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MODELLING AND SIMULATION OF CURRENT CONTROLLED D.C. MOTOR FED BY THYRISTOR CHOPPER

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ABSTRACT

This paper presents the application of a general method of modelling of control systems with static converters in the current control of a separately excited d.c. motor fed by a d.c. chopper. A non-linear discrete model is obtained, taking into consideration the three interavals of operation of chopper (duty, commutation and free-wheeling). To determine the P-I current controller parameters, the non linear model is linearised around an operating point and the characteristic equation of the system is obtained. To check the results a program of simulation is made taking into consideration all the internal and external events.

1. LIST OF SYMBOLS

i_c : Instantaneous value of commutating capacitor current.
t_b : Commutation interval of chopper.
t_a : Duty interval of chopper.
t_c : Free-wheeling interval of chopper.
k_i : Gain of current transducer.
τ, K_1 : Controller parameters.
v_c : Instantaneous value of commutating capacitor voltage.
v_o : Current reference.

2. INTRODUCTION

Static converters usually present a basic cycle which is divided in to several intervals. The problem is generaly how to define the final state as a function of the intial state which will be a discrete nonlinear recurrence equation. Here we define a linearised model around a steady state operating point to write the recurrence matrix M_R such as

$$\delta \underline{X}_{n+1} = M_R \quad \delta \underline{X}_n$$

where $\delta \underline{X}_{n+1}$ and $\delta \underline{X}_n$ are the variation of the final and intial states around the operating point .
Then the problem is now to write the relations between $\delta \underline{X}_n$ and $\delta \underline{X}_{n+1}$ for a system which presents various sturctures during a basic cycle.

The above regurious method of modelling applied in this paper has been used in many application for d.c. motor fed by thyristor bridge. References [1] and [2] present this method for current and speed control of D.C. motor. The classical controllers have been realized using operational amplifiers. This method also can be applied for modelling of such systems using digital controllers [3,4]. In the above application the systems have either one mode or two modes of operation (continnous or discontinnous current operation). In the case where the motor is fed by a d.c. chopper there are three modes of operation (duty, commutation and free-wheeling) which complicate the analysis. The first attempt for applying this method in the case of chopper was presented in [5] where the current controller is of the proportional type.
Then, the system becames a first order one. In this paper the current controller is of the proportional integral type, thus the systems is a second order one.

3. MODELLING PRINCIPLES [2,4]

The differential equation of one mode of operation is written as :

$$F_1 = \frac{d\underline{X}_1}{dt} + G_1\, \underline{X}_1 = \underline{V}_1(t) \qquad t \in [t_n, t'_n) \tag{1}$$

where $\underline{X}_1$ is the state varible of this mode

t_n, t'_n are the biging and the end instants for this mode.

Then the state equation is :

$$\frac{d\underline{X}_1}{dt} = A_1\, \underline{X}_1 + \underline{B}_1(t) \tag{2}$$

where $A_1 = -F_1^{-1}\, G_1$ and $\underline{B}_1 = F_1^{-1}\, \underline{V}_1$

From the solution of equation (2) the state variable at the end of the mode can be written in the following form,

$$\underline{X}_1(t'_n) = e^{(t'_n - t_n)A_1}\, \underline{X}_1(t_n) + \int_{t_n}^{t'_n} e^{(t'_n - t)A_1}\, \underline{B}_1(t)\, dt \tag{3}$$

multiplying both sides of equation (3) by $e^{-t'_n A_1}$ we get

$$\underline{E}_1(X_1(t'_n),\ t'_n) = \underline{E}_1(X_1(t_n),\ t_n) \tag{4}$$

with

$$\underline{E}(X_1,\ t) = e^{-tA_1}\, \underline{X}_1 - \int_0^t e^{-\ A_1}\, \underline{B}(\tau)\, d\tau \tag{5}$$

