PERFORMANCE OF RECTANGULAR PLANE FIN

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أداء ريشة مستوية مستطيلة المقطع

الخلاصة:

يقدم هذا البحث دراسة لأداء ريشة مستوية مستطيلة المقطع تمتد من حائط مستوى. تم عمل برنامج حاسب آلى لحساب كمية الحرارة المفقودة من ريشة لها نسب أطوال ورقم بيوت Biot number مختلفة. ثلاثة ريش لها نسب أطوال ٥ ، ١٠ و ٢٠ تم دراستها. كل ريشة تم دراستها فى أربع حالات من رقم بيوت Biot number مختلفة هى ٠,٠١ ، ٠,١ ، ١ و ١٠ . تم تمثيل توزيع درجات الحرارة داخل الريش. كما تم تمثيل كفاءة الريشة مع رقم بيوت Biot number لعدة قيم لنسب الأطوال وعمل خريطة يمكن بواسطتها تعيين الأبعاد المثلى للريشة (الطول والسمك) اذا ما تم تعيين كل من درجة حرارة السطح ودرجة حرارة المائع المحيط بالريشة وكفاءة الريشة والخواص الحرارية للمادة المصنع منها الريشة. كما وجد أن الريش الأقل طولا وسمكا أكثر كفاءة.

ABSTRACT:

This investigation is concerned with the performance of a rectangular plane cooling fin at steady state conditions. A numerical approach is developed to calculate the heat transfer from a cooling fin for different values of Biot number and aspect ratios. Three fins of aspect ratios 5, 10, and 20 are studied. Each fin is examined for four cases of Biot number (Bi = 0.01, 0.1, 1.0, and 10). Temperature distribution inside the fins was obtained. Also a chart relating Biot number and fin efficiency with aspect ratio as a parameter was plotted. Thus, for specified values of fin efficiency; base temperature, fluid temperature, and thermal properties of the fin material; a chart relating fin efficiency versus Biot number with aspect ratio as a parameter can be represented, from which the optimum length and thicknees can be obtained. The study indicated that short and thin fins are more efficient.

1. INTRODUCTION:

Heat transfer can be cheaply and often very satisfactorily augmented by simply using radiating fins to extend the available surface area. There are many engineering applications in which radiating fins are used; radiating fins are employed in all air cooled reciprocating engines, they frequently appear on all forms of electrical machinery and component of electronic circuits and they are an essential feature of the modern compact heat exchangers. Radiating fins, also, play an important role in thermal control of spacecraft, space vehicles and satellites.

Since the weight and material cost are the major design considerations in heat exchangers, it is important to have minimum fin mass in these applications. Several investigators have analyzed the problem of minimization of fin mass in the past [1-8]. Relatively, a few work discussed the optimum design of radiating fin array considering mutual interactions between

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the radiator elements. Sparrow et al.[9] presented the optimum design of radiatively interacting longitudinal fins without considering the fin-to-base mutual irradiation. Later Sparrow and Eckert [10] emphasized the importance of mutual radiation interaction between fin and its base surface. Schnurr et al. [11] employed a nonlinear optimization technique to determine the minimum weight design for straight and circular fins of rectangular and triangular profiles, protruding from a cylinder, considering the fin-to-fin and the fin-to-base radiation reactions. Chung and Zhang [12] determined the optimum shape and minimum mass of thin fin accounting for fin-to-base interaction based on a variational calculus approach. Chung and Zhang [13] later extended their analysis to minimize the weight of a radiating straight fin array projecting from a cylindrical surface considering both fin-to-fin and fin-to-base interactions. Krishnaprakas [14] determined minimum mass design of a straight rectangular fin array extending from a plan wall considering fin-to-fin and fin-to-base radiation interactions. Correlations in terms of dimensionless numbers are presented to facilitate an easy design, i.e., to select the optimum fin length, thickness, and spacing. The performance of plane finned heat exchangers under dehumidifying conditions were studied by Wang et al. [15]. Systematic studies of continuous fin-and-tube tube heat exchangers under dehumidifying conditions were reported in their study. The heat exchangers consist of nine fin-and-tube heat exchangers having plane fins. The effects of fin spacing, the number of tube row, and inlet conditions were investigated. Data were presented in terms of the j and friction factors. It was found that the inconsistencies in the open literature may be associated with the wet fin efficiency. A correlation is proposed for the present plate fin configuration; this correlation can describe 92 percent of j, and approximately 91 percent of the f data within ±10 percent.

