On The Estimation of Parameters for the Inverse Weibul Distribution

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Abstract:

In this paper, we use the least absolute deviation and Newton Raphson methods, to estimate the parameters of inverse Weibull (IW) distribution, also we compare between these methods by using mean absolute percentage error of the reliability function, numerical example is given to clarify the use of these methods.

Introduction:

The two parameters inverse Weibull distribution

$$f(t) = B\alpha^{-B} t^{-B-1} exp\left[-1/(\alpha t)^B\right] \qquad t \geq 0 \qquad \alpha, B \rangle 0 \qquad (1)$$

is used as a model in the analysis of life testing data, it is also suitable model to describe mechanical degration phenomena. The probability of failure before time t is given by the inverse Weibull commutative distribution function defined by:

$$F(t) = exp\left[-1/(\alpha t)^B\right] \qquad (2)$$

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In R. Calabria and Pulcini (1989) the statistical properties of the ML estimators of the parameters β and α, and the reliability function R(t) for a complete I.W. sample have been investigated. Tables of lower confidence limits for R(t) have been provided. In P. Erto (1989) the estimators for β, α and R(t) obtained by using the plotting positions technique, also the L. S. estimators and their statistical properties have been provided. R. Calabria and least squares methods to estimate the inverse Weibull distribution parameters. In Calabria and Pulcini (1994), Bayes prediction intervals are derived, both when no prior information is available and when prior information on the unreliability level at a fixed time is introduced.

1. Least absolute deviation method, (LAD):

The reliability function of (1) is given by:

$$R(t) = 1 - e^{-(\alpha t)^{-\beta}} \tag{3}$$

$$= 1 - e^{-\theta t^{-\beta}}$$

$$e^{-\theta t^{-\beta}} = 1 - R(t) \tag{4}$$

Taking the logarithm of both sides of (4)

Ln θ - B Ln t = Ln (- Ln [1- R(t)])

Or:

$$\text{Ln } t = \frac{\text{Ln}\theta}{\beta} - \frac{1}{\beta}\text{Ln }(-\text{Ln}[1 - R(t)])$$

$$y_i = B_o + B_1 X_i \tag{5}$$

Where $y_i = \text{Ln } t_i$, $\quad B_o = \frac{\text{Ln}\theta}{\beta}$ $\quad$ and $\quad B_1 = \frac{-1}{\beta}$

And $\quad x_i = \text{Ln }(-\text{Ln }[1- R(t)])$, $\quad \theta = \alpha^{-\beta}$

Consider the linear regression problem in the form of n observations (y_i, X_i), for i= 1, 2, ...,n. the regression model is given by (5), where B_o and B_1

are the parameters, the problem is to find $\hat{\beta}_o$ and $\hat{\beta}_1$ estimations of β_o and β_1 so that the dependent variables y_i cause predicted as

$$\hat{y}_i = \hat{\beta}_o + \hat{\beta}_1 X_i \qquad (6)$$

Also, the problem is to find the values of the parameters β_o and β_i, which will minimize the least absolute deviations (LAD) so that the problem is

$$\mathbf{Min.} \sum_{i=1}^{n} \left| y_i - (\beta_o + \beta_1 X_i) \right| \qquad (7)$$

Problem (7) have been transferred to linear programming formulation by Charnes, Cooper and Ferguson (1955), so the problem (7) is equivalent to:

$$\text{Minimize } \sum_{i=1}^{n} (P_i + N_i)$$

Subject to

$\beta_o + \beta_i X_i + P_i - N_i = y_i$, $\quad i=1, 2, \ldots\ldots, n.$

$P_i \geq 0$, $\quad N_i \geq 0$, $\quad i=1, 2, \ldots.., n.$

Where P_i, N_i, are the positive and negative errors and the problem is to find the values of β_o and β_1 that minimize the errors.

The standard tableau form of this problem is shown in the table (1)

Table (1): The LAD problem in tableau form

P1	P2	Pn	N1	N2.....	Nn	B0	B1	RHS
1			-1			1	X1	Y1
	1			-1		1	X2	Y2
						:	:	:
						:	Xn-1	Yn-1
		1			1-	1	Xn	Yn
1	1	1	1	1	1	0	0	Z(min)

2. Newton Raphson method (NR):

The Newton Raphson method is one algorithm for minimizing the sum of the residuals between data and nonlinear equations Newton's method for system uses the nxn Jacobian matrix in the vector situation and substitute by the derivative with the inversion of the Jacobian matrix, this method for finding the solution θ to the nonlinear system of equations represented by the vector equations.

