# PROBABILISTIC MODEL FOR FIELD CONCRETE STRENGTH

## استخدام النماذج الاحتمالية لقياس جودة الخرسانة المصبوبة في مواقع الإنشاء

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**ملخص البحث:-** هذا البحث يتعلق بتقييم وحساب جودة مقاومة الخرسانة المصبوبة في مواقع الإنشـــاء فـــي نطـــاق محافظة الدقهلية خلال الأعوام من عام ١٩٩٤ حتى ٢٠٠٠ وعينات من الخرسانة ببعض المحافظات المجاورة. الـهدف الرئيسي من البحث هو دراسة التغيرات في مقاومة الخرسانة المنتجة في العديد من مواقع الإنشاء المختلفة. باســـتخدام النموذج الاحتمالي (Probabilistic Model) لنتائج اختبار الضغط على المكعبات الخرسانية المصبوبة بـــالمواقع والمختبرة في معمل مقاومة المواد بجامعة المنصورة لوصف التغيرات فى مقاومة الضغط بين المواقع المختلفـــة. تـــم تجميع ٨٥٩ عينة خرسانية بطريقة عشوائية من مواقع الإنشاء المختلفة داخل حدود منطقـــة البحـــث لاختبارهـــا فـــي الضغط. أظهرت النتائج أن العينات الخرسانية أعطت تطابق مع التوزيع الطبيعي. وكذلك أظهرت نتـــائج التحليـــل أن معاملات الاختلاف كبيرة مقارنة بالحدود العادية. كما أوضحت النتائج ان النسبة بين متوسط الإجـــهاد للعينـــات إلـــى الإجهاد التصميمي تتراوح بين ٠,٩٩ – ١,٠٥ وكان معامل الاختلاف يتراوح بين ٢٧,٤٧% – ٣٥,٨١% معتمدة على مواقع اخذ العينات. تم التحقق من النموذج المستخدم باستخدام الاختبارات الإحصائية اللابارامترية ( $\chi^2$ ، كلوموجروف – سمي‌رنوف ) عند مستوى دلالة ٥% . النموذج المستخدم فى هذا البحث يساعد على التنبؤ بمســـتوى أداء العنـــاصر الإنشائية كما يخدم أيضا مستويات الثبات.

## ABSTRACT

The paper deals with the evaluation of field concrete strength placed on construction sites in Dakahilia, Egypt in the years 1994 – 2000. The main purpose of this research was to study the variability of concrete strength produced in different location on sites. Using the information of cube compressive strength data, probabilistic models have been developed to describe the variability of concrete compressive strength between locations. A total 859 concrete samples were randomly collected from construction sites for strength testing at strength of material laboratory, Mansoura University. The result indicate that concrete is well modeled by the normal distributions. Also, the results of the analysis showed the coefficient of variation are higher for this study than normal. The mean –to-nominal strength ratio varies between 0.99 and 1.05, whereas the coefficient of variation is in the range of 27.47% – 35.81%, depending on the location of concrete. The models are verified by $\chi 2$ and Kolmogorov – Smirnov goodness of fit tests at the 5% significance level. The models developed in this paper are useful for predicting the performance of structural element and assessing their reliability levels.

**KEYWORDS:** Compressive strength; Controls; Probabilistic models; Statistics; Reliability; Probability; Coefficient of variation; Normal probability paper.