Equation (4) is a symmetrical recurrence equation between the state at the end of the mode and the state at its begining. This $\underline{E}$ function represents a very useful tool for the modelling of the considered system. This function is invariant on the trajectory $(\underline{X}(t),\ t)$. Its variations verifies :

$$\delta\underline{E}_1(n') = \delta\underline{E}_1(n) \tag{6}$$

with $\delta\underline{E} = e^{-tA}\, [\ \delta\underline{X} - \underline{Y}\ \delta t]$ (7)

where $\underline{Y} = A\, \underline{X} + \underline{B}$

4. DESCRIPTION OF THE SYSTEM

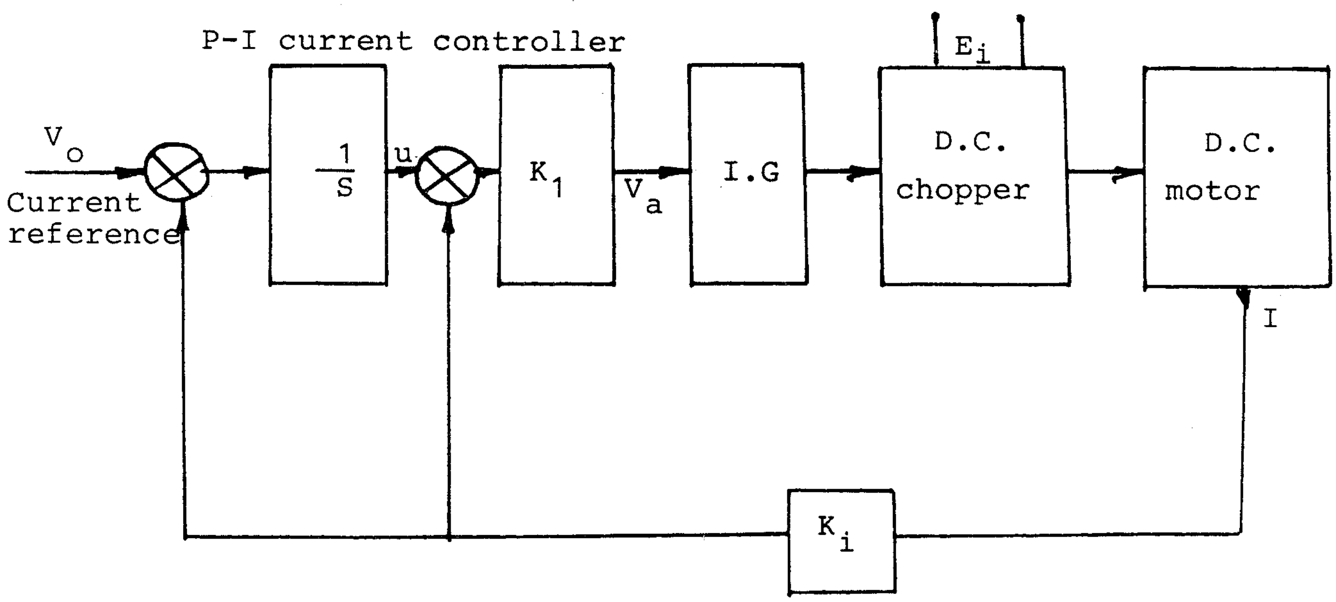


Figure (1) System block diagram

As shown in Figure (1), the system consists of separiately exited D.C. motor fed by a D.C. chopper (Figure 2). The auxiliary thyristor T_2 is fired at constant instants (t_n). But the main thyristor T_1 is fired when the control voltage equals the timing voltage (Figure 3). The control voltage is the output of the P-I controller.

