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ANALYSIS OF THE 2-D TRANSIENT HEAT CONDUCTION IN A COMPOSITE FINITE CYLINDER WITH HEAT GENERATION

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التوصيل الحرارى الغير مستقر ثنائى البعد فى أسطوانه
مركبه و محدودة الطول مع وجود مصدر حراره داخلى

يهدف البحث الى الحصول على التطور الزمنى لتوزيع درجات الحراره فى الاتجاهين القطرى والمحورى نتيجة لانتقال الحراره بالتوصيل الغير مستقر فى نظام اسطوانى محدود الطول ، يتكون هذا النظام من اسطوانه داخليه مصمته محاطه باسطوانه خارجيه، ويفترض أن الاسطوانه الداخليه بها مصدر حرارى ، كما أفترض ان هذه الاسطوانه المركبه تبرد بالحمل بواسطة مائع ذى درجة حراره معينه عند الدخول . ونظرا لعدم وجود حل رياضى تحليلى لهذه المشكله فلقد تم تم الحصول على التطور الزمنى لدرجات الحراره بتطبيق طريقة الفروق المحدوده، وفيها تم تحويل المعادله التفاضليه الاطيه لانتقال الحراره بالتوصيل الى معادله جبريه خطيه تميزه أنها عامه بدون أبعاد . وبتطبيق هذه المعادله على كل عنصر من عناصر الاسطوانه حيث قسمت فى الاتجاهين r,z نحصل على مجموعه من المعادلات الخطيه التى تم حلها بالطريقه الصريحه . وفى هذا البحث تم أخذ المقاومه الحراريه للتلامس بين الاسطوانتين الخارجيه والداخليه فى الاعتبار . وتدل النتائج على أن التطور الزمنى وتوزيع درجات الحراره فى الاتجاهين القطرى والمحورى يعتمد على عدة عوامل مثل عدد Biot ونسبة الانتشار الحرارى لمادتى الاسطوانتين و نسبة معامل التوصيل الحرارى لهما . وتدل النتائج على صلاحية ودقة الحل المقدم .

ABSTRACT
The objective of this paper is to study the 2-D transient temperature behavior in a composite finite cylinder with internal heat generation. For this purpose, a new dimensionless 2-D finite difference technique in the axial and radial directions is developed. The developed technique is then applied to obtain the time development of temperature profiles in a complex composite cylinder.

INTRODUCTION
The problem of heat conduction in rectangular fins both in steady-state and transient case receives recently great interest [1,2]. A single analytical 1-D transient heat conduction equation has been developed which is applicable for cartesian, cylindrical, and spherical coordinates [3]. The effect of temperature dependent thermophysical material properties has been considered in [4]. On the other hand, there is little activity on the 2-D heat conduction in cylindrical coordinates.

In nuclear industry, the transient temperature behavior of the fuel is of vital importance [5-10]. In light water reactors such as pressurized water reactor (PWR), the fuel rods are cooled convectively through the heat transfer into the cooling fluid. Therefore, the problem of heat conduction in the cylindrical fuel rod has been studied explicitly under predescribed conditions and assumptions. Conduction in the radial direction is usually taken into account [6,7,8]. With nonuniform cooling, conduction in the azimuthal direction must be considered. The steady-state heat conduction in the radial and azimuthal directions has been considered in [9], while the transient case was studied in [10]. The general equation for heat conduction is

$$\nabla^2 T^* + \frac{q'''}{k} = \frac{\rho c}{k} \frac{\partial T^*}{\partial t^*} \quad (1)$$

Unfortunately, the exact analytical solution for Eq.1 for the 2-D case is formidable. Analytical solutions are obtained only for the 1-D conduction and simple boundary conditions [2,11].
In the present work, a new dimensionless finite difference technique is developed for the problem of transient heat conduction in the radial and axial directions in a composite cylinder with internal heat generation. The new technique is general and simple.

