed-loop eigenvalues:
$\lambda_{1,2} = 0.667 \pm J\ 0.225$
$\lambda_3 = 0.17$
$\lambda_4 = 0.935$
$\lambda_5 = 0.972$

Feedback matrix

$$F_1 = \begin{matrix} -50.64 & 19.78 & -61.38 & 1.203 & 61.80 \\ 3.359 & -0.233 & -0.774 & -3.678 & -4.330 \end{matrix}$$

$$F_2 = \begin{matrix} 0.018 & -0.024 \\ 0.000 & -0.003 \end{matrix}$$

Fig.3 shows that the non-linear time response of the system with and without controller. The result illustrates that the controller provides substantial improvements in performance compared with the open-loop result as well as considerable damping of oscillation. The results were obtained by solving the system non-linear equations by using the Runge-Kutta numerical technique with an integration step of 0.002 second, and 1906 ICL computer.

6. CONCLUSION

The paper presented a proportional plus integral multi-variable control low along with an algorithm for

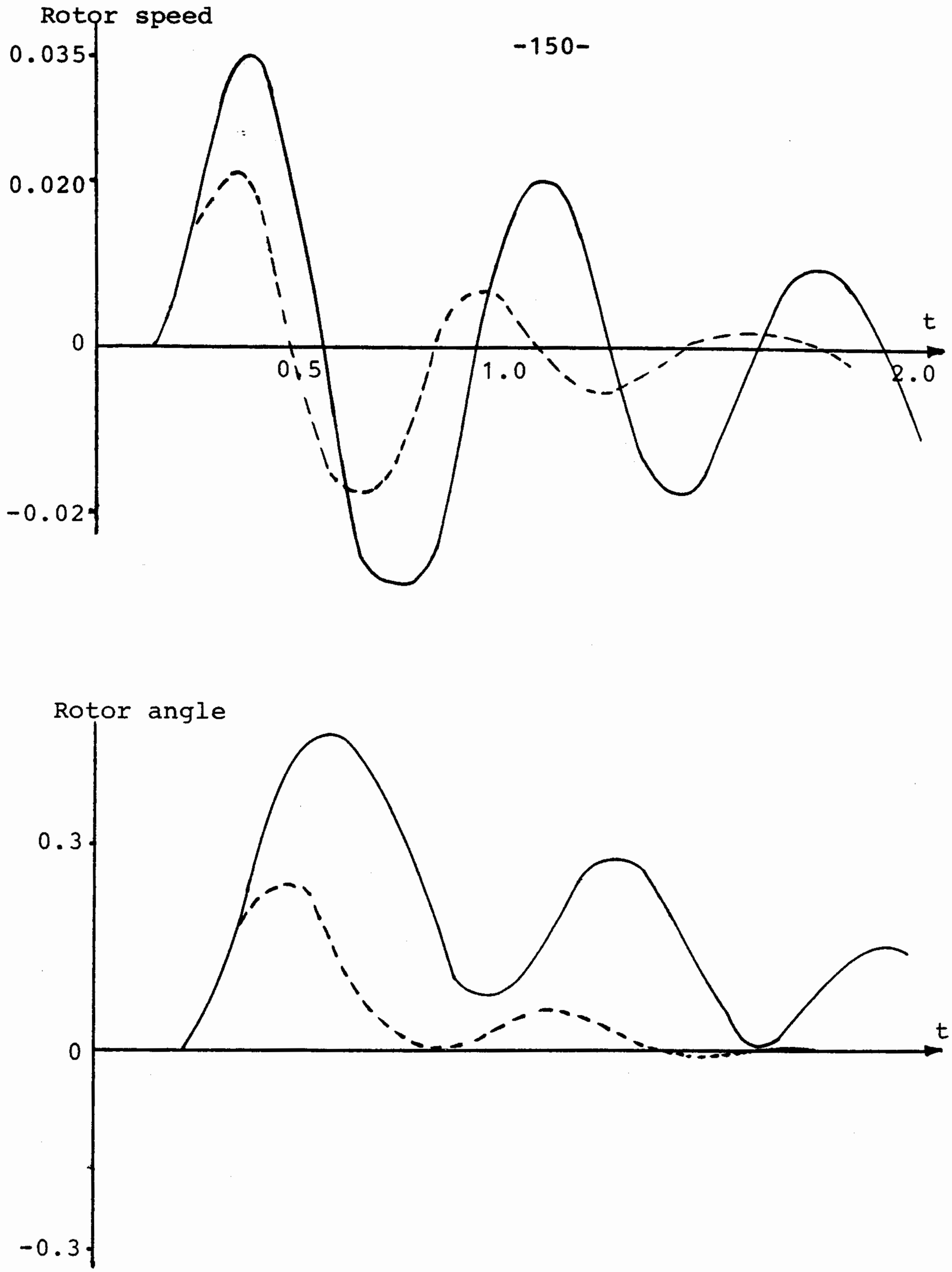


Fig.3. Response to a Step Change in input

——— Open-loop response

----- Digital controller

designing such controllers. The application of this technique is shown to improve the performance considerably.The controller can be implemented in practice using a mini-computer or microprocessor where the control signals are to be applied at specified instants.