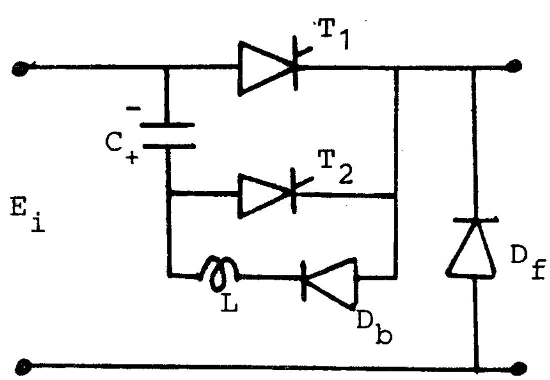
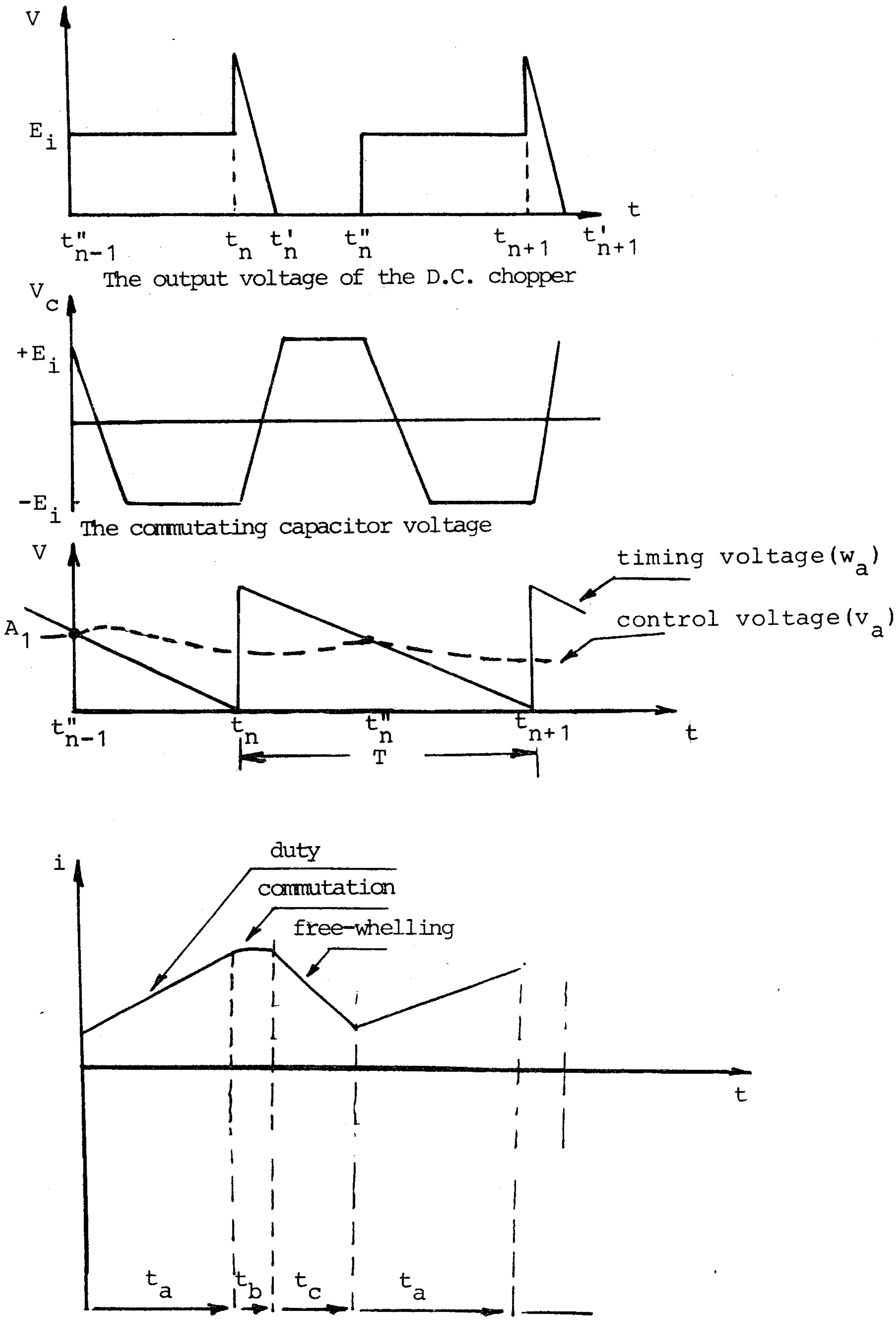


Figure (2) Equivalent circuit D.C. chopper



Figure(3) Defintions of some variables

5. PERFORMANCE EQUATIONS

The wave forms of the armature voltage and current are shown in Figure (3). The system has three distinet modes of operation. The performance equations for each mode can be expressed in the following way.

6. DUTY INTERVAL (MODE A)

This interval has a duration t_a. It starts at the instant t''_{n-1} when the control voltage (V_a) equals the timing voltage (W_a) as shown in Figure (3). At this instant the main thyristor T_1 is gated on. The duty interval ends at fixed instant t_b which is the beging of commutation.