The purpose of the present investigation is to develop a new different approach for determining the performance of a straight rectangular cooling fin extending from a plane wall.

2. MATHEMATICAL MODEL:

Figure (1) illustrates a plane fin of width l and height h. Assume that the base temperature T_b of the fin is known which is often in practice. For example, in the case of the air cooled engine, it is known as the engine block temperature. Also, it will be supposed that thc heat transfer coefficient α between the fin surface and the surroundings at temperature T_f is known and uniform which suffices, for the present purpose, to simply be taken α as specified.

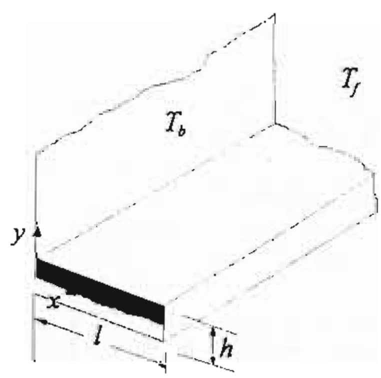

Figure (1) Schematic of rectangular plane fin.

The symmetry of fin allows the calculations to be limited to the half of the fin cross section. Calculations are performed in the rectangular region bounded by the vertical right-hand wall of the fin, the horizontal upper wall of the fin, the axis of symmetry in the lower side and the vertical wall of the base in the left-hand side. A uniform vertical temperature T_b is prescribed across the vertical plane, and a uniform temperature T_f is set along the walls of the fin surrounded by the fluid. These specifications lead to the boundary values of the temperature for this domain, indicated in figure (2).

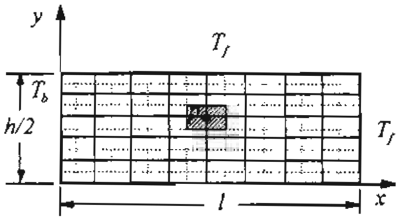

Figure (2) Domain of solution.

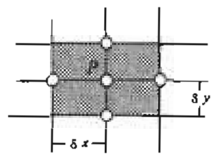

Figure (3) Typical control volume

In this case, the temperature variation in the z-direction is mush smaller than in the other two directions. This allows the case to be treated as two-dimensional heat conduction. Applying the fundamental law of heat conduction leads to a differential equation in T. The detailed derivation is given in [13], [14]. The resultant equation is:

$$\rho c \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right] - \frac{\partial}{\partial y}\left[k\frac{\partial T}{\partial y}\right] - s = 0 \qquad \text{.............. (1)}$$

where ρ is the density of the fin material, c its specific heat at constant volume, k its thermal conductivity and s is the rate at which heat is being internally generated in the element per unit volume.

For the steady heat conduction and fin with uniform thermal conductivity with no heat generation, the relevant form of the heat conduction equ. (1) reduced to:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad \text{.............................. (2)}$$

Two dimensionless parameters govern the radiating fin heat transfer, namely the aspect ratio; $Asp = l/h$; and Biot number; $Bi = \alpha L/k$; where the characteristic length L may be either h or l. Since the Biot number measures the ratio of resistance to conduction within the fin to the resistance to convection from its surface, and since it will be concerned with determining when the cross-fin conduction resistance causes significant temperature gradient in that direction, it will be useful using the definition as $Bi = \alpha h/k$.

One-dimensional fin analysis ("thin fin" approximation) is widely used by engineers to estimate fin heat transfer because of the relative ease of analytical solution. Schneider [16] gives the solution for the heat transfer rate $Q_{1\text{-}D}$ from the plane fin which is:

$$\frac{q_{1D}}{\alpha\left(T_b - T_f\right)} = \sqrt{2Bi}\,\frac{\sqrt{Bi/2} + \tanh\left(\sqrt{2Bi}\; l/h\right)}{1 + \sqrt{Bi/2}\,\tanh\left(\sqrt{2Bi}\; l/h\right)} \qquad \text{....... (3)}$$

It is common practice to express the effectiveness of radiating fin (fin efficiency η_f) as the ratio of heat loss from the fin, Q_{act}, to heat which would be transferred if the entire fin area were at the base temperature, Q_{IDEAL}.
Thus:

$$\eta_f = \frac{Q_{act}}{Q_{IDEAL}} \qquad ......(4)$$

Q_{act} and η_f will be obtained from numerical solution of equation (2) and compare it with Q_{IDEAL} to determine the conditions of validity of equ. (3).