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## INTRODUCTION

Structural reliability theory has a several important applications in the field of civil engineering. The uncertain quantities, that is loads, material properties and dimensions, are modeled as random variables (1-3). These quantities are fully described by probability distribution function for one quantity or by joint probability function for all quantities. Generally, these probability density functions are rarely available, and it is common to describe these quantities by their main descriptors as follows: mean values, standard deviations, and by coefficients of correlation between two random variables. The variability of concrete strength depends on the quality control of the concreting operations (4). A review of literature (5, 6, 7) indicates that the coefficient of variation of field-cast laboratory-cured specimens is in many cases, between 15 and 20%, which suggests that 20% is a reasonable maximum value for average controls. Korzekwa and Mames (8) found the results of coefficient of variation of factory-cast laboratory-cured specimens to range from 16 to 20 % for average control and to be smaller than 15% for above average quality control of concrete.
This paper presents probabilistic models of the compressive strength of concrete produced in Dakahilia governorate, Egypt. The models are verified by $\chi^2$ and Kolmogorov-Simirnov goodness of fit tests as the 5% significance level. The models developed are essential for predicting the variation in the performance of structural elements and assessing their reliability levels.
Dakahilia was divided to five location, namely East (E-28), West (W-28), Middle M-28, south (S-28), North (N-28). Moreover, some samples was collected from outside Dakahilia (O-28). One age of concrete, 28 days, was used from the construction sites in Dakahilia, in the years 1994 – 2000, are tested. 859 concrete cube was taken from different sites and location and tested at strength of material laboratory, Mansoura University.

## EXPERIMENTAL PROGRAMME

The main purpose of the experimental programme was to develop probabilistic models of the compressive strength of concrete in Dakahilia, Egypt.
Concrete samples were randomly collected from different sites throughout Dakahilia. A total of 859 concrete samples were collected from construction sites after the curing process. At each site, three concrete sample were taken from one batch. Each sample comprised three standard cube (150 × 150 × 150 mm), which after 24 hours were cured in site and transferred to laboratory at 28 days to testing for compressive strength. The concrete samples were classified according to ages (28 days).

## PROBABILISTIC ANALYSIS

The probabilistic models (9-10) for the strength of the two ages of concrete were developed by the following four main steps

1. The reliability and homogeneity of data were checked
2. The data were plotted on normal probability forms.
3. A linear regression analysis was performed.
4. The validity of the developed models was determined.

*Reliability and homogeneity data*

In order for test results to provide representative probabilistic models of the distribution functions, they must be reliable and form a homogeneous set. Concrete samples were collected from construction sites after curing. Three cubes were tested from each sample in order to check reliability of the test process. Theoretically, the strength of the two cubes should be identical. If the difference between the results for the three cubes exceeded 10% of

the strength, human error in the preparation and testing of the cube had likely occurred and the results was rejected.

*Normal probability papers*

Normal probability paper (*NPP*) is a special form on which the normal cumulative distribution function is represented by straight line. This type of form is usually employed when modeling normal and log-normal distribution functions. The relation between the two models can be stated as follows: if a parameter has log-normal distribution its natural logarithm has a normal distribution (11, 12)

*Regression analysis*

A regression analysis was performed between the strength parameter x and the inverse of the standard normal distribution function z. An equation for the line of the best fit can be developed using the well-known least-squares method:

$$X = az + b \qquad (1)$$

Where a and b are constant, which are calculated using linear regression analysis on NPP.

*Model Validation*

The validity of the models obtained was checked using $\chi^2$ and Kolmogorov - Smirnov goodness of fit tests as the 5% significance level. The statistics $D_1$ and $D_2$ for the $\chi^2$ and Kolmogorov - Smirnov goodness of fit tests, respectively, were calculated as follows: (8-9)

$$D_1 = \sum_{i=1}^{k}\left[\frac{(Oi - Ei)^2}{Ei}\right] \qquad (2)$$

Where $O_i$ and $E_i$ are the observed and expected number of occurrences in the $i$th interval, respectively, and k is the number of intervals; and

$$D_2 = \max_{i=1}^{n}\left[\left|\frac{i}{n} - Fx\left(X^{(i)}\right)\right|\right] \qquad (3)$$

$D_2$ is the largest of the absolute values of the n differences between the hypothesized distribution function $F_x(X^{(i)})$ and the observed cumulative histogram $i/n$.

The values of $D_1$ and $D_2$ thus obtained were compared with the corresponding critical values $D_{1c}$ and $D_{2c}$ at the 5% significance level.

## RESULTS, ANALYSIS AND DISCUSSION

The modeling process was performed for the six location of concrete samples. (Figs. 1-6). The probabilistic models of the variation of concrete strength are listed in Table 1. The models were checked using $\chi^2$ and Kolmogorov – Smirnov goodness of fit test at 5% significance level. Table 2 show the results of the $\chi^2$ test for concrete at different location.