The differential equations of this mode are (Figure 4)

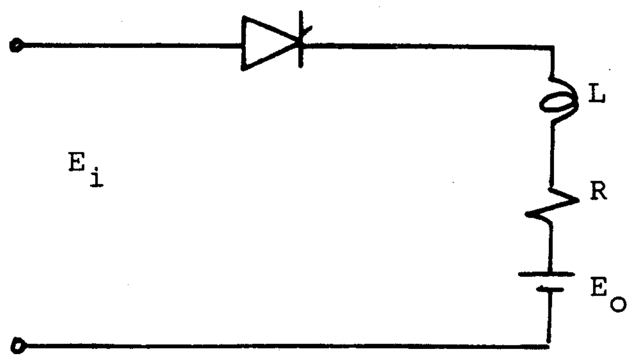


Figure (4) Equivalent circuit with duty interval.

$$L\frac{di}{dt} + R\,i = E_i - E_o \qquad (8)$$

$$\tau\frac{du}{dt} + K_i\,i = V_o \qquad (9)$$

Equations (8) and (9) can be written in matrix form of Equation (1) as :

$$F_a = \begin{bmatrix} L & 0 \\ 0 & \tau \end{bmatrix}, \quad G = \begin{bmatrix} R & 0 \\ K_i & 0 \end{bmatrix}, \quad V_a = \begin{bmatrix} E_i - E_o \\ V_o \end{bmatrix}, X_a = \begin{bmatrix} i \\ u \end{bmatrix}$$

And if these equations are written in the form of state Equation (2), we get

$$A_a = \begin{bmatrix} -R/L & 0 \\ -K_i/\tau & 0 \end{bmatrix}, \quad B_a \doteq \begin{bmatrix} (E_i - E_o)/L \\ V_o/\tau \end{bmatrix}$$

7. COMMUTATION INTERVAL (MODE B)

The duration of this interval is t_b. It begins when thyristor T_2 is fired by a timer at fixed instant t_n and ends when the voltage across the capacitor equals $(+E_i)$ at instant t_n'. The equations of the system in this mode are deduced from Figure (5) and Figure (1).

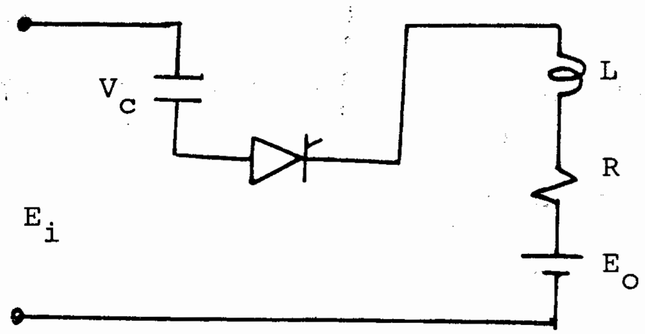


Figure (5) Equivalent circuil with commutation interval.

$$L \frac{di}{dt} + R_i + V_c = E_i - E_o \tag{10}$$

$$C \frac{dv_c}{dt} - i = 0 \tag{11}$$

$$\tau \frac{du}{dt} + K_i\ i = V_o \tag{12}$$

Similarly, as in mode (a), following Equations are obtained:

$$X_b = \begin{bmatrix} i \\ V_c \\ u \end{bmatrix}, \quad F_b = \begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & \tau \end{bmatrix}, \quad G_b = \begin{bmatrix} r & 1 & 0 \\ -1 & 0 & 0 \\ K_i & 0 & 0 \end{bmatrix}$$

$$V_b = \begin{bmatrix} E_i - E_o \\ 0 \\ V_o \end{bmatrix}, \quad A_b = \begin{bmatrix} -R/L & -1/L & 0 \\ 1/C & 0 & 0 \\ -K_i/\tau & 0 & 0 \end{bmatrix}, \quad B_b = \begin{bmatrix} (E_1 - E_o)/L \\ 0 \\ V_o/\tau \end{bmatrix}$$

8. FREE-WHEELING INTERVAL (MODE C)

When the voltage across the capacitor becomes $(+E_i)$, thyristor T_2 is turned OFF, and as T_1 was also OFF, then free-wheeling interval begins (t'_n). The chopper equivalent circuit becomes as shown in Figure 6.