$F(x) = 0$, has the form

$\theta^{k} = \theta^{k-1} - [J(\theta)k-1]^{-1} F(\theta^{k-1})$

For $K \geq 1$

Given the initial approximation $\theta^{(o)}$ to the solution θ, the initial values of θ can be assumed to be zero. Where $J(t)$ is the first order derivative of $R(t)$ in the matrix form given by:

$$J(t)=\begin{bmatrix} \frac{\partial R_{1(t)}}{\partial \alpha} & \frac{\partial R_{i}(t)}{\partial \beta} \\ \frac{\partial R_{2}(t)}{\partial \alpha} & \frac{\partial R_{2}(t)}{\partial \beta} \end{bmatrix}$$

$$R(t) = 1 - e^{-(\alpha t)^{-B}}$$

$$\frac{\partial R(t)}{\partial \alpha} = B\,\alpha^{-B-1} t^{-B} e^{-(\alpha t)^{-B}}$$

$$\frac{\partial R(t)}{\partial \alpha} = \ln(\alpha t)\ (\alpha t)^{-B}\ e^{-(\alpha t)^{-B}}$$

and $\frac{\partial R(t)}{\partial a_k}$ is the particle derivative of the function $R(t)$ with respect to the K^{th} parameter evaluated at the i^{th} data point.

To compute the accuracy of estimates of parameters and reliability function we use the mean absolute percentage error (MAPE) defined by

$$\text{MAPE} = \frac{1}{n}\sum|Z_t| \qquad (8)$$

Where

$$Z_t = \left(\frac{R(t) - R_e(t)}{R(t)}\right) * 100$$

And

R(t) is actual reliability function. $R_e(t)$ is the estimated reliability function

Numerical Example:

Consider the following 10 simulated data from an inverse Weibull distribution with parameters $\alpha = 0.01$ and $\beta = 2$ listed in the following table (4)

Table (1):

55.3	66.9	87.0	112.5	118.4
129.1	141.5	167.1	245.4	335.5

The maximum likelihood estimates are $\hat{\alpha} = 0.0102$ and $\hat{\beta} = 2.152$

The values of x_i and y_i in equation (6) are given by the table (2)

Table (2)

x	y
1.18	4.01
0.8	4.2
0.28	4.47
-0.24	4.72
-0.34	4.77
-0.51	4.86
-0.69	4.95
-3.01	6.11
-1.8	5.5
-2.42	5.82

The corresponding tableau of the tableau (1) is given by

Table (3):

P1	P2	Pn	N1	N2.....	Nn	B0	B1	RHS
1			-1			1	1.18	4.01
	1			-1		1	0.8	4.2
						1	0.28	4.47
						1	-0.24	4.72
		1			1-	1	-0.34	4.77
						1	-0.51	4.86
						1	-0.69	4.95
						1	-3.01	6.11
						1	-1.8	5.5
						1	-2.42	5.82
1	1	1	1	1	1	0	0	Z (min)

The values of β_0 and β_1 are computed with the aid of the computer program (1), then $\hat{\alpha}$ and $\hat{\beta}$ can be obtained.

The estimates $\hat{\alpha}$, $\hat{\beta}$ from Newton Raphson method are obtained after 4 iteration by the computer program (2), as shown in table (4).

The reliability estimates R_1, R_2, R_3 and R_4, are computed from relation (3) at different values of the estimated parameters $\hat{\alpha}$ and $\hat{\beta}$, while MAPE is computed from the relation (8), as shown in table (5)

Table (4) The Parameters Estimates

Method	No outliers		Outliers	
	α	β	α	β
LAD	0.0102	19912	0.0101	1.9952
NR	0.1553	1.704	0.1668	1.811

Table (5) Reliability Estimates

T	True R(t)	R(t)			
		R1	R2	R3	R4
1	0.96199	0.95616	0.02530	0.95910	0.01773
2	0.89294	0.88239	0.01835	0.88766	0.01259
3	0.73318	0.71876	0.01177	0.72592	0.00784
4	0.54621	0.53210	0.00761	0.53931	0.00493
5	0.51000	0.49681	0.00698	0.10336	0.00410
6	0.45119	0.43903	0.00602	0.44107	0.00384
7	0.39313	0.38221	0.00516	0.38765	0.00326
8	0.30102	0.04697	0.00389	0.04759	0.00241
9	0.15300	0.14862	0.00202	0.11083	0.00120
10	0.08101	0.08270	0.00119	0.08387	0.00068
MAPE		2.2423	98.42947	1.1176	98.98219

R_1 is Reliability estimates from LAD in the absence of outliers

R_2 is the reliability estimates from Newton Raphson method in the absence of outliers.