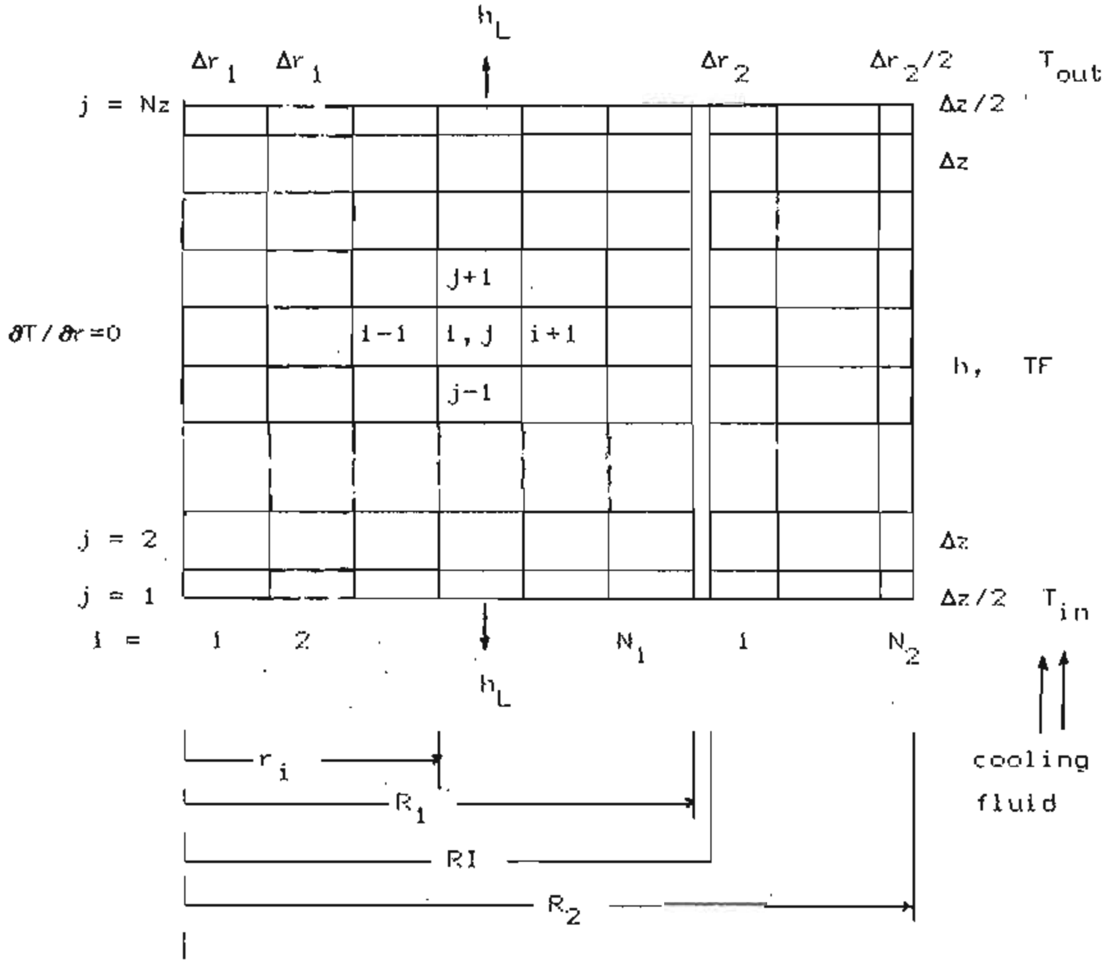


Fig.(1) Composite cylindrical system

PHYSICAL FORMULATION AND MATHEMATICAL MODEL

Figure (1) represents a composite cylindrical system. The heat is generated in the inner solid cylinder of radius R_1. The material of this cylinder may be an electric conductor, nuclear reactive material or chemical reactor. This inner cylinder is encapsulated within another cylinder having an inside radius RI and outside radius R_2. The outer cylinder represents the cladding material of a fuel rod or the insulator of an electric cable. The contact resistance has been considered through the heat transfer coefficient in a very thin gap between the two materials. The system is cooled by convection into the surrounding fluid which has temperature that varies along the axial direction of the system from T^*_{in} to T^*_{out}. Numerical solution of Eq.1 is obtained through transforming it into algebraic equation. Therefore, the dimensionless time variable t and the dimensionless space variables r,z are broken into discrete intervals Δt, Δr, and ΔZ, as shown in Fig.(1). The inner solid cylinder is divided in the radial direction into N_1 layer and the outer cylinder into N_2 layer. The entire system is then divided in the axial direction into NZ divisions. Applying the principle of energy balance for each nodal point i,j and rearranging, one gets the following dimensionless discretization equation :

$$T^{t+\Delta t}_{i,j} = a_o + a_1 T^t_{i-1,j} + a_2 T^t_{i+1,j} + a_3\, T^t_{i,j-1} + a_4\, T^t_{i,j+1} + a_5 T^t_{i,j} \qquad (2)$$

where $T = \dfrac{T^* - T^*_{in}}{\Delta T^*_g}$, $\Delta T^*_g = q_c''' R_1^2/k_1$, $a_o = (q'''/q_c''')\Delta t$

and $\Delta t = (\Delta Fo_r)_1 = \dfrac{k_1\ \Delta t^*}{\rho_1 c_1 R_1^2}$

The following boundary conditions are applicable for the considered case :