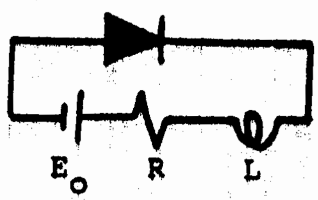


Figure (6) Equivalent circuit with free wheeling interval

The equations is this mode are

$$L \frac{di}{dt} + R\, i = -E_o \qquad (13)$$

$$\tau \frac{du}{dt} + K_i\, i = V_o \qquad (14)$$

Similarly we can be written

$$X_c = \begin{bmatrix} i \\ u \end{bmatrix}, \quad F_c = \begin{bmatrix} L & 0 \\ 0 & \end{bmatrix}, \quad G_c = \begin{bmatrix} R & 0 \\ K_i & 0 \end{bmatrix}$$

$$V_c = \begin{bmatrix} -E_o \\ V_o \end{bmatrix}, \quad A_c = \begin{bmatrix} -R/L & 0 \\ -K_i/\tau & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} -E_o/L \\ V_o/ \end{bmatrix}$$

The free wheeling interval ends when the main thyristor T_1 is turned ON. This occurs when the control voltage equals the timing voltage

$$V_a(t_n'') = W_a(t_n'') \tag{15}$$

The control voltage is the output of controller. Its equation can be written directly from Figure (1), as :

$$V_a(t) = K_1[u(t) - K_i \; i(t)] = P_r^T \underline{X}_c \tag{16}$$

with $P_r^T = (-K_1 K_i \quad K_i)$

The linear timing voltage equation can be determined from Figure (3)

$$W_a(t) = A_1 \left[\frac{T - (t_n'' - t_n)}{T} \right] \tag{17}$$

A_1 : is the slope of the timing voltage

T : is the period of one basic cycle = $t_a + t_b + t_c$

t_c : is the free wheeling duration = $t_n'' - t_n'$

9. NON LINEAR MODEL OF THE SYSTEM

Using the $\underline{E}$-function we can find the recurrence relations for each mode,

$$\underline{E}_a\,(t_n'') = \underline{E}_a(t_{n+1}) \tag{18}$$

$$\underline{E}_b\,(t_n) = \underline{E}_b\,(t_n') \tag{19}$$

$$\underline{E}_c\,(t_n') = \underline{E}_c(t_n'') \tag{20}$$

each of the above recurrence equations represents the relation between the state at the beging and the end of the same mode. To join these three recurrences together to get the total rercurrence relation of complete basic cycle (modes a, b, c), we fined the relations between the states at the end of one mode and begining of the next mode.

$$\underline{X}_a(t''_n) = H_{ac}\,\underline{X}_c(t''_n) \qquad (23)$$

with

$$H_{ba} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad H_b = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \quad G_{cb} = C_{bc}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$H_{ac} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

10. THE LINEARISED MODEL

In the following analysis we try to establish a linearised model of the system about a steady state operating point defined by

$$t''_{n-1} = t''_n = t''_o$$

$$t'_n = t'_{n+1} = t'_o$$

$$t_n = t_{n+1} = t_o$$

$$i(t''_{n-1}) = i(t''_n) = i''_o$$

Refering to equation (6), equations(18), (19) and (20) can be written is linearised form as follows :

$$R_a : \quad \delta\underline{E}_a(t''_n) = \delta\underline{E}_a(t_{n+1}) \qquad (24)$$

$$R_b : \quad \delta\underline{E}_b(t_n) = \delta\underline{E}_b(t'_n) \qquad (25)$$

$$R_c : \quad \delta\underline{E}_c(t'_n) = \delta\underline{E}_c(t''_n) \qquad (26)$$

The joining relations between the end of each mode and the beging of the next mode (R_{ab}, R_{bc}, R_{ca}) can be determined using Equations (7), (21), (22), (23) and the controller equation and timing voltage Equations(18) and (19).