3. NUMERICAL SCHEME:

A rectilinear grid is imposed on the domain of interest to be used in the computer solution. The temperatures are calculated at the discrete nodes. Uniform grid spacing in both the horizontal and the vertical directions are used in the calculation procedure.

Each grid node is imagined as being surrounded by its own control volume and connected to the neighboring nodes by boundaries defined as the perpendicular bisectors of the grid lines. Figure (3) shows node *P* and its surrounding nodes.

An algebraic, finite difference counterpart of the differential equation (2) is derived for the representative cluster of grid nodes shown in figure (3). The procedure was made as follows: equation (2) was integrated, over the control volume surrounding *P* and simultaneously averaged over a finite increment of time δt, then the remaining integrals will be replaced by algebraic approximations [17]. This transfers the partial differential equation (pde) (2) into an algebraic equation.

One such equation can be written for each interior cell, yielding a set of simultaneous algebraic equations, whose number equals to the number of unknown temperatures (implicit scheme). The aim now is to solve these simultaneous equations.

In that program, a combination of iteration by lines and block adjustments is used. The aim is to achieve computational efficiency without excessive complication and computer storage.

The line iteration procedure involves simultaneous solution for the temperatures along each grid line, while the temperatures along neighboring lines are temporarily taken as known temperatures. The simultaneous solution is achieved by a particular form of Gaussian elimination known as "Thomas Algorithm" [18], or "Tri-Diagonal Matrix Algorithm" (TDMA). This procedure is applied along north-south grid lines, starting at the west-most one and sweeping eastwards.

The action of the line iterative procedure is to sweep the errors in the prevailing solution of the temperature field to the boundaries, were they are reduced or eliminated by the boundary conditions. The errors are not reduced to zero in just one iteration. That is revealed by consideration of the residual sources R_P of the finite-difference equations [19].

The residual sources R_P for the cells must ideally be zero, if the prevailing temperature field were without errors which can be achieved by the application of the TDMA to a grid line.

Unfortunately there are some circumstances in which the rate of reduction of the residual sources R_P by the line-iteration procedure becomes unacceptably slow. This may happened when the resistance to the heat transfer at the boundaries is large. This may happen with conductive cooling when Biot number, $Bi = \alpha h/k$, is small. Since it is the changes in boundary heat flow which effectively remove the errors, the effect of high resistance is to cause them to be reflected back into the interior field; without significant reduction.

An effective solution for this problem is to use a procedure which, by simultaneously adjusting the temperatures on each line by uniform increments for each line (the value of the increment varying from line to another) causes the residual sources R_P to sum to zero along every line, and hence over the entire field. This procedure is known as the block adjustment procedure, the details of the procedure is described in [20].

3.1. GRID SIZE AND NUMBER OF ITERATIONS:

The recommended grid for the present calculations is a uniform grid with spacing in the *y*-direction of 7 nodes and expands at a constant rate in the x-direction, with 12 nodes. This expansion factor varies according to the value of Biot number. This type of grid is employed because in general the temperature gradient ($\partial T/\partial x$) along the fin decreases with x and increases with *Bi*.

Changing the grid size to 15 nodes in the horizontal direction and 10 nodes in the vertical direction, does not affect the accuracy of the solution.

Number of iterations were changing from one test run to another according to Biot number. Cases of low Biot numbers needed small number of iterations while ones of high Biot needed more number of iterations to obtain a converging solution. For example, 5 interation were needed for Bi = 0.01 while the number of iterations increased to 12 for Bi = 10.