1- at r = 0 (the innermost nodal points i = 1), we have

$$(\partial T/\partial r)_{r=0} = 0.0 \qquad (3)$$

2- at $r = R_2/R_1$ (the outermost nodal points $i = N_1 + N_2$), we have

$$(\partial T/\partial r)_{r=R_2/R_1} = -\ (hR_1/k_2).(T_{N,j} - TF_j) \qquad (4)$$

3- at z = 0 (bottom nodal points j = 1), we have

$$(\partial T/\partial z)_{Z=0} = (h_L L/k).(T_{i,1} - T_{in}) \qquad (5)$$

4- at z = 1 (the top nodal points j = NZ), we have

$$(\partial T/\partial z)_{Z=1} = -\ (h_L L/k).(T_{i,NZ} - T_{out}) \qquad (6)$$

where k in Eqs.5 and 6 is K_1 for material 1 and K_2 for material 2. The coefficients a_1, a_2, a_3, and a_4 which satisfy the above boundary conditions are listed in the tables below, where :

$X_1 = 2\ \Delta r_1\ N_1^2\ \Delta t$, $\qquad X_2 = 2.\ \Delta r_2\ N_2^2\ (DR)\ (RD)\ \Delta t$

$Y_1 = \varepsilon^2\ Nz^2\ \Delta t$, $\qquad Y_2 = Y_1\ (DR)$

$$E = \left[\ 0.5 + \frac{N_1}{H_g} + \frac{0.5\ N_1}{N_2(CR)(RD)}\ \right], \quad \text{and} \quad A(i) = R_i^2 - R_{i-1}^2$$

The effect of the contact resistance at the interface between the first material and the inner surface of the second material is taken into account through the coefficients a_1 and a_2 for both layers N_1 and N_1+1.

Coefficients of the nodal points in the inner cylinder

	Interior nodes	Last column $i = N_1$	Bottom layer $j = 1$	Top layer $j = Nz$
a_1	$\frac{X_1 r_{i-1}}{A(i)}$	$\frac{X_1 r_{i-1}}{A(i)}$	$\frac{X_1 r_{i-1}}{A(i)}$	$\frac{X_1 r_{i-1}}{A(i)}$
a_2	$\frac{X_1 r_i}{A(i)}$	$\frac{X_1 r_i}{A(i)\ E}$	$\frac{X_1 r_i}{A(i)}$	$\frac{X_1 r_i}{A(i)}$
a_3	Y_1	Y_1	$\frac{Y_1 H_{L1}}{Nz}$	Y_1
a_4	Y_1	Y_1	Y_1	$\frac{Y_1 H_{L1}}{Nz}$

Coefficients of the nodal points in the outer cylinder

	Interior nodes	First column $i = N_1+1$	Last column $i = N_1+N_2$	Bottom layer $j = 1$	Top layer $j = Nz$
a_1	$\frac{X_2 r_{i-1}}{A(i)}$	$\frac{X_2 r_{i-i}}{A(i)}$	$\frac{X_2 r_{i-1}}{A(i)}$	$\frac{X_2 r_{i-1}}{A(i)}$	$\frac{X_2 r_{i-1}}{A(i)}$
a_2	$\frac{X_2 r_i}{A(i)}$	$\frac{X_2 r_i}{A(i)\ E}$	$\frac{X_2\ Bi\ \Delta r_2}{A(i)}$	$\frac{X_2 r_i}{A(i)}$	$\frac{X_2 r_i}{A(i)}$
a_3	Y_2	Y_2	Y_2	$\frac{Y_2 H_{L2}}{Nz}$	Y_2
a_4	Y_2	Y_2	Y_2	Y_2	$\frac{Y_2 H_{L2}}{Nz}$