$$R_{ab} \; : \; \delta \underline{E}_b(t_n) = e^{-t_o A_b} \, e^{t_o A_a} \, H_{ba} \, \delta \underline{E}_a(t_n) \tag{27}$$

$$R_{bc} \; : \; \delta \underline{E}_c(t'_n) = e^{-t'_o A_c} \, F_c^{-1} \, D_{cb} \, F_b \, e^{t'_o A_b} \, \delta \underline{E}_b(t'_n) \tag{28}$$

$$R_{ca} \; : \; \delta \underline{E}_a(t''_n) = e^{-t''_o A_a} \, M_B \, e^{t''_o A_c} \, \delta \underline{E}_c(t''_n) \tag{29}$$

$$\text{where : } M_B = H_{ac} + \frac{H_{ac} \, Y''_c - Y''_a}{A_n - Y''_c \, P_r^T} \, P_r^T \tag{30}$$

Using the linearised recurrence equations(24 to 26) and the joing relation (27 to 29) knowing that $\delta t_n = \delta t_{n+1} = 0$ (fixed instants) and $\delta E_a(t_{n+1}) = e^{-t_o A_a} \, \delta X_a(t_{n+1})$
The linearised recurrence equation of the system over one complete cycle can be written as :

$$\delta \underline{X}_a(t_{n+1}) = M_R \, \delta \underline{X}_a(t_n) \tag{31}$$

where the recurrence matrix M_R is

$$M_R = e^{t_a A_a} \, M_B \, e^{t_c A_c} \, F_c^{-1} \, D_{cb} \, F_b \, e^{t_b A_b} \, H_{ba} \tag{32}$$

with

$$e^{t_a A_a} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} , \quad e^{t_b A_b} = \begin{bmatrix} \psi_{11} & \psi_{12} & 0 \\ \psi_{21} & \psi_{22} & 0 \\ \omega_1 & \omega_2 & 1 \end{bmatrix}$$

$$e^{t_c A_c} = \begin{bmatrix} \phi'_{11} & \phi'_{12} \\ \phi'_{21} & \phi'_{22} \end{bmatrix}$$

$$M_B = \begin{bmatrix} 1 + (DE_i K_1 K_i)/L & -DE_i K_1/L \\ 0 & 1 \end{bmatrix}$$

The expression for all coefficients are given in Appendix. Equation (31) can be written in another form as,

$$\begin{bmatrix} \delta i(n+1) \\ \delta u(n+1) \end{bmatrix} = M_R \begin{bmatrix} \delta i_n \\ \delta u_n \end{bmatrix} \tag{33}$$

Using the Z transform, the characteristic equation of the system can be determined

$$Z^2 - Z(A_{24} + A_{13}\,\psi_{11} + A_{14}\,\omega_1) + \psi_{11}(A_{24}A_{13} - A_{14}A_{23}) = 0 \tag{34}$$

11. CHOICE OF THE POLES OF THE SYSTEM

The characteristic Equation (34) is a second order equation which has two roots Z_1 and Z_2 representing the poles of the system where

$$Z_1 + Z_2 = A_{24} + A_{13}\,\psi_{11} + A_{14}\,\omega_1 \tag{35}$$

$$Z_1 Z_2 = \psi_{11}(A_{24}\,A_{13} - A_{14}\,A_{23}) \tag{36}$$

choosing the poles Z_1 and Z_2 for required response, the parameters of the controller K_1 and τ_1 can then be determined

$$K_1 = A_n \left[\frac{1}{K_i(R\,i''_o + E_o)/L + E_i K_i / \{L[(Z_1 Z_2/\phi_{11}\,\phi'_{11}\,\psi_{11}) - 1]\}} \right. \tag{37}$$

$$\tau = D\,K_1\,B\,/\,N \tag{38}$$

with $B = L\,K_i(\phi_{11}-1)E_i/R + \phi_{11}E_i L\,K_i(\phi'_{11}-1)\psi_{11}/R - \phi_{11}E_i K_i C\psi_{21}$

$$\mu = L + \phi_{11}\phi'_{11}\psi_{11}\,L + \phi_{11}\,D\,E_i K_1 K_i \phi'_{11}\psi_{11} - L(Z_1 + Z_2)$$