R_3 is reliability estimate from LAD in the presence of outliers

R_4 is reliability estimates from NR in the presence of outliers.

Conclusion:

From table (4) the parameters estimates from least absolute deviation method are close to the true values and following table (5) the MAPE from least absolute deviation method are less than MAPE from Newton Raphson method. Also least absolute deviation method is easy to use, and takes little time on computer, also it is more accurate as compared with Newton Raphson method, because the MAPE from it is less than MAPE from the Newton Raphson method.

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Computer program (1)

```
      DIMENSION X(10), Y(10), B(10, 10), A (10,10) ELERR(10,10)
      OPEN (5, FILE = 'AM.DAT')
      OPEN (6, FILE = 'AM.OUT')
      N=10
      READ (5, *), X(I), Y(I), I=1,N)
      DO 10 J = J+1,N
      IF (X(I).EQ X(J)) THEN
      B(I,J) = 100000.0
      ELSE
      B(I,J) = (Y(J)-Y(I))/(X(J)-X(I))
      ENDIF
      A(I,J) = Y(I)-B(I,J)*X(I)
      SUM = 0.0
      DO 30 K = 1,N
30    SUM = SUM + ABS(Y(K)-A(I,J)+B(I,J)*X(K)))
      ELERR(I,J) = SUM
10    CONTINUE
      L = 1
      M= 2
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```
      AMIN=ELERR(1,2)
      DO 40I = 1, N-1
      DO40J = I+1,N
      IF (AMIN. LT. ELERR(I,J)) GO TO 40
      AMIN=ELERR(I,J)
      L=I
      M=J
40    CONTINUE
      WRITE(6,7) A(L,M), B(L,M)
7     FORMAT(2X,2F10.4)
      STOP
      end
```