4. SOLUTION PROCEDURE:

The solution procedure starts from the initial distribution of the temperature T at time $t = t_o$. The initial distribution consists of the boundary condition values seted according to the heat transfer rate and other node values which are assumed initially to be at the base temperature T_b. Then the procedure advances to the next time level $t = t_o + \delta t$. For steady state condition, such as the case in question, only just one time step with δt effectively set equal to infinity can be performed. The procedure first evaluates the coefficients of the partial differential equations (pde) basing them on the prevailing temperatures as an initial estimate. Then, the temperatures are updated by first performing a line-iteration sweep and then making the block adjustments. The result so obtained is examined for satisfactoriness, by checking if the residual source sum R_p is sufficiently small. If it is unsatisfactory, the cycle is repeated from coefficients calculation stage, until an acceptable solution is obtained. This provides the temperature field.

In practice, the quantities of interest are the temperature distribution, actual rate of heat transfer $Q_{act.}$, ideal rate of heat transfer if entire fin area were at base temperature, Q_{IDEAL}, and fin efficiency η_f. Arrangements are therefore made in the program to calculate these quantities once a converged solution has been obtained for the temperature.

5. RESULTS AND DISCUSSION:

Test runs were performed for three fins of aspect ratios of 5, 10 and 20 in each case of Biot numbers of 10^{-2}, 10^{-1}, 1.0 and 10. In order to determine the fin efficiency η_f and the ratio of the computed heat transfer to 'one dimensional' heat transfers for each case. Temperature distribution inside the fin was also reported.

The values of quantities l, h, , k, T_b and T_f are arbitrary values. In this study the following values are used: h = 0.002 m, k = 52 W/m.°k (steel) , T_b = 100° C and T_f = 20° C. The aspect ratio and Biot number can then be specified through l and α respectively.

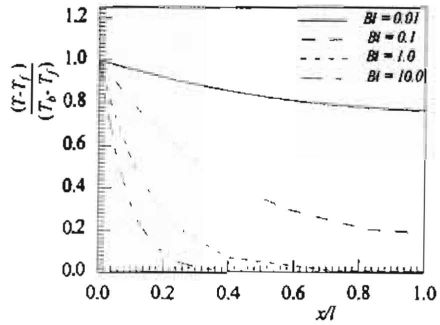

Figure (4) Temperature distribution at symmetry plane for a fin of Asp = 5

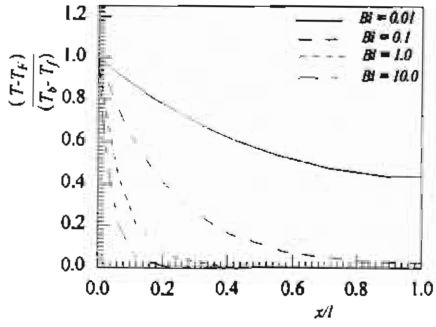

Figure (5) Temperature distribution at symmetry plane for a fin of Asp = 10

Figures (4 to 6) represent the temperature distributions at the y=0 symmetry plane of the fin with the dimensionless length x/l for three fins of aspect ratios 5, 10, and 20. For each fin, four Biot numbers (Bi = 0.01, 0.1, 1.0, and 10.) were plotted. The figures indicated that, increasing aspect ratio, *Asp*, increases the temperature gradient. It also indicated that the temperature gradient becomes more steep with increasing Biot number.

Physically, Biot number *Bi* represents the ratio of the thermal resistance of the block to that of the fluid layer. When *Bi* is very large, the convection layer offers negligible resistance compared with the internal resistance of the block and this causes the temperature gradient to be very steep. At the other extreme, when *Bi* is very small, implying that the convection layer is now the dominant resistance that decreases the temperature gradient. This result indicates that thin fins are more effective than thick ones.

Figure (7) represents the fin efficiency η_f versus Biot number, Bi, for the three aspect ratios, *Asp*= 5, 10, and 20. The figure indicated that, for the same Aspect ratio, the efficiency decreases as Biot number increases. This agrees with the result obtained from figures (4-6), since increasing Biot number increases the block resistance with respect to the surface layer resistance which increases the temperature gradient that, in turn, reduces the surface temperature. This causes the rate of convective heat transfer to be reduced that reduces the fin efficiency. The figure, also, indicated that the fin efficiency decreases with increasing the aspect ratio. This result indicates that the short fins are more efficient than long ones. Preparing a chart, such as this shown in figure (7), is very

