

Post graduate Exam (Basic Engineering Sciences)
Branch: Engineering Mathematics (Master 500)

Menoufia University
Faculty of Engineering
Shebin El-Kom

Academic Year: 2016-2017

Department: Basic Eng. Sci.



Subject: Basic Topics in Algebra

Code : BES 502

Time Allowed: 3 hours

Date: 31 / 5 / 2017

Max Marks: 100

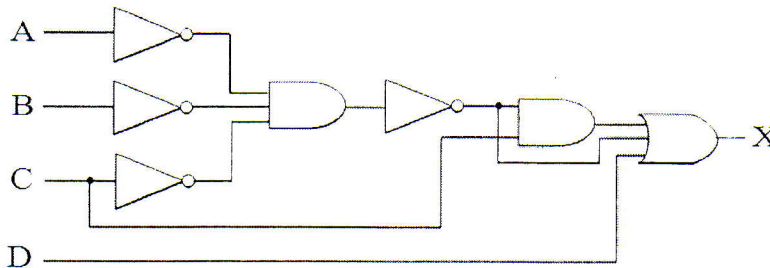
Answer all the following questions:

- Q.1** (A) Define the following: Vector space, Span, Basis, and Orthonormality.
(B) Classify tensors according to its order, and compare between symmetric and anti-symmetric tensors. then Show that any tensor can be represented by a linear combination of symmetric and anti-symmetric parts.
(C) Expand the equations: $A_{ij}x_i x_j$ ($i, j = 1, 2, 3$)
(D) State the definition of a group, then show that each following set or subset forms a group or not:
1. The subset $\{1, -1, i, -i\}$ of the complex numbers under complex multiplication.
 2. The set of even integers under addition.
 3. The set of odd integers under addition.
 4. The set of integers under subtraction.

[Q.1 (40 mark)]

- Q.2** (A) Apply DeMorgn's theorem to the following expression:
$$\overline{A + B\bar{C} + D(E + \bar{F})}$$

(B) Simplify $\overline{AB + \bar{A}B + AB}$.
(C) Reduce the combinational logic circuit in the following figure to a minimum form.



[Q.2 (30 mark)]

- Q.3** (A) Prove that Eigenvectors for a real symmetric matrix which belong to different eigenvalues are necessarily orthogonal.

(B) Consider $A = \begin{pmatrix} 3 & -2 \\ 0 & 3 \\ 4 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$ and $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

1. Find the QR factorization of Matrix ,
2. Solve for $\min \|A X - B\|$.

(C) Consider $A = \begin{pmatrix} 1 & 6 & 2 \\ 2 & 12 & 5 \\ -1 & -3 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 9 \\ -4 \\ 17 \end{pmatrix}$ and $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

- i) Show that A does not have an LU decomposition ,
- ii) Re-order the rows of A and find an LU decomposition of the new matrix ,
- iii) Hence solve $A X = B$ by applying LU decomposition.

[Q.3 (30 mark)]

Good Luck