Computer program (2)

```
      PARAMETER (N=10, NITAMAX=5)
      DOUBLE PRECISION T(N), V(N), VM(N), E(N), W(N), ER(N)
      DOUBLE PRECISION AA, BETA, Z
      LOGICAL OUTERR
      OPEN (5, FILE = 'WI3.DAT')
      OPEN (6, FILE = 'WI3.OUT')
      DO 1 i =1, 10
1     READ(5, *) T(I), V(I)
      WRITE (6, *)'INPUT THE INITIAL VALUES     I     T     V'
      DO 2 I = 1, 10
      WRITE (6, 202) I, T(I), V(I)
      WRITE (6, *) 'GAUSS NEWTON METHOD'
      DO 30 I = 1, N
      W(I) = 1.0
30    CONTINUE
      CALL REGRES (W, T, V, AA, BETA, N)
      DO 40 NITER = 1, NITMAX
      CALL GAUSNW (AA, BETA, T,E,N)
      OUTERR = (NITER. EQ. NITMAX)
      CALL CALERR (AA, BETA, V, T, N, VM, W, E, Z, OUTERR)
      WRITE (6, 100) 'AA=' , AA, 'BETA=', BETA, 'Z=', Z
40    CONTINUE
      STOP 'END OF THE PROGRAM'
100   FORMAT (/, 1X, 3(3X, A, G12.4))
101   FORMAT (2F10.4)
200   FORMAT (I3,2x, 2F10.4)
201   FORMAT (/1X, 70(' ') //1X, A/)
202   FORMAT (I5, I10, F20.4)
      END
      DOUBLE PRECISION FUNCTION DEALPA (AA, BETA, T)
      DOUBLE PRECISION AA, BETA, T
      DEALPA=DEXP(-(AA*T)**(-BETA))*BETA(AA*(-BETA-1))*
      (T**(-BETA))
      RETURN
```

```
      END
      DOUBLE PRECISION FUNCTION DEBETA (AA, BETA, T)
      DOUBLE PRECISION AA, BETA, T
      DEBETA=DEXP (-(AA*T)**(-BETA))*DLOG(AA*T)* (AA*T) ***(BETA)
      RETURN
      END
      SUBROUTINE GAUSNW (AA, BETA, T, E, N)
      DOUBLE PRECISION AA, BETA, T(N), E(N)
      DOUBLE PRECISION DELTAA, DELTAB, DET
      DOUBLE PRECISIO a(2,2), B(2)
      INTEGER  N
      I = I +1
      DO 11 J = 1, 2
      B(J) = 0.0
      DO 11  K = 1, 2
      A(J,K) = 0.0
11    CONTINUE
      I = I + 1
      DO 20  I = 1, N
      A(1, 1) = A(1,1) + DEALPA (AA, BETA, T(I)) **2
      A(1,2)=A(1,2)+DEALPA(AA,BETA,T(I))*DEBETA(AA,BETA,T(I))
      A (2,2)=A(2,2)+DEBETA(AA,BETA, T(I))**2
      B(1) = B(1)+DEALPA(AA, BETA, T(I))**E(I)
      B(2)=B(2)+DEBETA(AA, BETA, T(I))*E(I)
20    CONTINUE
      DET=A(1,1)*A(2,2)-A(1,2)*(1,3)
      DELTAA=-(A(1,2)*B(2)-A(2,2)*B(1))/DET
      DELTAB=(A(1,1)*B(2)-A(1,2)*B(1))/DET
      AA = AA + DELTAA
      BETA = BETA+DELTAB
      RETURN
      END
      SUBROUTINE CALLERR (AA,BETA,V,T,N,V,M,W,E,Z,OUTERR)
      PARAMETER (NMAX =10)
      LOGICAL OUTERR
      N=10
      DOUBLE PRECISION AA, BETA, T(N), E(N)
      DOUBLE PRECISION V(N), W(N), VM(N), ER(NMAX),Z
      INTEGER N,I
      IF (N.GT.NMAX) STOP 'CALERR: INCREASE NMAX VALUE'
      IF (OUTERR) WRITE (*, 100)
      Z= 0.0
      DO 12  I = 1, N
      VM(I) = DEXP (-(AA T(I))*BETA)
      E(I) = V(I)-VM(I)
      ER(I) = E(I)/V(I)
      Z=Z+E(I)/V(I)
      Z=Z+E(I) **2
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```
      IF (OUTERR) WRITE (6, 200) T(I), V(I), VM(I), E(I), ER(I)
12    CONTINUE
100   FORMAT (/T11, 'T', T25, 'V', T39, 'VM', T10, 'E', T65, 'ER'/)
200   FORMAT (3F14.4, 2E15. 4)
      RETURN
      END
      SUBROUTINE REGRES (W, T, V, AA, BETA, N)
      PARAMETER (NMAX = 10)
      LOGICAL OUTERR
      N = 10
      DOUBLE PRECISION AA, BETA
      DOUBLE PRECISION V(N), T(N), W(N)
      DOUBLE PRECISION A(2,2), B(2), C(2), DET
      INTEGER N, I
      A(1, 1) = 0.0
      A(1,2) = 0.0
      A(2,2)= 0.0
      B(1) = 0.0
      B(2) = 0.0
      DO  13  I =1, N
      A(1,1) = A(1,1) + W(I)
      A(1,2) = A(1,2) + W(I) * DLOG(T(I))
      A(2,2) = A(2,2) + W(I) * DLOG (T(I))**2
      B(1) = B(1) + W(I)*DLOG (DLOG(1/V(I)))
      B(2)= B(2) + W(I)*DLOG(DLOG(1/V(I)))*DLOG (T(I))
13    CONTINUE
      DET = A(1,1)*A(2,2) -A(1,2)*A(1,2)
      C(1) = (B(1)*A(2,2)-B(2)*A(1,2))/DET
      C(2) = (B(2)*A(1,1)-B(1)*A(1,2))/DET
      BETA=C(2)
      AA=DEXP (C(1)/C(2))
      RETURN
      END
```

ملخص البحث

تقدير معالم توزيع وابيل العكسى

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فى هذا البحث تم استخدام طريقة أقل الانحرافات المطلقة وطريقة نيوتن رافسون لتقدير معالم توزيع وابيل العكسى فى حالة وجود نقط شاذة وفى حالة عدم وجود نقط شاذة وقد تم عمل مقارنة بين هذه الطرق فى الحالتين وتبين الآتى من خلال مثال عددى:

١- باستخدام طريقة أقل الانحرافات المطلقة فإن قيم المعالم تكون قريبة جداً من القيم الحقيقية للتوزيع عن قيم المعالم المقدرة بطريقة نيوتن رافسون.

٢- طريقة أقل الانحرافات المطلقة تأخذ وقت قليل على الكمبيوتر عن طريقة نيوتن رافسون.

٣- متوسط الخطأ المئوى المطلق لدالة الاعتمادية فى طريقة أقل الانحرافات أقل منه فى طريقة نيوتن رافسون خاصة فى وجود نقط شاذة.