

A COMPUTER AIDED DESIGN OF WEDGE-TYPE  
ROTOR-BARS IN SQUIRREL-CAGE INDUCTION MOTOR

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ABSTRACT

The deep-bar rotor induction motor is one of the most popular motors in the high starting-torque group of three-phase squirrel-cage induction motors. This motor is commonly fitted with deep-bars, of the wedge-form, on the rotor-side. Optimising the dimensions of these bars is a tedious work, especially when the motor is required to exert a starting-torque which exceeds the normal range.

This paper presents a computer aided design of the wedge-type rotor-bar to fulfill this requirement. The design concept, relating the bar-dimensions to the basic group of the motor specifications, is given by taking into consideration the skin-effect just at starting. The iterative method employed to obtain the proper cross-section area of the conductor, as well as the method of checking the preliminary dimensions, are also explained in this paper.

The design process is written in form of a digital program which gives, for a given output power, the proper bar-dimensions for an extended range of starting-torque ratios. The results show that for a given output power, a starting-torque ratio higher than usual can be obtained on the expense of a little increase in the starting-current ratio.

0.0 NOMENCLATURE

- $V_1$  := rated phase voltage of stator, volts;  
 $I_n$  := rated phase current, amps;  
 $I_{2,sc}$  := phase short-circuit current of rotor at  $s=1$ , amps;  
 $I_\mu$  := per-phase magnetising-current, amps;  
 $i_\mu$  := relative magnetising-current =  $I_\mu / I_n$   
 $P_n$  := rated output power, kW;  
 $f_1, f_2$  := stator-and rotor-frequency, respectively;  
 $s$  := rotor-slip, per unit;  
STR := starting-torque ratio;

- SCR := starting-current ratio;  
 $S_2$  := number of rotor slots;  
 $b_1$  := the upper bar-width,mm;  
 $b_0$  := the lower bar-width,mm;  
 $b_{0.5}$  := the bar-width at the middle of  $h_{pr}$ ,mm;  
 $b_i$  := the lower width of the equivalent slot,mm;  
 $g$  := conductivity of the bar-material,  $m/(mm^2 \cdot ohm)$ ;  
 $H$  := the actual bar-height,mm;  
 $h_{pr}$  := penetration depth due to resistance variation;  
 $h_{pi}$  := penetration depth due to inductance variation;  
 $J_2$  := rotor-side current-density under running conditions;  
 $K_{w1}$  := fundamental winding-factor of stator;  
 $(K_r)_{s=1}$  := resistance coefficient at starting =  $r_{ac}/r_{dc}$ ;  
 $(K_l)_{s=1}$  := inductance coefficient at starting =  $\lambda_{ac}/\lambda_{dc}$ ;  
 $l$  := length of iron-core without air-ducts,m;  
 $(P_c)_{1,sc}$  and  $(P_c)_{2,sc}$  := stator and rotor copper-losses, respectively, both are determined at  $s = 1$ ;  
 $r_b$  := bar-resistance, including the appertaining end-ring portion;  
 $r_{bo}$  := resistance of the bar-portion outside the iron-core;  
 $r_{bi}$  := resistance of the bar-portion embeded in the iron-core;  
 $r_{ac}$  := bar-resistance at  $s = 1$  ;  
 $r_{dc}$  := bar-resistance under running conditions;  
 $Z_1$  := number of stator-conductors per phase;  
 $\beta_0$  or  $\beta_1$  := the ratio  $b_1/b_0$  or  $b_1/b_i$ , respectively;  
 $\lambda_{ac}$  or  $\lambda_{dc}$  := specific permeance of the equivalent or actual slot, respectively;

## 1.0 INTRODUCTION

High starting-torque squirrel-cage induction-motors are usually fitted with deep-bar rotors. They are more simpler than the double-cage rotors [1]. Under the well known shapes of the deep bars; the wedge-bar finds a good resonance and common use. A deep-bar rotor can readily be designed to have an effective resistance at starting several times greater than its dc resistance. As the motor accelerates, the motor frequency decreases and the rotor effective resistance decreases accordingly to reach almost its dc value at rated slip.

The machine designer makes here use of the inductive effect of

the slot leakage flux on the current distribution in the deep-bar in order to ensure the above variations in the rotor resistance [3,4]. This effect is basically the same as the skin and proximity effects in any system of conductors having alternating currents. At starting where the rotor-frequency is equal to the stator-frequency, the bar-current is forced toward the top of the bar resulting in an increase in the effective-resistance and a smaller decrease in the effective leakage inductance of the bar. Since the distortion in the current distribution depends on an inductive effect, the effective rotor-resistance is function of the rotor frequency which varies from  $f_2=f_1$  at starting to  $f_2=sf_1$  under running conditions. This resistance is also function of the permeability and conductivity of the bar-material, as well as of the bar-dimensions and the wedge-ratio  $\beta$ , [2,6]. Accordingly, the machine designer is obliged to determine these dimensions precisely in order to get the proposed starting conditions, and a good performance at rated slip. It is a tedious work especially when the motor is required to exert a higher starting-torque which exceeds the normal range.

The following study aims to obtain a computer aided design of the wedge-type rotor-bar. For this purpose the main group of motor specifications are assumed to be known. In addition, the study assumes that the main dimensions, as well as the electric design of the stator-side, are settled. For each STR within an extended range, the proposed design is to be processed in two stages. In the first stage the preliminary dimensions of the bar are to be determined. In the second stage, these dimensions are to be checked and adjusted in order to get their final or proper values.

## 2.0 PRELIMINARY BAR-DIMENSIONS

These dimensions must satisfy the main group of motor specifications; such as :  $P_n$  , SCR , and STR. It follows now a brief discussion of the basic relations pertaining this stage of the design process. The discussion is ended with the relevant design steps.