For all nodal points $a_o = (q'''/q_c''')\Delta t$, and

$$a_5 = 1. - a_1 - a_2 - a_3 - a_4$$

Applying Eq.2 to each nodal point one gets a system of finite difference dimensionless equations. In this stage, The solution of this system can be performed using either the explicit or the implicit scheme [12]. In this work, the explicit scheme is used because of its simplicity although it is conditionally stable. As an illustrative example, the transient temperature profile of nuclear fuel rod of a PWR has been calculated. Such a composite cylinder system is chosen here because of its complexity as described in the Appendix, where the following data are valid:

$R_1 = 1$, RI = 1.021, $R_2 = 1.18$, $\varepsilon = 0.001$, RD = 6.29, CR = 6, DR = 9, $H_g = 6.44$, HH = 1.0, M = 2.55, and BI = 12.55.

For this specific problem, the heat generation rate is a sinsoidal function in axial coordinate z, where

$$\frac{q'''}{q_c'''} = \sin(\pi z) \qquad (7)$$

To get the dimensionless temperature of the cooling fluid at any level z, the following dimensionless relation is then obtained from the heat balance :

$$TF_{j+1} = M \, \Delta Z \, (T_{N,j} - TF_j) + TF_j \qquad (8)$$

Other systems are easier to deal with, such as electric cables, chemical reactors etc..

RESULTS AND DISCUSSION

Since there is no analytical solution of Eq.1 for the 2-D case in cylindrical bodies, then there are no reference data for comparison. Forthunatly, the radius to height ratio of the considered example is too small (ε = 0.001), which makes reasonable comparison between the numerically obtained values (at large time) and the 1-D steady-state values. The 1-D steady-state analytical solution of the considered example has been obtained in the Appendix.

Referring to Fig.2, the dimensionless temperature at the centerline T_o is 0.352 at t = 2 (which is steady-state value). The physical centerline temperature is then given by

$$T_o^* = T_o \left(\frac{q_c''' R_1^2}{k_1}\right) + T_{in}^*$$

$$= 0.352 \times 3045.7 + T_{in}^* = 1072 + T_{in}^* \text{ degree}$$

According to the 1-D analytical solution given in the Appendix, the corresponding value is $T_o^* = 1074 + T_{in}^*$. Comparison between the two values indicates well agreement which proves validity of the proposed numerical technique.

Figure 3 illustrates the radial temperature profiles at the bottom (z = 0.0), center (z = 0.5), and top (z = 1.0) of the composite cylinder. It is clear that the temperature distribution at the center is higher because the heat generation rate is a sinsoidal function in z with its maximum value q_c''' at the z = 0.5. Another important result is that the radial temperature profile at the top of the composite cylinder is higher than that at the bottom. This is expected since the temperature of the cooling fluid rises in the direction of z.

Figures 3 and 4 indicate the effect of Biot number on the radial temperature profiles at different times (0.1 and 2.0). It is clear that the effect of Biot number on the temperature of the outer cylinder is faster than its effect on the inner one. In addition, low Biot numbers exhibit higher temperature level. The time development of the radial temperature profiles for low Biot number (BI = 0.1) is given on Fig.6.

To examine the effect of the material thermophysical properties, the radial temperature profiles are plotted on Fig.7 for different