The results illustrate substantial improvements in system stability and damping out power system oscillations when the controller is introduced. The techniques introduced in the paper are quite general and could be applied to other systems than that studied here.

7. *REFERENCES*

1] Anderson, J.H., and Raina, V.M., "Power system excitation and governor control using optimal control theory" Int. J. control, Vol. 19, pp. 289 - 308, 1974.

2] Newton,M.E. and Hogg,B.W., "Optimal control of a microalternator system" IEEE Trans. PAS-95, pp. 1822 - 1831, 1976.

3] Osheba,S.M. and Hogg,B.W., "Multi-variables controller for a turbogonerator" IEEE winter power meeting, 1980, New-York A 80-106-5.

4] Osheba, S.M. and Hogg,B.W., "Performance of state-space controllers for turbogenerators in multi-machine power systems" IEEE Trans. PAS-101, 1982, pp. 3276 - 3287.

5] Osheba,S.M.and Hogg,B.W.,"Equivalent turbogenerator model for design of multi-variable controllers in multi-machine systems" Int. J. of Electric Power & energy system, Vol. 6, 1984, pp. 89 - 101.

6] Osheba, S.M., "Decenteratized control scheme for turbogenerators in power systems" Proc. 1st Eurpean worshop on RICLS, patras university, Greece, July 9-12, 1984 pp. 296 0 304.

7] Hamdy A.M. Moussa and Yao-non-Ya, "Optimal power system stabilization through excitation and/or Governor control", 1971 IEEE PES summer meeting & International symposium on high power testing, portland, are., paper No.TP581-PWR.

APPENDIX (I)

An Algorithm to Compute $\lambda_{i,q}$

The composite matrix G is known to have the following properties[7]

1) The 2n eigenvalues of G are symmetrically located with respect to both real and imaginary axes of the complex Z-plane (inside the unit circle). Let the eigenvalue vector Λ of G be partitioned as

$$= \left[\Lambda_I \ , \ \Lambda_{II}\right]^T = \begin{bmatrix} \Lambda_I \\ \Lambda_{II} \end{bmatrix} \qquad (1)$$

where Λ_I has positive real parts (stable eigenvalue) and Λ_{II} has negative real parts (inside the unit circle). Then we have

$$\Lambda_I = - \Lambda_{II}$$

2) The eigenvalues with the positive real parts of B are the same as those of the optimal closed-loop system, i.e.

$$\Lambda_I = [\lambda_1 \ , \ \dots \ , \ \lambda_i \ , \ \dots \ , \ \lambda_n]^t$$

3) The eigenvector matrix of G, can be written as

$$Z = \begin{bmatrix} Z_I & Z_{III} \\ Z_{II} & Z_{IV} \end{bmatrix} \qquad (2)$$

and the first column of the eigenvector matrix Z corresponding to the stable eigenvalues Λ_I.

4) The eigenvector of G^t may be written as

$$V = \begin{bmatrix} Z_{IV} & Z_{II} \\ -Z_{III} & -Z_I \end{bmatrix} \qquad (3)$$

Let an eigenvector of the stable eigenvalue Z_i of G be

$$Z_i = \begin{bmatrix} Z_{Ii} \\ Z_{IIi} \end{bmatrix} = [Z_{Ii} \ , \ Z_{IIi}]^t \qquad (4)$$

and that of G^T be

$$V_i = \begin{matrix} Z_{IVi} \\ -Z_{IIIi} \end{matrix} = [Z_{IVi} \quad , \quad -Z_{IIIi}]^t \qquad (5)$$

the following is obtained

$$\Delta \lambda_i = \frac{1}{C_i} \quad V_i^t \quad \Delta G \; Z_i$$

where,

$$C_i = V_i^t \; Z_i$$

Since in the chosen case

$$\Delta G = \begin{matrix} 0 & 0 \\ -\Delta Q & 0 \end{matrix}$$

then we get

$$\Delta \lambda_i = \frac{1}{C_i} \quad Z_{IIIi}^t \; \Delta Q \; Z_{Ii}$$

and for the diagonal changes in Q the following is obtained

$$\Delta \lambda_i = \lambda_{i,q}^t \cdot \Delta q$$

where

$$\lambda_{i,q} = [\lambda_{i,q1} \quad , \quad \lambda_{i,q2}, \quad \cdots \quad , \quad \lambda_{i,qn}]^t$$

and

$$i,qJ = \frac{1}{C_i} \quad Z_{IIIi}(J) \; Z_{Ii}(j)$$

where,

$$C_i = \overset{n}{\underset{j=1}{}} \quad [Z_{IVi}(j) \; Z_{Ii}(j) \; -Z_{IIIi}(j) \; Z_{IIi}(j)]$$