### 2.1 Relation Between $(K_r)_{s=1}$ and H

The tendency of the bar-current to flow through the upper-portion during the starting-period, results in an equivalent-bar and in an equivalent-slot [3]. They determine the effective bar resistance and the slot-inductance during this period, respectively. The depth of either the equivalent-bar or the equivalent-slot is equal to  $h_{pr}$  or  $h_{pi}$  , respectively:

$$h_{pr} \approx (10 \text{ mm}) / \sqrt{s \cdot (f_2/50) \cdot (g/50)} \quad (1)$$

$$h_{pi} \approx (15 \text{ mm}) / \sqrt{s \cdot (f_2/50) \cdot (g/50)} \quad (2)$$

Both depths are measured from the conductor upper-edge. Under running conditions, either of them becomes equal to the actual bar-depth,  $H$ .

Copper is usually used to build deep-bar cages because of its, relatively, high conductivity. At starting,  $s=1$ , and taking the effects due to temperature variations during this period into consideration;  $g$  can be taken, in average, equal to 50. Also, under the assumption that the supply frequency is 50 Hz, at  $s=1$  the rotor frequency  $f_2=f_1=50$  Hz. Accordingly

$$(h_{pr})_{s=1} \approx 10 \text{ mm} \quad (3-a)$$

$$(h_{pi})_{s=1} \approx 15 \text{ mm} \quad (3-b)$$

The adjustment of STR to suit a proposed value depends mainly on the adjustment of the existing bar-resistance at  $s=1$ ,  $r_{ac}$ . This resistance can be related to the bar-resistance under running conditions,  $r_{dc}$ , by the resistance coefficient  $(K_r)_{s=1}$  which is equal to the ratio  $r_{ac}/r_{dc}$ . This coefficient gives, at starting, how many times the resistance of the bar, as well as of the rotor, is greater than its resistance under running conditions.

Consequently, the starting-torque-ratio depends on  $(K_r)_{s=1}$ . Larger values of STR require also that  $(K_r)_{s=1}$  must be larger. It can be determined by the ratio of the actual-bar area to the equivalent-bar area:

$$(K_r)_{s=1} = (H \cdot (b_1 + b_0)/2) / ((h_{pr})_{s=1} \cdot b_{0.5}) \quad (4)$$

As  $(h_{pr})_{s=1} = 10$  mm, the width  $b_{0.5}$  can be determined by:

$$b_{0.5} = b_1 + 5(b_0 - b_1)/H \quad (5)$$

Substituting for  $b_{0.5}$  in equation(4), taking into account that  $\beta = b_1/b_0$ , an explicit relation for  $H$  in terms of  $\beta$  and  $(K_r)_{s=1}$  can be derived:

$$a.H^2 + b.H + c = 0 \quad (6)$$

where

$$a = (1 + \beta)/20 \quad (7-a)$$

$$b = -\beta \cdot (K_r)_{s=1} \quad (7-b)$$

$$c = -5(1 - \beta) \cdot (K_r)_{s=1} \quad (7-c)$$

This second order simultaneous equation has two solutions; from

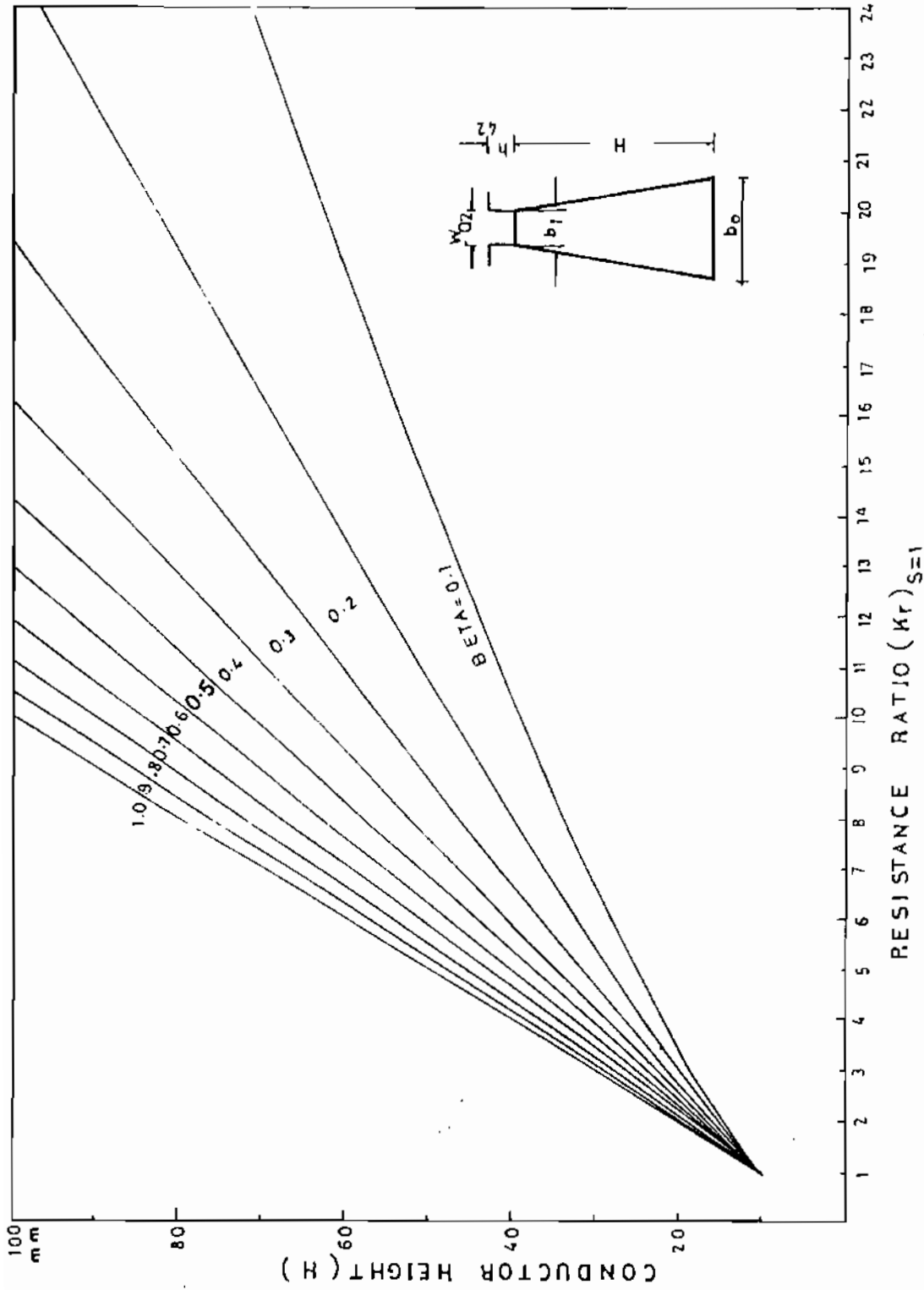


Fig.(1): Variation Of Conductor-Height With ( $K_r$ )<sub>S=1</sub>.

which the non-zero positive solution is accepted. Figure(1) shows the family of curves which gives the relation between  $H$  and  $(K_r)_{s=1}$  while  $\beta$  is taken as a parameter. This relation is nearly linear, especially for  $\beta \gg 0.5$ . The conductor height  $H$  will be equal to  $(h_{pr})_{s=1} = 10$  mm for  $(K_r)_{s=1} = 1$ ; irrespective of the value of  $\beta$ . Equation(6) is helpful in obtaining the wedge height for a given  $\beta$  and a predetermined resistance-coefficient  $(K_r)_{s=1}$  which corresponds to a given STR.

## 2.2 Relation between $(K_r)_{s=1}$ and STR

The starting-torque ratio can be expressed as the ratio of the rotor copper-loss at  $s=1$ , to the rated output power [3,5]. Thereby, the stand-still rotor copper-loss can be stated by the following relation:

$$(S_2 \cdot I_{2,sc}^2 \cdot r_b) 10^{-3} = (STR) \cdot P_n \quad (8)$$

in which, the bar-resistance at starting is:

$$r_b = r_{bo} + r_{bi} \cdot (K_r)_{s=1} \quad (9)$$

Substituting eq.(9) in eq.(8), a relation for  $(K_r)_{s=1}$  can be derived as:

$$(K_r)_{s=1} = (P_n \cdot 10^3) \cdot (STR) / (S_2 \cdot I_{2,sc}^2 \cdot r_{bi}) - (r_{bo}/r_{bi}) \quad (10)$$

This equation relates  $(K_r)_{s=1}$  to the main group of motor specifications:  $P_n$ , STR, and  $(SCR)_{s=1}$  SCR. The starting-current ratio takes a part in the determination of  $I_{2,sc}$ .

## 2.3 Determination of $I_{2,sc}$ and $I_{do}$

A consequent result of the skin-effect on the rotor parameters is that they are no more constant during the starting period. Accordingly, the rotor-current (in turn the stator-current) will assume different loci between  $s=1$  and  $s \approx 0$ . These loci are convergent [3] and assume nearly the same circle as  $s$  approaching zero. From all corresponding circle-diagrams, the circle-diagram at  $s=1$  will be taken to determine the preliminary  $I_{2,sc}$  and  $I_{do}$ .

Figure(2) shows the basic-sketch of this circle-diagram, in which:

$$(P_c)_{1,sc} = 3 \cdot I_{1,sc}^2 \cdot r_1 \quad (11-a)$$

$$(P_c)_{2,sc} = (STR) \cdot P_n \cdot 10^3 \quad (11-b)$$

$$I_{1,sc} = (SCR) \cdot I_n \quad (12)$$

$$I_{\mu} = i_{\mu} \cdot I_n \quad (13)$$

where  $r_1 :=$  the per phase stator-resistance, and  $i_{\mu} = 0.45-0.55$  for high starting-torque squirrel-cage induction motors.

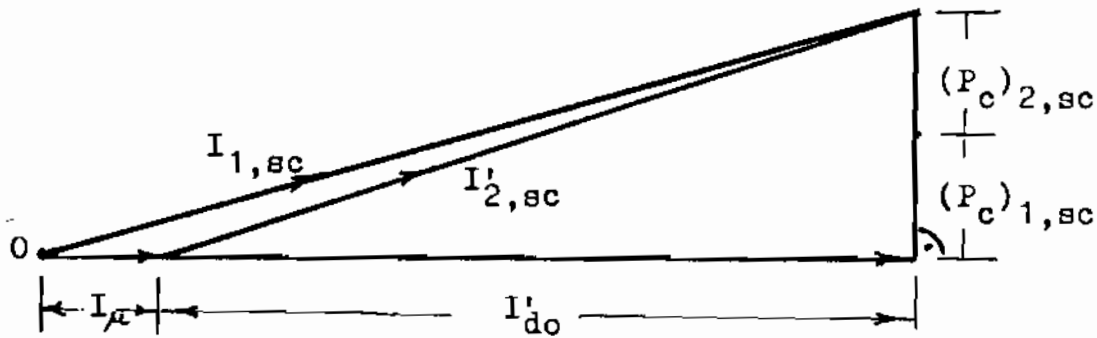


Fig.(2): Basic-sketch of circle-diagram at  $s=1$ .