Df equals the number of categories minus one. Small significance values (<.05) indicate that the observed distribution does not conform to the hypothesized distribution. In the present work, the significance level is greater than .05, which indicates that the models of the variation in strength listed in Table 2. are acceptable at 95% confidence level. The distribution of compressive strength does not differ from the distribution hypothesized.

Table 1. Strength distribution models and their statistics for concrete

| Location | Distribution type | Distribution function | λ | $V_c$ % | P % | count |
|---|---|---|---|---|---|---|
| E-28 | Normal | X = 31.411 + 8.63z | 1.04 | 27.47 | 17.765 | 162 |
| W-28 | Normal | X = 31.792 + 8.628z | 1.05 | 27.13 | 17.952 | 93 |
| M-28 | Normal | X = 30.418 + 8.512z | 1.01 | 27.98 | 17.729 | 211 |
| N-28 | Normal | X = 29.94 + 10.332z | 0.99 | 34.5 | 15.076 | 126 |
| S-28 | Normal | X = 30.534 + 9.19z | 1.01 | 30.09 | 14.067 | 150 |
| O-28 | Normal | X = 30.322 + 10.861z | 1.01 | 35.81 | 11.133 | 117 |

Table 2. Results of $\chi^2$ test for concrete

| Location | E-28 | W-28 | M-28 | N-28 | S-28 | O-28 |
|---|---|---|---|---|---|---|
| Chi | 72.716 | 17.00 | 125.256 | 27.81 | 46.333 | 37.744 |
| Df | 96 | 65 | 109 | 94 | 94 | 70 |
| Asymp Sig. | 0.963 | 1.00 | 0.137 | 1.00 | 1.00 | 0.999 |

The Kolmogorov-Smirnov test compares an observed cumulative distribution function to a theoretical cumulative distribution. The theoretical distribution can be normal, uniform, or Poisson. In this work, the normal distribution is selected. Parameters of the theoretical distribution are estimated from the observed data. Absolute indicates the largest absolute difference between the theoretical cumulative distribution and the observed cumulative distribution function. Large significance values (>0.05) indicate that the observed distribution corresponds to the theoretical distribution.

In this work, the significance level is greater than 0.05, as shown in Table 3. Thus the distribution of compressive strength resembles a normal distribution.

Table 3. Results Kolmogorov – Smirnov test for concrete

| Concrete location | E-28 | W-28 | M-28 | N-28 | S-28 | O-28 |
|---|---|---|---|---|---|---|
| Most Extreme Differences | | | | | | |
| Absolute | 0.077 | 0.067 | 0.039 | 0.061 | 0.037 | 0.083 |
| Positive | 0.068 | 0.067 | 0.039 | 0.061 | 0.032 | 0.047 |
| Negative | -0.077 | -0.052 | -0.023 | -0.046 | -0.037 | -0.083 |
| Kolmogorov – Smirnov | 0.986 | 0.645 | 0.573 | 0.688 | 0.458 | 0.893 |
| Asymp Sig. | 0.285 | 0.8 | 0.898 | 0.732 | 0.985 | 0.402 |

The mean / nominal strength λ are 1.04, 1.05, 1.01, 0.99, 1.01, 1.01 for concrete sample E-28, W-28, -M-28, N-28, S-28, O-28 respectively; λ may be constant or rarely changed at constant nominal strength.

The coefficient of variation $V_c$ for concrete E-28, W-28, -M-28, N-28, S-28, O-28 are 27.47%, 27.13%, 27.98%, 34.5%, 30.09%, 35.81% respectively. Concrete produced at S-28, O-28 is a poor quality compared with other location in this study, which adversely affects the capacity and durability of structures containing it.

The six percentiles of the distribution functions are much smaller than the nominal strengths. This can be attributed to poor quality control in the production process and may be change in the concrete proportions resulting from extra water.