APPENDIX (II)

The coefficient of matrix A,B are

$$K_1 = \frac{X_T + X_d - X_{ad}^2}{X_{fd}} \qquad K_2 = \frac{X_{ad}}{X_{fd} \cdot K_1}$$

$$K_3 = (1 - \frac{X_T}{K_1}) \; V_b \qquad K_4 = X_T \; K_2$$

$$K_5 = \frac{-X_q V_b}{X_q + X_T} \qquad K_6 = \frac{V_b}{X_q + X_T}$$

$$K_7 = K_3 K_6 + \frac{K_5}{K_1} V_b \qquad K_8 = K_4 K_6 - K_2 K_5$$

$$K_9 = -\frac{\omega_o}{2H} (K_7 \cos(2\delta) + K_8 \psi_{fd} \cos(\delta))$$

$$K_{10} = -\frac{\omega_o}{2H} (K_8 \sin\delta) \qquad K_{11} = \frac{1.0}{X_{fd}} + \frac{X_{ad}K_2}{X_{fd}}$$

$$K_{12} = \left(\frac{X_{ad}V_b}{X_{fd}K_1}\right) \sin\delta \qquad K_{13} = -\omega_o r_{fd} K_{11}$$

$$K_{14} = -\omega_o r_{fd} K_{12} \qquad K_{15} = \frac{\omega_o r_{fd}}{X_{ad}}$$

$$K_{16} = \frac{V_b X_q}{X_q + X_T} \qquad K_{17} = \frac{X_T - X_{ad}}{X_{fd}(X_T + X_d) - \frac{X_{ad}^2}{X_{fd}}}$$

$$K_{18} = \frac{2 K_4^2 \psi_{fd} + 2 K_3 K_4 \cos\delta}{2 V_T}$$

$$K_{19} = \frac{K_{16}^2 + K_{18}^2 \sin(2\delta) - 2 K_{17} K_{18} \psi_{fd} \sin(\delta)}{2 V_T}$$

$$K_{20} = \frac{2 K_{17}^2 \psi_{fd} + 2 K_{17} K_{18} \cos(\delta)}{2 V_T}$$

$$K_{21} = -\frac{1}{T_G} \qquad K_{22} = -\frac{1}{T_h}$$

$$K_{23} = -K_{22} \qquad K_{24} = \frac{1}{T_G}$$

The system data are as follows :

$H = 3.25$, $\omega_o = 314$, $R_{fd} = 0.0014$, $R_a = 0.005$,
$R_{TL} = 0.025$, $R_{KD} = 0.0078$, $R_{KQ} = 0.0084$, $R_{TR} = 0.0$,
$K_D = 0.017$, $V_b = 1.0$, $T_G = 0.1$, $T_h = 0.59$,
$X_d = 2.0$, $X_q = 1.91$, $X_{ad} = 1,86$, $X_{aq} = 1.77$,
$X_{KD} = 1.94$, $X_{Kq} = 1.96$, $X_{fd} = 1.97$, $X_{TR} = 0.1$,
$X_{TL} = 0.35$, $X_{af} = 1.86$.

NOMENCLATURE

General

ϕ	: Transition matrix.
Δ	: Driving matrix.
X	: State vector.
U	: Control vector.
Q	: Positive definite weighting matrix.
R	: Positive definite weighting matrix.
q	: Vector diagonal elements of Q.
F	: Feedback gain matrix.
$\lambda=E+jH$	: Eigenvalue vector.
$\lambda_{i,q}$	: Sensitivity vector of the eigenvalue λ_i w.r.t q.
S	: Sensitivily matrix.
G	: Composite matrix.
Λ,Z,V	: Eigenvalue vector, elgenvector matrices of G,G^T.

LIST OF SYMBOLS

δ	: Rotor angle.
$P\delta$	: Rotor speed deviation, $P = \frac{d}{dt}$.
ψ_{fd}	: Field flux linkage.
G_v	: Position of inlet valves.
T_M	: Generator shaft torque.
U_e	: Exciter input.
U_g	: Governor input.
P_b,Q_b	: Busbar real power, busbar reactive power.
T	: Sampling interval is 20 ms.
H	: Inertia constant.
V_t	: Terminal voltage.
ω_o	: Angular frequency of infinite busbar.
R_{fd}	: Field resistance.
R_a	: Stator resistance.
R_{TL}	: Line resistance.
R_{KD},R_{KQ}	: Resistance of d-and q-axis damper windings.
R_{TR}	: Transformer resistance.
V_b	: Infinite-busbar voltage.

T_G, T_h : Equivalent governor and turbine time constants.

X_d, X_q : Synchronous reactance in d-and q-axis damper windings.

X_{ad}, X_{aq} : Mutual reactance between armature and field winding.

X_{KD}, X_{KQ} : Self reactance in d-and q-axis damper windings.

X_{fd} : Self reactance of field winding.

X_T : Transformer and line reactances .

R_T : Transformer and line resistances.