Taking the appropriate scales for both current and power, the above sketch can be constructed to obtain  $I'_{2,sc}$  and  $I'_{do}$ ; either graphically or digitally. The preliminary  $I_{2,sc}$  and  $I_{do}$  can be determined as follows:

$$I_{2,sc} = u \cdot I'_{2,sc} \quad \text{and} \quad I_{do} = u \cdot I'_{do} \quad (14)$$

where  $u$  is the transformation-ratio [3] :

$$u = (Z_1 \cdot K_{w1} / (S_2/3)) \cdot (1 + 0.5 I_{\mu} / I'_{do}) \quad (15)$$

#### 2.4 Cross-Section Area of the Bar

The determination of both the dc resistances:  $r_{bo}$  and  $r_{bi}$  requires the knowledge of the cross-section area of the bar  $A_b$ . For first trial a preliminary value of this area,  $A'_b$ , will be taken as a ratio of the gross cross-section areas of the stator-slot conductors  $A_{S,1}$  :

$$A'_b = \nu' \cdot A_{S,1} \quad , \text{mm}^2 \quad (16)$$

The ratio  $\nu'$  is to be determined by:

$$\nu' = \nu \cdot (y_{S,2} / y_{S,1}) \quad (17)$$

where  $y_{S,1}$  and  $y_{S,2}$  are the slot-pitch of the stator and rotor, respectively, and  $\nu$  is the ratio between the rotor-side to the stator-side copper-sheet heights. It can be taken equal to about 0.7 to 0.8 .

Equation(16) gives a preliminary value of the bar cross-section

area which will be used to start the digital computation of the preliminary dimensions:  $H$ ,  $b_0$ , and  $b_1$ . With the resulting bar dimensions, a new value  $A_b$  will be determined to start with it again the calculation process of the preliminary dimensions. Such an iteration procedure will be stopped when the difference between two successive values of  $A_b$  is too small.

### 2.5 Bar-Width at the Middle of $h_{pr}$

The knowledge of this width,  $b_{0.5}$ , enables to determine the upper and lower widths,  $b_1$  and  $b_0$  respectively, for given  $B$  and  $H$ ; according to equation(5). Of course, it must ensure the main group of motor specifications:  $P_n$ , STR and SCR. An explicit relation for  $b_{0.5}$  in term of these specifications can be derived as follows: Having  $(K_r)_{s=1}$  from eq.(4) into eq.(9), the bar-resistance can be rewritten as:

$$r_b = r_{bo} + 1/((h_{pr})_{s=1} \cdot b_{0.5} \cdot g) \quad .$$

Substituting this relation in eq.(8) and solving for  $b_{0.5}$ , it results that:

$$b_{0.5} = 1/((h_{pr})_{s=1} \cdot g) / (STR \cdot P_n \cdot 10^3 / (S_2 \cdot I_{2,sc}^2) - r_{bo}) \quad (18)$$

### 2.6 Design Procedure

In order to get the preliminary bar-dimensions, assume  $B$  to have a proper value of 0.5 and process the following design steps:

#### 1 st Step:

1. Construct the basic-sketch of the circle-diagram at  $s=1$ ; according to eqs. (11), (12) and (13). Determine  $I'_{2,sc}$  and  $I'_{do}$  from it.
2. Determine the transformation ratio "u", by which the preliminary currents  $I_{2,sc}$  and  $I_{do}$  can be determined.

#### 2 nd Step:

1. Determine the preliminary cross-section area  $A'_b$ , according to eq. (16), thereby the dc resistances  $r_{bo}$  and  $r_{bi}$  can be determined.
2. Determine  $(K_r)_{s=1}$ ; using equation(10).
3. Determine the preliminary  $H$  by solving equation(6).
4. If  $H$  is greater than its upper limit ( $= 80$  mm):
  - (a) reduce  $B$  by a suitable increment, then go back to point (3) in the second step and process forward,
  - or (b) increase SCR by a suitable increment, then go back to the first step and process forward.

#### 3 rd Step:

1. Determine the bar-width  $b_{0.5}$ ; according to equation(18).



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2. Equation(5) yields that:

$$b_o = b_{0.5} / (\beta + 5(1 - \beta)/H) \text{ ,mm} \quad (19)$$

Use this relation to obtain  $b_o$  , thereby:

$$b_1 = \beta \cdot b_o \text{ ,mm} \quad (20)$$

This way, the preliminary bar-dimensions are determined to correspond to  $A'_b$  .

3. Approach with the above bar-dimensions ,through an iterative process, the proper cross-section area  $A_b$  . Check the the difference between two successive values of  $A_b$  . If the difference is too large , put  $A'_b = A_b$  then go back to the second step and process forward. Check also for  $J_2$  .

### 3.0 PROPER SLOT- AND BAR-DIMENSIONS

In the above analysis the basic-sketch of the circle-diagram at  $s=1$  has been constructed to obtain  $I_{2,sc}$  and  $I_{do}$  in accordance with the motor specificatios  $P_n$ ,SCR and STR. The first current has been used while determining the preliminary bar-dimensions. They are also the dimensions of the rotor-slot portion subjected to the bar-current. Final dimensions of the rotor-slot, including the tooth-tip, can not be decided before the diameter current is checked and settled values of the bar-dimensions are obtained. This time, the diameter-current is to be determined through the ideal reactance  $X_1$  at  $s=1$  ; as follows:

$$\text{and} \quad I_1 = V_1 / X_1 \quad (21-a)$$

$$I_d = I_1 - I_{\mu} \quad (21-b)$$

In this relation  $X_1 = x_1 + x'_{20}$  ,where  $x_1$  is the per phase leakage reactance of the stator and  $x'_{20}$  is the per phase leakage reactance of the rotor referred to the stator-side. The resulting diameter-current,  $I_d$ , must be the proper value which ensures also the proposed motor specifications. Therefore, it will be compared with  $I_{do}$  and an adjustment procedure may be carried out to get  $I_d$  equal to  $I_{do}$  .