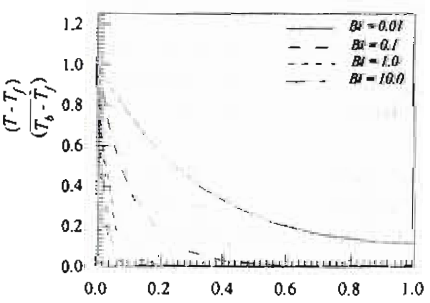

Figure (6) Temperature distribution at symmetry plane for a fin of Asp = 20

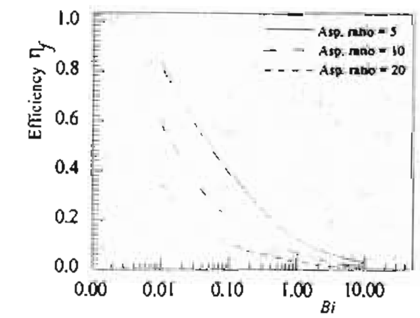

Figure (7) Fin efficiency versus Biot number For different aspect ratios.

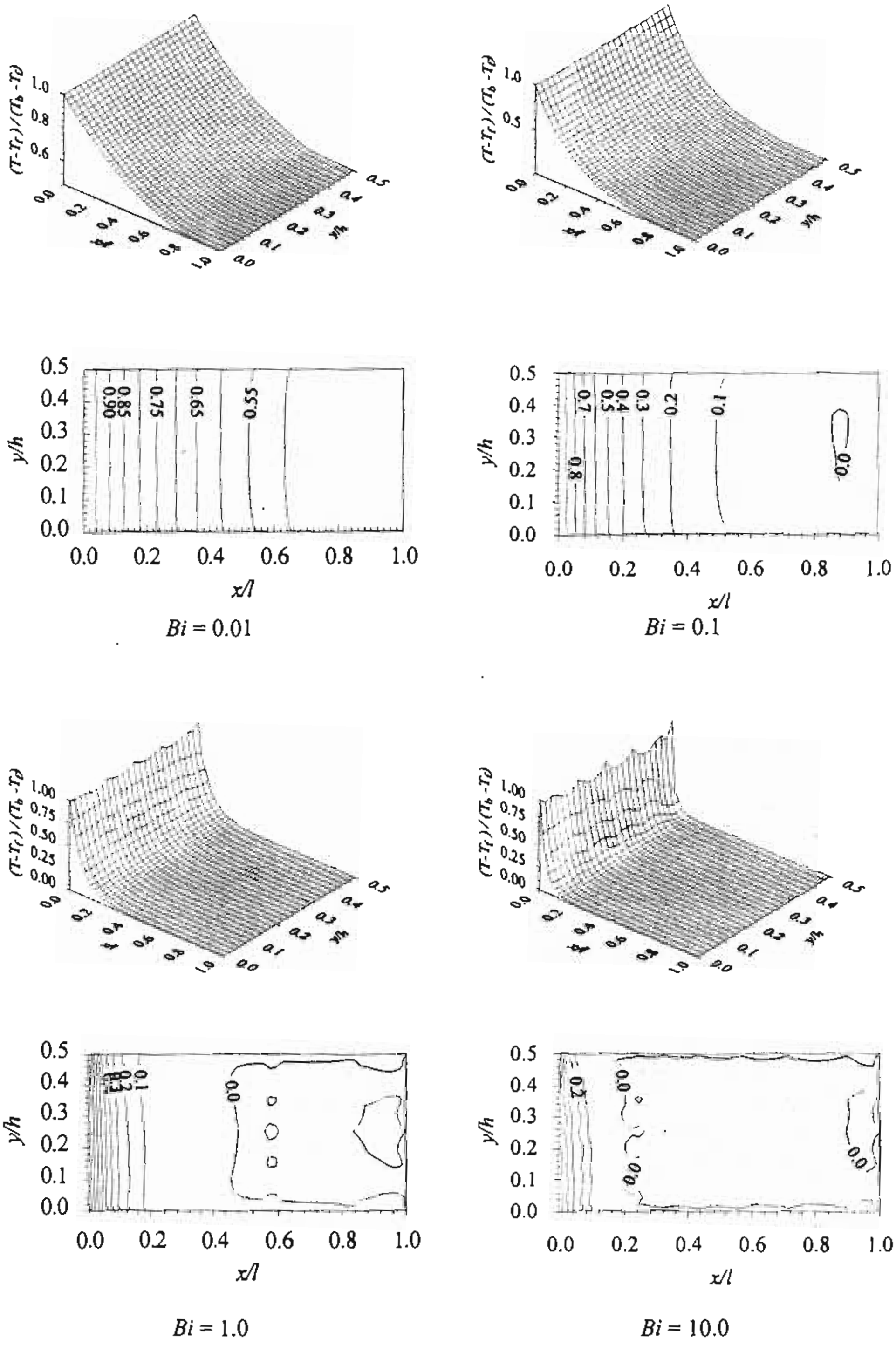

Figure (8) Response surface and contours of constant temperature for a fin of Asp = 10.0

useful for designing an economical fin i.e. one formed from a minimum mass of material, since if both Biot number and fin efficiency are defined, the aspect ratio can be obtained from the chart, shown in figure(7), from which minimum length of the fin can also be obtained.

Figure (8) shows the response surface and the contours of constant temperature for a fin of aspect ratio, *Asp.*=10, for Biot numbers, Bi = 0.01, 0.1, 1.0, and 10. The response surface showed that the temperature gradient becomes more steep as Biot number increases. The contours of constant temperature indicated that lines of constant temperature becomes more dense as Biot number increases leaving behind 58% of the fin surface at zero temperature gradient for Biot number 1.0 and 80% of the fin surface at zero temperature gradient for the case of Biot number 10 which means that the convective rate of heat transfer reduces with increasing Biot number which reduces the fin efficiency as well.

Figure (9) shows the change in the ratio between the calculated rate of heat transfer Q_{Act} to that obtained from the one-dimensional analysis, Q_{1-D}, with Biot number for the three aspect ratios (*Asp*= 5, 10, and 20). The figure shows that the ratio (Q_{Act}/Q_{1-D}) is the same for the three aspect ratios since all the curves coincide with each other. It also indicated that the ratio decreases with increasing Biot number. The figure shows that the range of Biot number for which the one-dimensional analysis is satisfactory lies between 0.01 and 1.0 within 5% accuracy.