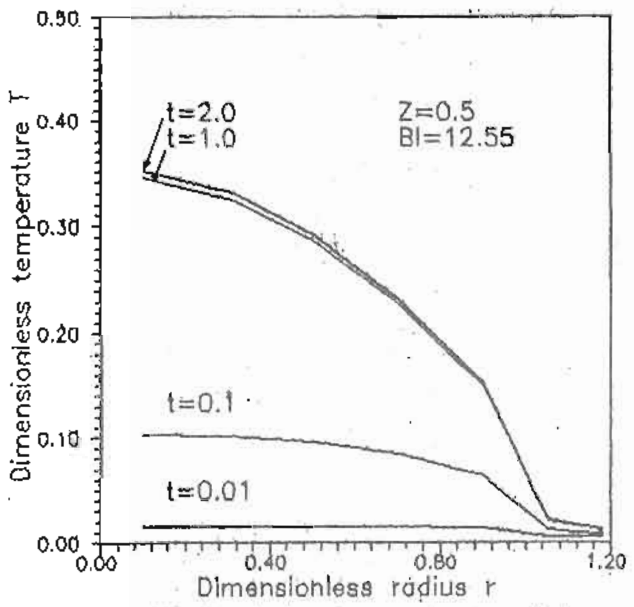


Fig.(2) Time development of radial temperature profile

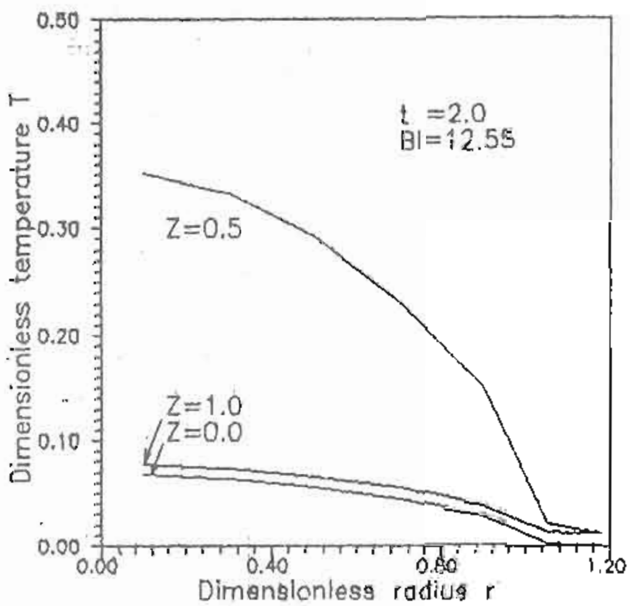


Fig.(3) Radial temperature distributions at different axial distances Z

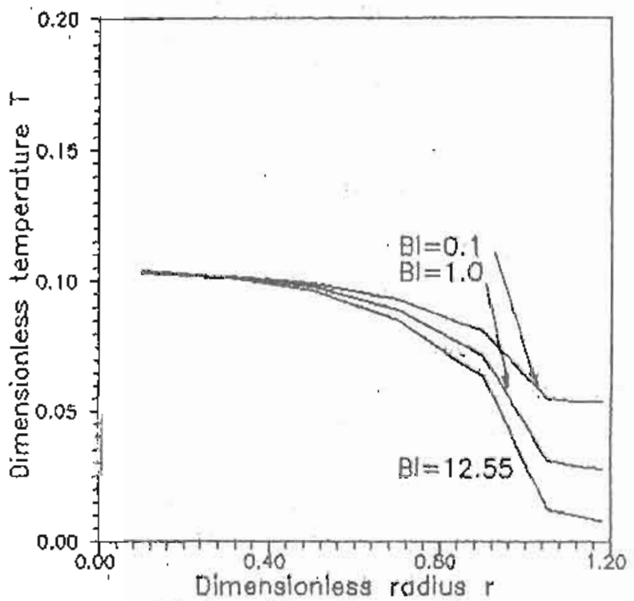


Fig.(4) Effect of Biot number on radial temperature pofile (t=0.1, z=0.5)

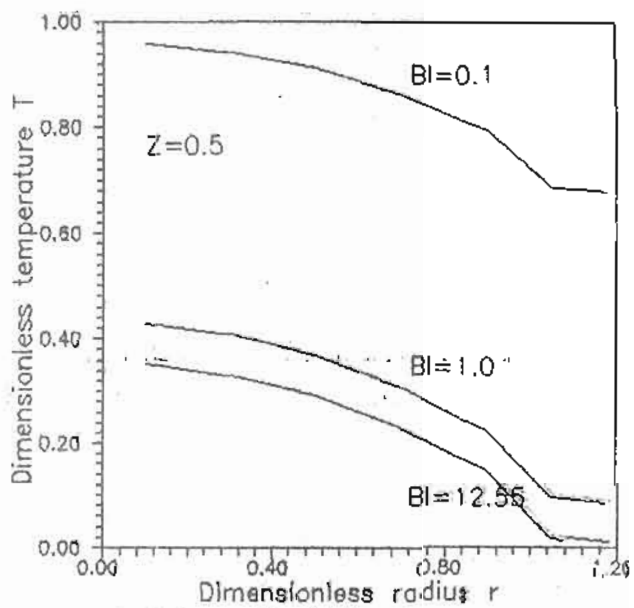


Fig.(5) Effect of Biot number on radial temperature pofile (t = 2.0)