### 3.1 The Inductance Coefficient ( $K_i$ )<sub>s=1</sub>

As the rotor-slot is a tapered-slot, its leakage-reactance consists mainly of two parts: the leakage-reactance of the tooth tip, and the leakage-reactance of the lower slot-portion which is subjected to the bar-current. At starting,  $s=1$ , the second

part of this reactance is function of the specific permeance of the equivalent slot  $\lambda_{ac}$ , which can be determined by:

$$\lambda_{ac} = y(\beta_i) \cdot (h_{pi})_{s=1} / (3 \cdot b_1) \quad (22)$$

Under running conditions, the same part is function of the specific permeance of the actual slot  $\lambda_{dc}$ :

$$\lambda_{dc} = y(\beta) \cdot H / (3 \cdot b_1) \quad (23)$$

In this equation,  $y(\beta)$  is the form-factor which can be determined by:

$$y(\beta) = 3\beta / ((1 - \beta)(1 - \beta^2)) (\ln(1/\beta) / (1 - \beta^2) - 0.75 + 0.25\beta^2) \quad (24)$$

Similarly,  $y(\beta_i)$  can be determined by substituting  $\beta_i$  instead of  $\beta$  in the same relation, eq.(24). The lower-width of the equivalent slot, required to get  $\beta_i$ , can be determined by:

$$b_i = b_1 + (b_0 - b_1) \cdot (h_{pi})_{s=1} / H \quad (25)$$

The form-factor gives the specific permeance of the current subjected slot-portion having a tapered-form, in term of the specific permeance of a current subjected slot-portion having a rectangular-form with the same height and upper-width.

Now, the ratio  $\lambda_{ac} / \lambda_{dc}$  is the inductance coefficient  $(K_i)_{s=1}$  which reflect the effect of crowding the bar-current towards the upper top at starting. According to eqs.(22) and (23), this coefficient can be determined by the following relation taking into consideration that  $(h_{pi})_{s=1} = 15 \text{ mm}$ :

$$(K_i)_{s=1} = (15/H) \cdot (y(\beta_i) / y(\beta)) \quad (26)$$

### 3.2 Variation Of $(K_i)_{s=1}$ With H

Equation (26) reveals that the relation between  $(K_i)_{s=1}$  and H is also dependent of the ratio  $y(\beta_i) / y(\beta)$ . Both form-factors are also function of H: In accordance with  $y(\beta)$ , it can be stated here that, for a given STR, any variation in the wedge-height H must be accompanied by a corresponding variation in the wedge-ratio  $\beta$  in order to hold the cross-section area of the bar constant at a proper value. Thereby, it can be ensured that the rotor copper losses under running conditions will not be affected by the proposed STR. In accordance with  $y(\beta_i)$ , the equivalent slot ratio  $\beta_i$  can be expressed at starting as a function of  $\beta$  and H:

$$\beta_i = \beta / (\beta + (1 - \beta) \cdot (h_{pi})_{s=1} / H) \quad (27)$$

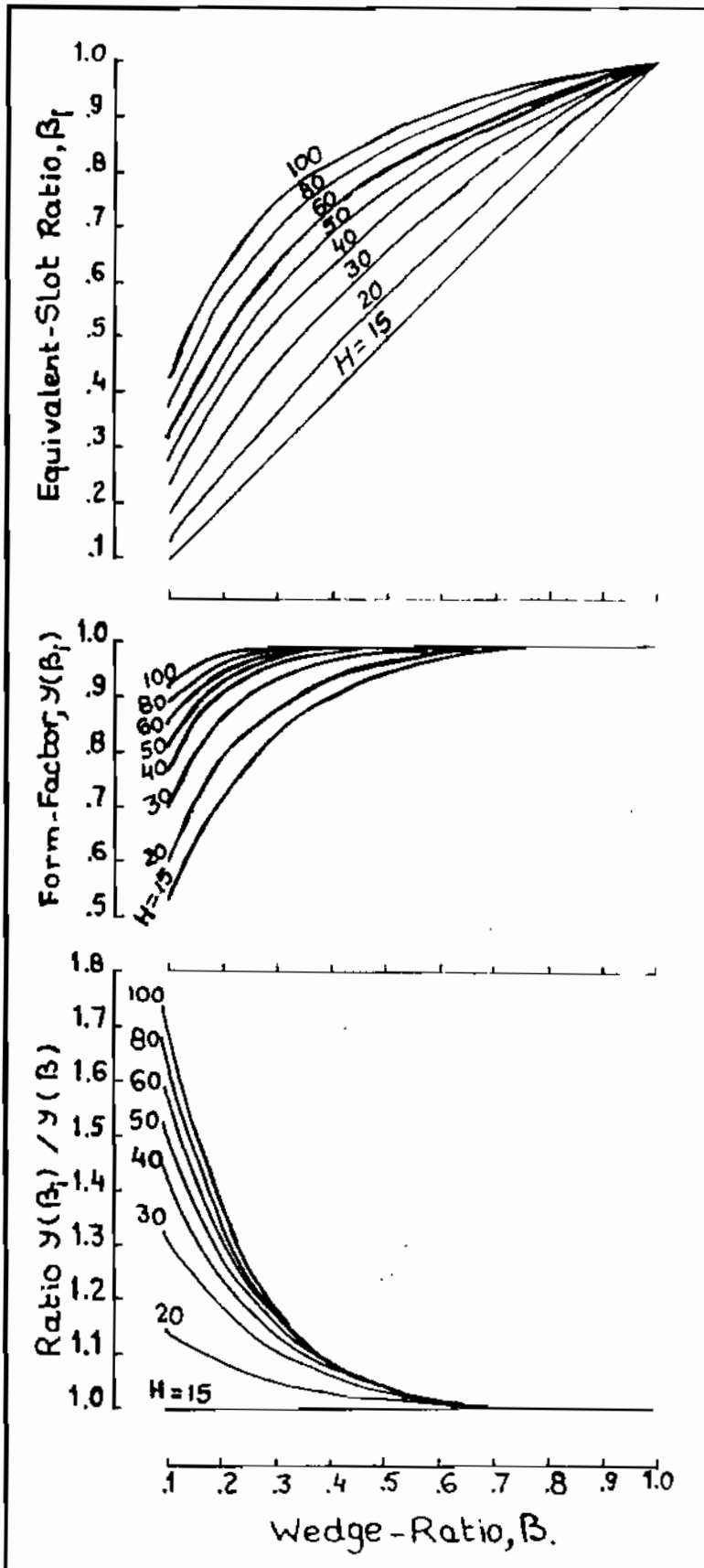


Figure (3)

Figure (4)

Figure (5)

Accordingly and for a given  $H$ , the relation between  $\beta_1$  and  $\beta$  at starting can be found, Fig.(3). The wedge-height  $H$  had been varied from  $H=(h_{pi})_{s=1}=15$  mm, to 100 mm. It is seen that for  $H=15$  mm, the relation between both ratios is linear;  $\beta_1=\beta$ . For  $H>15$  mm, the relation becomes non-linear and all curves of  $\beta_1$  will converge the same value  $\beta_1=1$  at  $\beta=1$ . This means that for a rectangular bar  $\beta_1=\beta=1$ , irrespective of  $H$ .