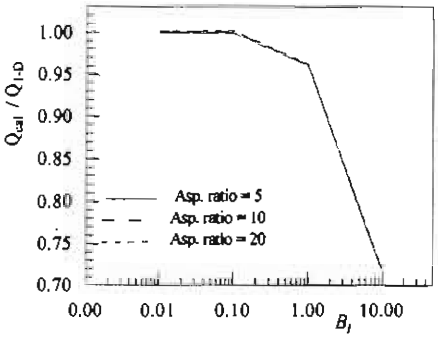

Figure (9) ratio between the calculated rate of heat transfer to that obtained from the one-dimensional analysis with Biot number.

6. CONCLUSIONS:

This investigation is concerned with the performance of a rectangular plane cooling fin at steady state conditions. Three fins of aspect ratios 5, 10, and 20 are studied. Each fin is examined for four cases of Biot number (Bi = 0.01, 0.1, 1.0, and 10). Temperature distribution and heat transfer rate for each case are calculated. The study reveals that temperature gradient decreases with decreasing both Biot number and aspect ratio which in turn increases the temperature of the fin surface, consequently, the convective heat transfer rate increases which increases the fin efficiency. It also indicated that the range of Biot number for which the fin performance is essentially that given by one dimensional analysis is ($0.01 \leq Bi \leq 1.0$) to within 5% accuracy.The study indicated that short and thin fins are mor efficient. A chart relating the fin efficiency and Biot number with aspect ratio as parameter is presented to facilitate an easy design, i.e., to select the optimum fin length and thickness.

7. NOMENCLATURE:

Asp	Aspect ratio = l/h.
Bi	Biot number = $\alpha h/k$.
h	Fin thickness.
k	Thermal conductivity.
l	Fin length.
$q_{act.}$	Actual rate of heat transfer.
$q_{Ideal.}$	Ideal rate of heat transfer.

$q_{1\text{-}D.}$	One-dimension rate of heat transfer.
t	Time.
δt	Time increment.
T	Temperature.
T_b	Base temperature.
T_f	Fluid temperature.
x	Longitudinal Cartesian coordinate.
y	Transverse Cartesian coordinate.
α	Heat transfer coefficient.
η_f	Fin efficiency.

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