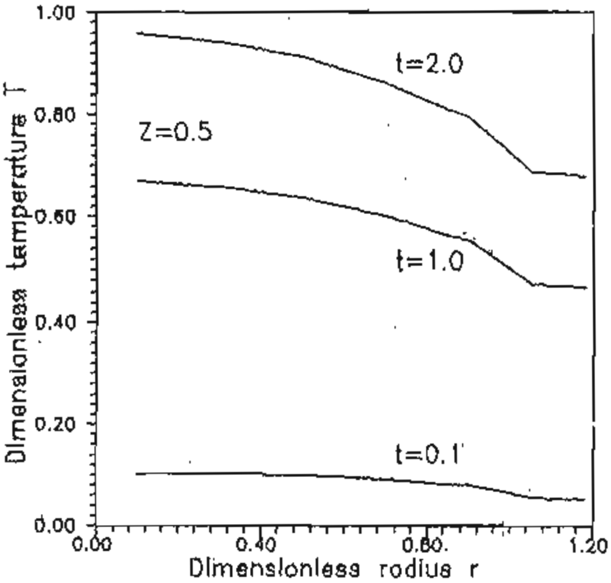


Fig.(6) Time development of radial temperature pofile (Bi=0.1)

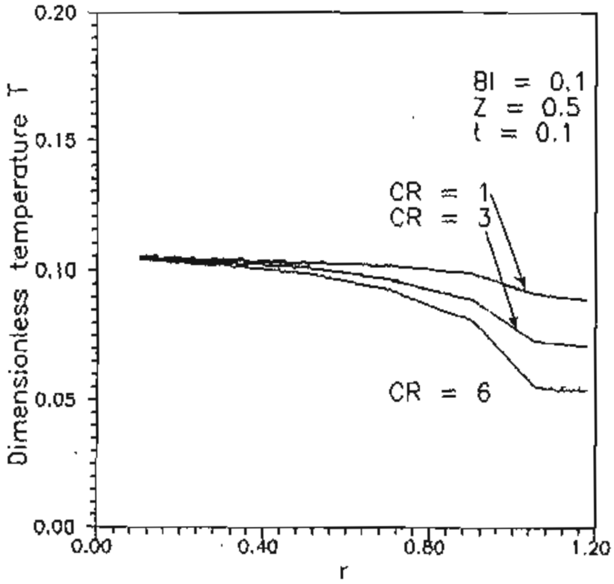


Fig.(7) Effect of conductivity ratio on temperature profile

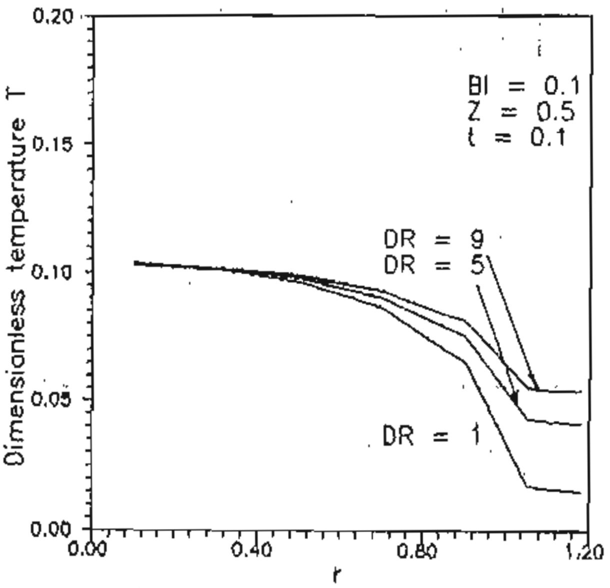


Fig.(8) Effect of diffusivity ratio on temperature profile

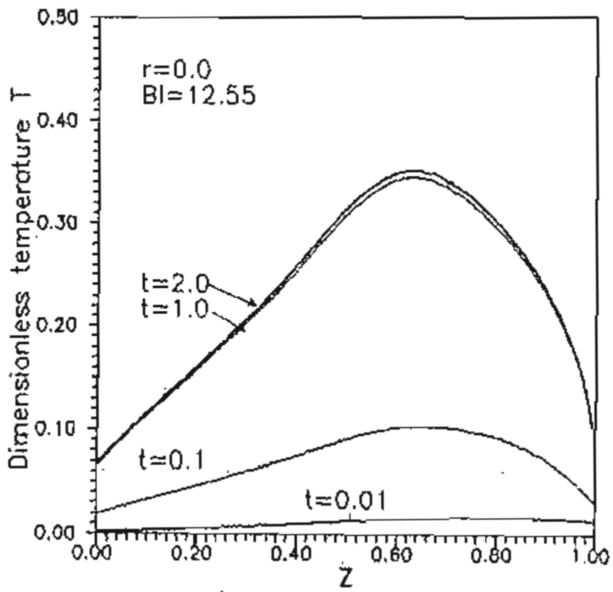


Fig.(9) Time development of axial temperature pofile

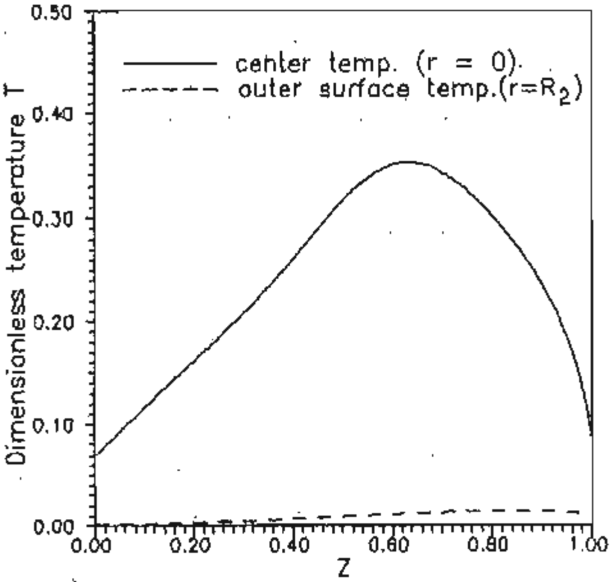