Figure(4) shows the relation between  $y(\beta_1)$  and the wedge-ratio  $\beta$ . As  $\beta_1=\beta$  for  $H=15$  mm, it can be concluded that for this bar-height  $y(\beta_1)=y(\beta)$ . For  $H>15$  mm and the same  $\beta$ ,  $y(\beta_1)$  will be larger than the corresponding  $y(\beta)$ ; but it converges unity for values of  $\beta>0.5$ , irrespective of  $H$ . Accordingly, the ratio  $y(\beta_1)/y(\beta)$  will be greater than 1 for  $H>15$  mm, Fig.(5); but it converges also unity for values of  $\beta>0.5$ , irrespective of  $H$ .

Now, as the variation-nature of the ratio  $y(\beta_1)/y(\beta)$  with  $\beta$  for a given  $H$  is explained, the variation of  $(K_1)_{s=1}$  with  $H$ , according to eq.(26), can be depicted as in Fig.(6). It is seen that  $(K_1)_{s=1} = 1$  for  $H=15$  mm and irrespective of  $\beta$ . This means that the variations in the slot-inductance, due to skin-effect, will not be existing. Recalling the value of  $(K_r)_{s=1}$  under the same conditions, it will be found to be equal 1. Therefore, wedge-bars must be constructed with  $H$  greater than 15 mm in order to maintain a corresponding increase in the starting-torque. Taking  $\beta$  as a parameter, Fig.(7) gives the relation between  $(K_1)_{s=1}$  and  $H$ . Both figures, Fig.(6) and (7), show that for a given  $H$  remarkable variations in  $(K_1)_{s=1}$  are not observed for the range  $0.5<\beta\leq 1.0$ . In this range of  $\beta$ , the coefficient  $(K_1)_{s=1}$  seems to be constant.

### 3.3 Adjustment Of The Diameter Current $I_d$

It is well known that the specific-permeances of the leakage paths existing in the machine, determine its ideal-reactance  $X_1 = x_1 + x'_{20}$ . An effective part of these permeances at  $s=1$  is the specific-permeance of the rotor-slot including the tooth tip:

$$\lambda'_{s2} = \left[ (K_1)_{s=1} \cdot y(\beta) \cdot H / (3 \cdot b_1) + (h_{42}/W_{02}) \right] \cdot (K_{w1}/q_2)^2 \quad (28)$$

, where  $q_2$  is the number of rotor-slots per pole per phase.

The adjustment of  $I_d$  to be equal to  $I_{d0}$  can be done by the adjustment of  $X_1$ , eqs.(21); through the adjustment of  $\lambda'_{s2}$ . The adjustment process of  $\lambda'_{s2}$ , which can be established by affecting

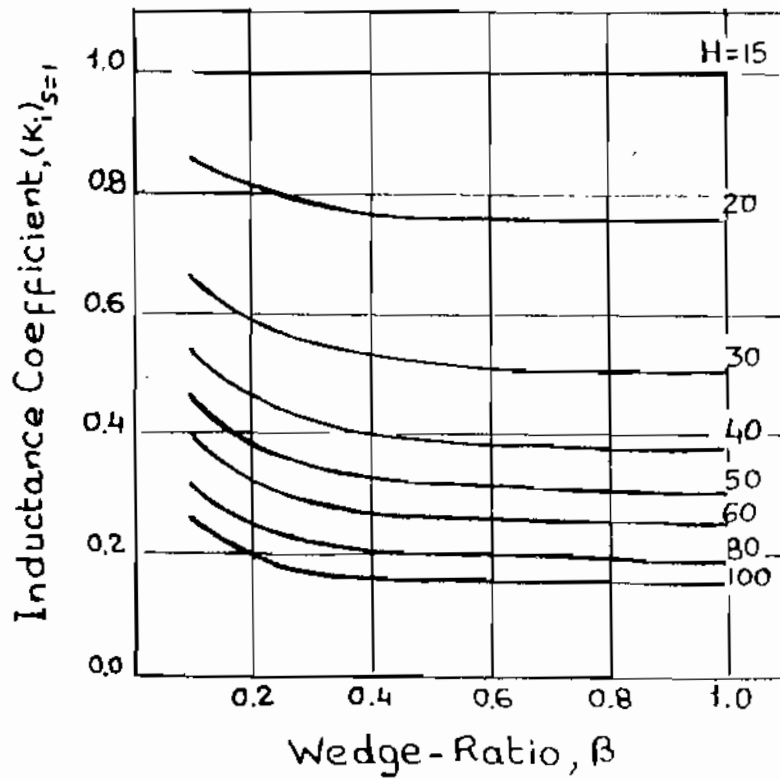


Fig.(6): Variation Of  $(K_1)_{S=1}$  With  $\beta$ .