## CONCLUSION

The experimental programme undertaken to estimate the statistical characteristics and probabilistic models of concrete in Dakahilia, Egypt for reliability and risk analysis are reported. The results indicate that concrete has normal distributions. The results obtained can be used to predict the statistical characteristics of structural elements and to calculate the partial safety factors. From the results, coefficient of variation of the strength of six locations was observed high, indicating the poor quality. Also The six percentiles of the distribution functions are much smaller than the nominal strengths, indicating the same poor quality.

## REFERENCES


1- Comite Euro-International DU Beton CEB-FIP Model Code 1990 – Design code Thomas Telford, London, 1993.

2- American Society of Civil Engineering. "Minimum design loads for buildings and other structures, ASCE, New York, 1993, Standard 7-93.

3- British Standards Institution, "Design of concrete structures Part 1: general rules and rules for buildings BSI, Milton Keynes, 1992, Eurocode 2.

4- Chmielewski T., Konopka E., " Statistical evaluations of field concrete strength". Magazine of concrete research, 1999, 51, No. 1, Feb., 45 -52.

5- Mirza S. A., Hatzinikolas M., Maggregor J. G., "Statistical descriptions of strength of concrete", Journal of the structural Division of the ASCE, 1979, 1021-1037.

6- Borcz A. and Jankowiak R., "Quality control of concrete produced in factories in the light and standard draft "concrete", Inzynieriai I Budownictwo, 1975, No. 1

7- M.M Kamal, A,H.A Abdel Reheem, " Quality control between theory and practice in building industry", Mansoura Engineering Journal, Vol. 12, No. 2, Dec. 1987.

8- Korzekwa S. T. and Mames J, "Appraisal of Concretes quality produced in factories", Przeglad Budowlany, 1976, No. 5.

9- Stewart M. G., "Workmanship and its Influence on Probabilistic Models of Concrete Compressive strength", ACI material Journal, July – August, 1995, 361- 381.

10-Rossi P., Wu X., "Probabilistie model for material behaviour analysis and appraisement of concrete structures", Magazine of concrete research, 1992, 44, No. 161, Dec. , pp. 271-280.

11-Stephen B. V., "Statistics for Engineering Problem Solving", 1994, PWS Publishing Company, USA.

12-Montgomery, Douglas C., "Applied Statistics and probability for Engineeris", second edition, 1999, John wiley & Sons, Inc., USA.


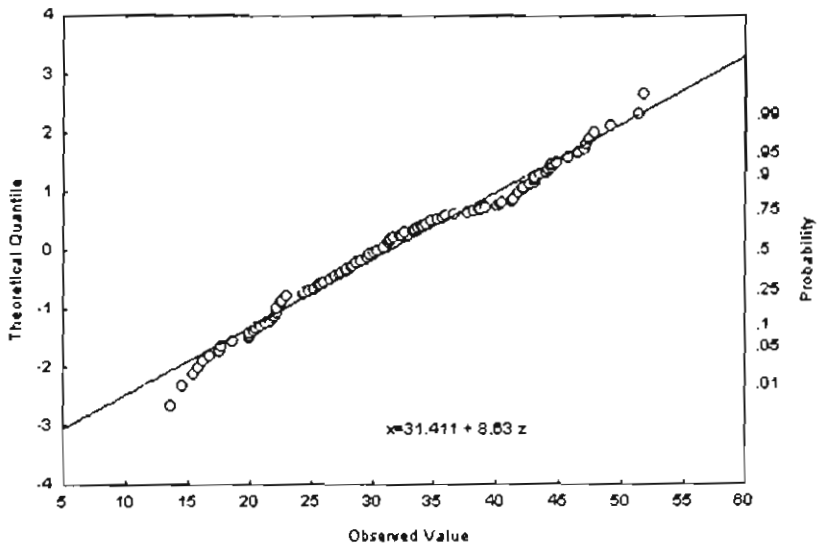


Fig. 1 Normal distribution function of the compressive strength (MPa) in concrete sample E-28