Fig.(10) Axial temperature distribution (t = 2.0, Bi = 12.55)

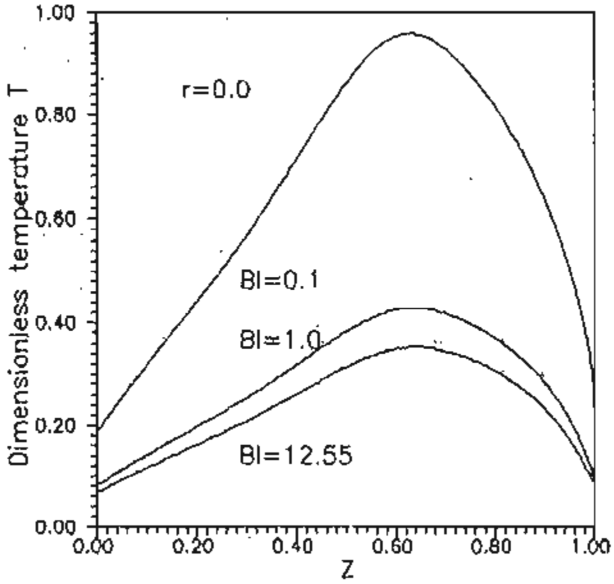


Fig.(11) Effect of Biot number on axial temperature pofile (t = 2.0)

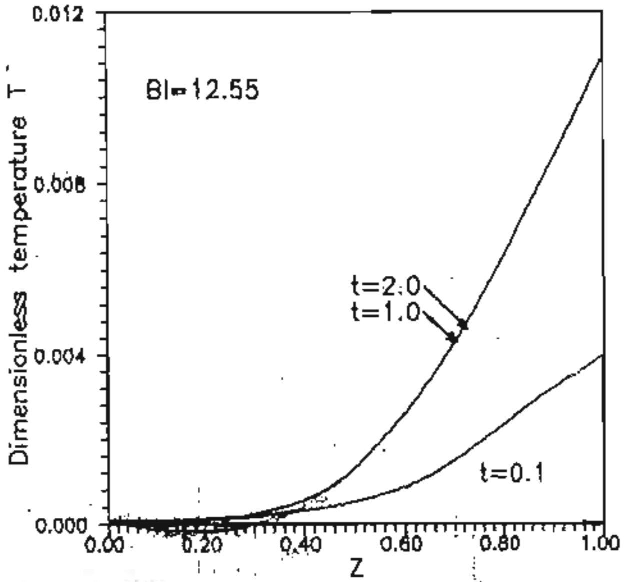


Fig.(12) Time development of temperature pofile of the cooling fluid

APPENDIX

Steady-state analytical solution of the 1-D heat conduction in composite cylinder with internal heat generation