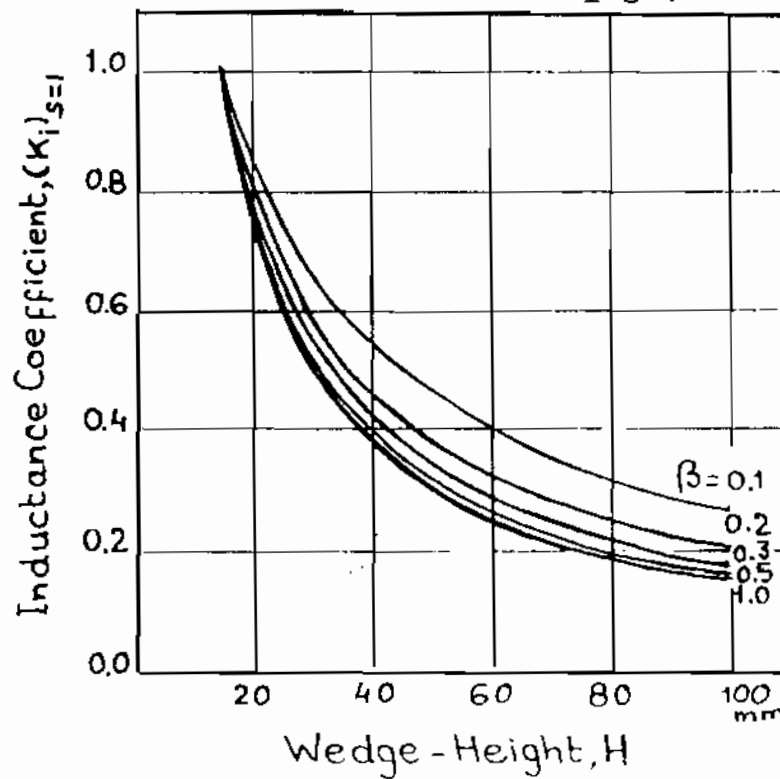


Fig.(7): Variation Of  $(K_1)_{S=1}$  With  $H$ .

its two components through either  $h_{42}$  or  $(K_1)_{s=1}$ , depends on whether  $I_d$  is greater than  $I_{d0}$  or smaller.

As the calculations have been started with  $\beta=0.5$ , the corresponding  $(K_1)_{s=1}$  is able to be increased or decreased by having an other suitable value of  $\beta$ ; in order to get the counter effect on  $I_d$ . In case of  $I_d > I_{d0}$ , it is more simpler to increase  $h_{42}$  in order to diminish  $I_d$ . Therefore,  $h_{42}$  is assumed to have, initially, a minimum value of 0.5 mm. Of course, the starting-torque ratio STR affects to a great extent the value of  $I_d$  compared with  $I_{d0}$ .

### 3.4 Design Procedure (Continuation)

The design-steps can be continued now to obtain the proper slot - and bar-dimensions:

#### 4 th Step:

1. Use the preliminary bar-dimensions to obtain  $b_1$ , eq.(25), then determine  $\beta_1$ .
2. Determine  $(K_1)_{s=1}$ , eq.(26), For this purpose either  $y(\beta_1)$  or  $y(\beta)$  can be determined by eq.(24).
3. Determine  $\lambda'_{s2}$ , eq.(28), which will be added to the specific permeances of the settled leakage paths in both machine -sides. Thereby  $X_i$  then  $I_d$  can be determined.

#### 5 th Step:

1. Compare  $I_d$  with  $I_{d0}$ . If  $I_d$  is tolerated within  $\pm 0.5$  amp from  $I_{d0}$ , accept the preliminary bar- and slot-dimensions to be the proper dimensions.
2. If  $I_d > I_{d0}$ :
  - (a) Modify the second term in  $\lambda'_{s2}$  to be larger by enlarging  $h_{42}$  by a suitable increment, then go back to point (4) in the 4 th step and process forward.
  - (b) If the upper limit of  $h_{42}$  (=5 mm) is reached and  $I_d$  is still greater than  $I_{d0}$ , modify the first term in  $\lambda'_{s2}$  to be larger through increasing  $(K_1)_{s=1}$ . For this purpose decrease  $\beta$  by an increment of 0.01, then go back to point (3) in the second step and process forward. In this case,  $\beta$  is not allowed to be less than 0.3.
3. If  $I_d < I_{d0}$ :
 

Modify the first term in  $\lambda'_{s2}$  to be smaller through decreasing  $(K_1)_{s=1}$ . For this purpose, increase  $\beta$  by an increment of 0.01, then go back to point (3) in the second step and process forward.

#### 4.0 DIGITAL PROGRAM AND RESULTS

A digital program is written on the basis of the above discussed design concept. As mentioned before the design process assumes settled electrical design of the stator-side for a given output. The program gives the proper dimensions of a group of rotor-bars corresponding to an extended range of STR. In addition, it gives for each starting-torque ratio the corresponding SCR,  $\beta$ ,  $(K_r)_{s=1}$ ,  $(K_1)_{s=1}$ ,  $I_{d0}$ ,  $I_d$ ,  $b_{0.5}$ ,  $h_{42}$ ,  $\nu'$ , and  $J_2$ .

The program is tested through the determination of the group of rotor-bars which may be suitable for a 500-kW, 6000-V, 50-Hz, 125-rpm motor and corresponds to a range of STR = 0.5 0.1 2.5 .

The results show that for this motor, the range of STR can be extended from 0.5 to 2.0 . Figure (8) shows the variations of some results: SCR ,  $(K_r)_{s=1}$  , H , and  $(K_1)_{s=1}$  with the possible range of STR. It is observed for the normal range  $0.5 \leq \text{STR} \leq 1.0$  that  $I_d$  is greater than  $I_{d0}$  . Consequently, the adjustment of  $I_d$  is possible by the controlling of  $h_{42}$  . This height changes from  $h_{42} = 5$  mm at STR = 0.5 , to reach a minimum value of 0.5 mm at STR = 1.0 . In this case SCR and  $\beta$  are hold constant at 3.5 and 0.5 , respectively. It is seen for this range of STR , Fig.(8), that either  $(K_r)_{s=1}$  or H varies linearly.