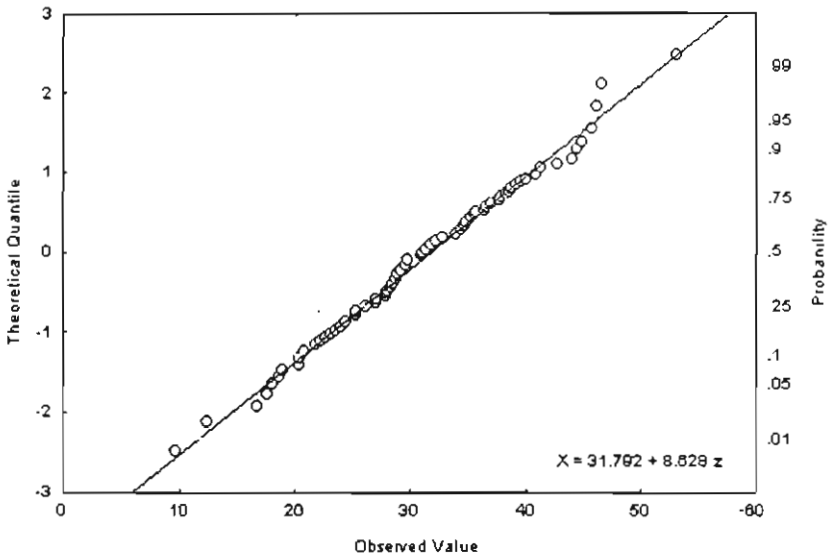


Fig. 2 Normal distribution function of the compressive strength (MPa) in concrete sample w-28

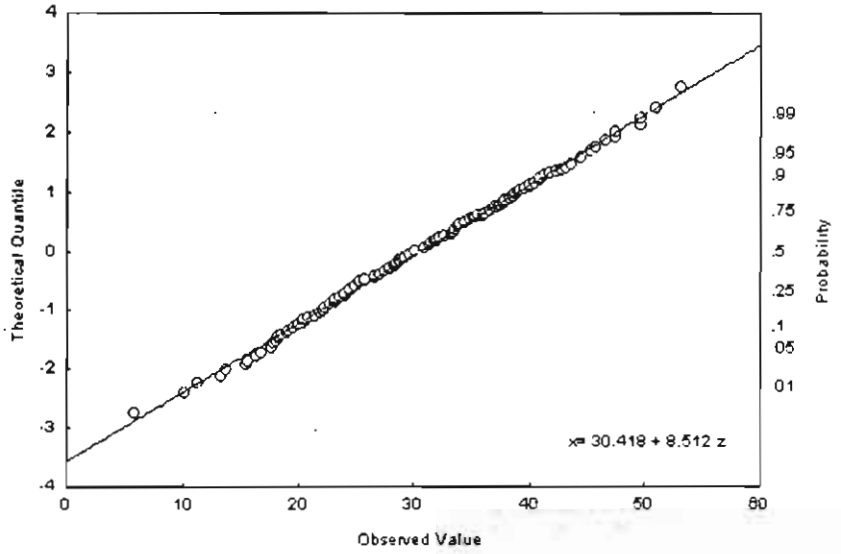


Fig. 3 Normal distribution function of the compressive strength (MPa) in concrete sample M-28