Thermal and hydraulic specifications of a KWU 1300 MWe PWR [13]

Heat generation rate at $z = L/2$ is $q''_c = 4.7\times10^{8}$ W/m^{3},

Inlet temperature T^*_{in} = 291 °C,

Mass flow rate m = 332×10^{-3} Kg/s,
Specific heat of coolant cp = 5.5 KJ/Kg.K,
Fuel is UO_2,

k_1 = 2.5 W/m.K, $\alpha_1 = 8.28\times10^{-7}$ m^2/s, $R_1 = 4.025\times10^{-3}$ m,

Cladding is Zircaloy-4,

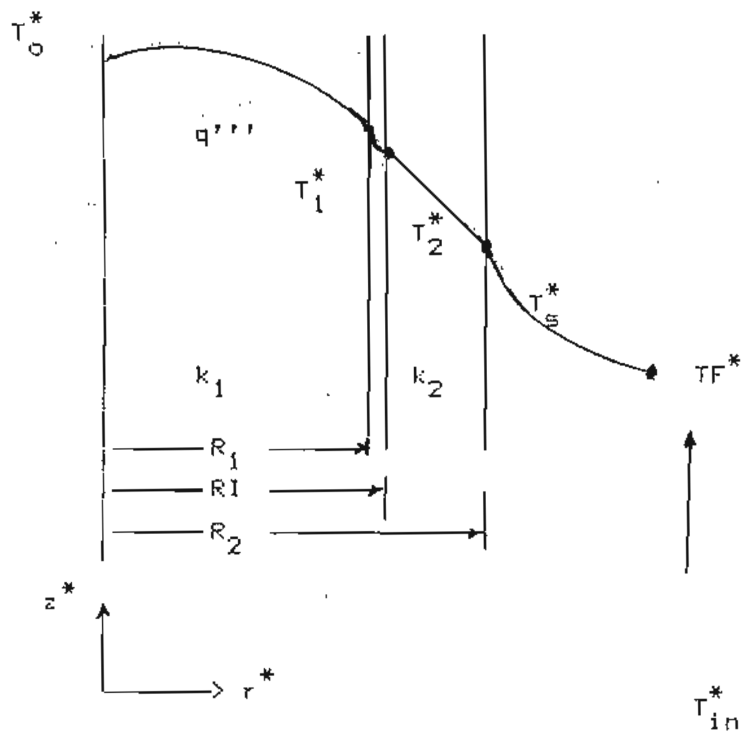
k_2 = 15.13 W/m.K, $\alpha_2 = 7.538\times10^{-6}$ m^2/s,

$RI = 4.11\times10^{-3}$ m, $R_2 = 4.75\times10^{-3}$ m,

Fuel rod active height L = 3.9 m,
Heat transfer coefficient in the gas gap h_g = 4000 W/m^2,

The heat transfer coefficient along the cooling channel h = 40000 W/m^{2}K as calculated using following Sieder-Tate correlation [5,11]

$$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}\ \left(\frac{\mu_w}{\mu}\right)^{0.14}$$



Considering the composite cylindrical system described in the above figure, under steady-state condition the centerline temperature T_o^* is given by:

$$T_o^* - TF^* = (q'''R_1^2/4k_1) + (q'''R_1/2h_g) + \frac{q'''R_1^2}{2}\left[\frac{1}{k_2} LN(R_2/RI) + \frac{1}{h R_2}\right] \quad (1)$$

where $q''' = q_c''' \sin(\pi z/L)$ (2)

Substituting for values of q_c''', R_1, RI, R_2, k_1, k_2, h_g, and h, we get

$$T_o^* - TF^* = 2.2433\times10^{-6}\ q''' = 1054 \text{ degree} \quad (3)$$

To determine TF^* consider the relation:

$$TF^* - T_{in}^* = \frac{L\ R_1^2\ q_c'''}{m\ cp}\left[1 - \cos(\pi z/L)\right] \quad (4)$$

At $z = L/2$, we get $TF^* - T_{in}^* = 20$ degree (5)

From Eqs.3 and 5, we get $T_o^* = 1074 + T_{in}^*$ degree (6)