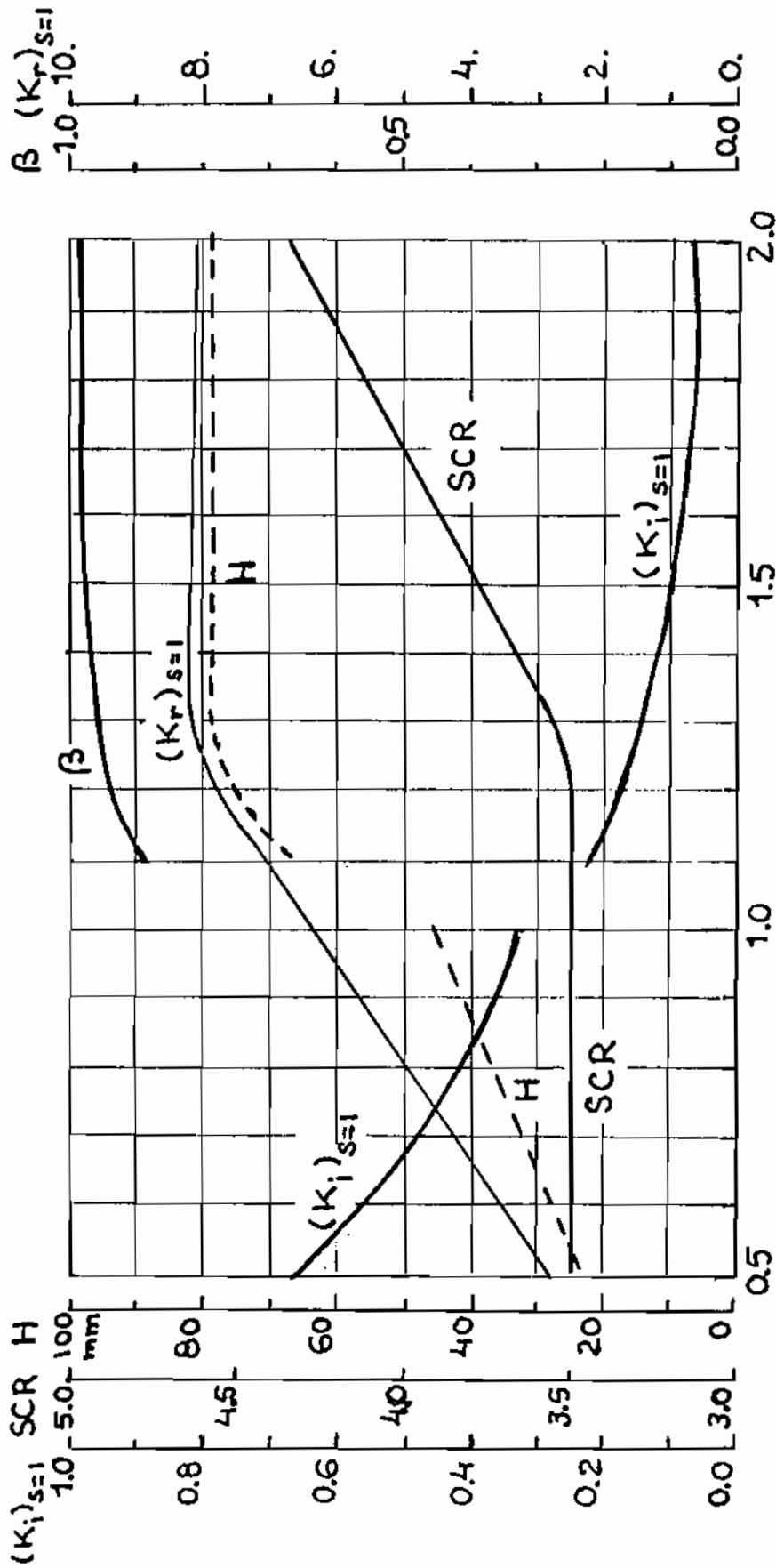
For higher starting-torque ratios  $1.0 \leq \text{STR} \leq 2.0$  ,  $I_d$  is almost smaller than  $I_{d0}$  . In this case, the adjustment of  $I_d$  is possible if SCR and  $\beta$  are allowed to increase. At first, the wedge-ratio  $\beta$  is allowed to increase. It follows that the resulting bar-height H may be greater than its maximum value. Consequently, SCR is allowed to increase in order to keep H within its probable value. It may be happen in this range of STR, that  $I_d$  is slightly greater than  $I_{d0}$  due to the increase of SCR. In this case a small increase in  $h_{42}$  adjusts  $I_d$  to its proper value.

It is seen in Fig.(8) that the starting-current ratio, SCR, is slightly increasing in the range  $1.0 \leq \text{STR} \leq 1.3$ . Thereafter, it will increase linearly to reach 4.3 at STR = 2.0 , while  $\beta$  , H , and  $(K_r)_{s=1}$  are nearly constant.

The results show also that for STR greater than 2.0 , it is not possible for this motor-example to adjust  $I_d$  , and at the same time to maintain  $\beta$  , SCR , and H within their accepted limits.

#### 5.0 CONCLUSIONS

The presented computer aided design of wedge-type rotor-bars makes it possible to find the probable range of starting-torque



### Starting Torque Ratio, STR

Fig.(8): Some results pertaining the example.



ratio, STR, for a three-phase deep-bar rotor induction motor for a given out-put power. This range is assumed to begin at  $STR=0.5$ , increasing in steps of 0.1. It is found for a 500-kW motor that the corresponding range of STR can be extended to become  $0.5 \leq STR \leq 2.0$ ; which is normally  $0.5 \leq STR \leq 1.2$ . The second half of this range is obtained on the expense of a small increase in the proposed starting-current ratio, which increases from 3.5 at  $STR=1.2$  to about 4.3 at  $STR=2.0$ . Accordingly, the starting torque can be doubled while the corresponding increase in the starting-current is about 23 % from its initial value. For each value within the probable range of STR, the computer program yields the relevant dimensions of the required rotor -bar and -slot as well as a group of check figures.

#### 6.0 REFERENCES

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