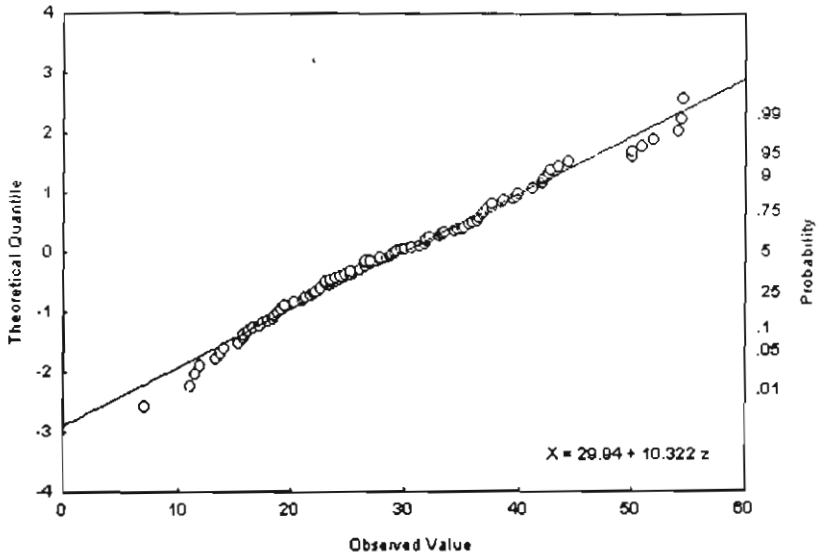


Fig. 4 Normal distribution function of the compressive strength (MPa) in concrete sample N-28

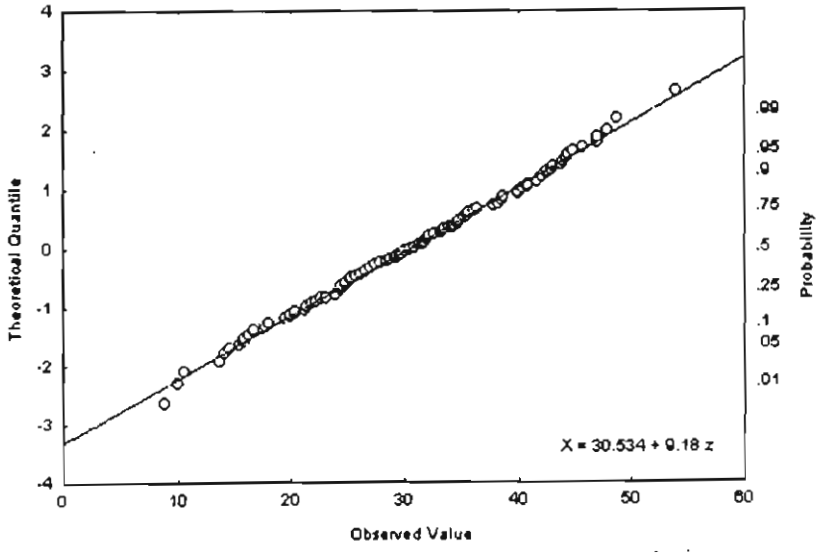


Fig. 5 Normal distribution function of the compressive strength (MPa) in concrete sample S-28

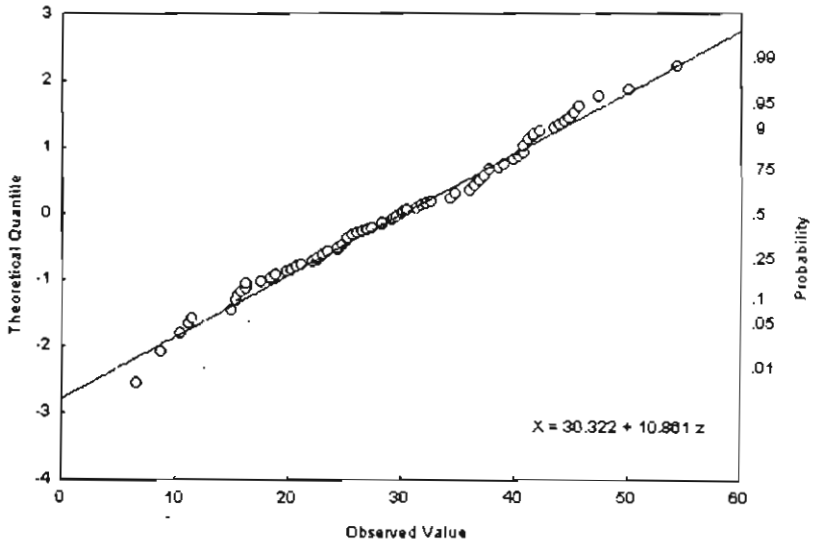


Fig. 6 Normal distribution function of the compressive strength (MPa) in concrete sample O-28