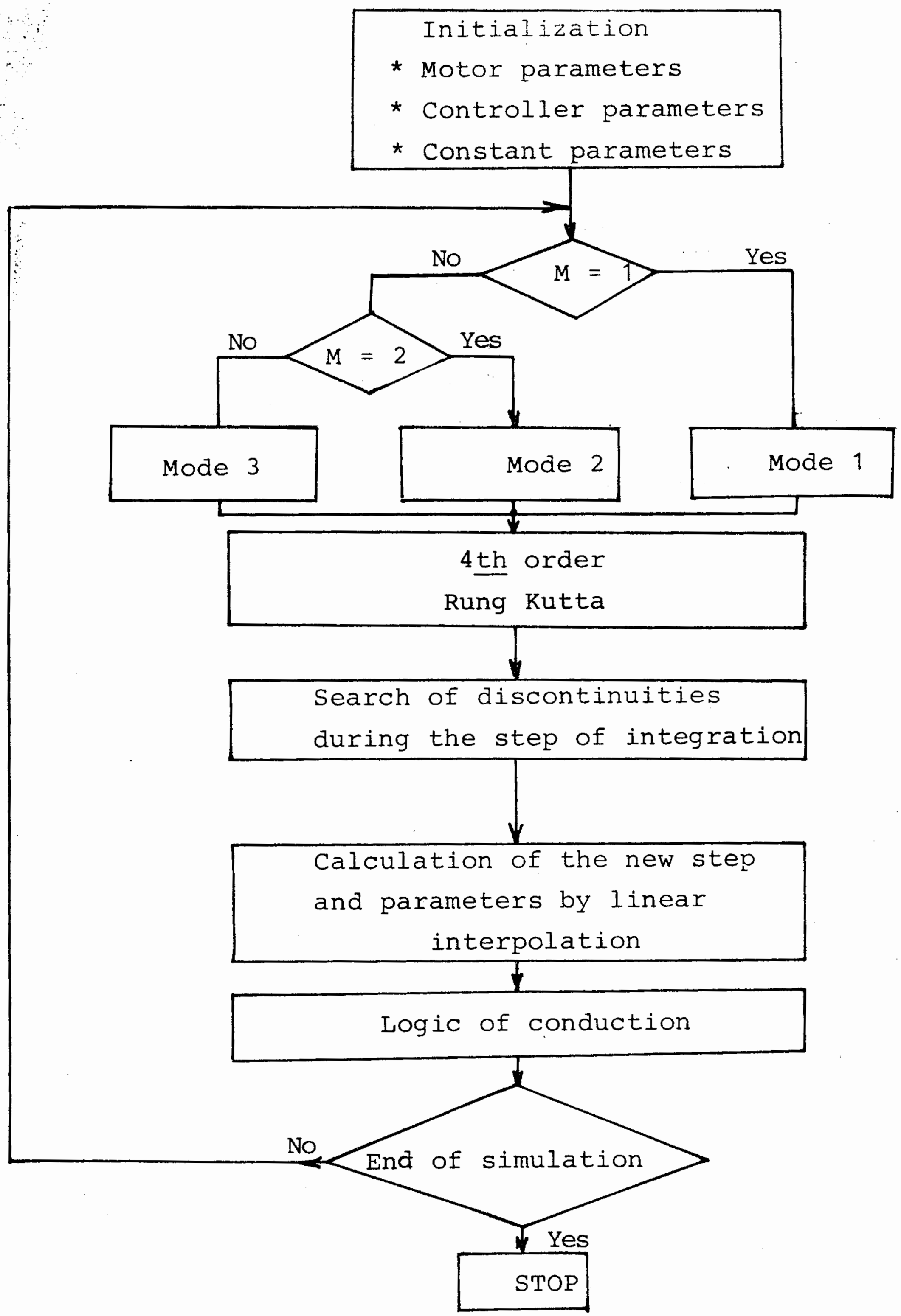
If the dead beat response is chose, then $Z_1 = Z_2 = 0$

$$K_1 = \frac{L\,A_n}{K_i[R\,i''_o + E_o - E_i]} \tag{39}$$

$$\tau = -[A_{10} + \phi_{11}K_{13}\,\psi_{11} + \phi_{11}\,K_{11}]/K_i \tag{40}$$

12. SIMULATION

A program of simulation of the system is mode taking into consideration the different modes of operations and the internal and external events. This program is based on Rung-Kutta numerical method the flow chart is given in Fig.(7).



Figure(7) Principal flow chart of the program of simulation.

13. RESULTS

The d.c. motor has the following characteristic

$R = 5.27$ ohm, $L = 72.6$ mH , $K_i = 1$ volt/Amp.

$E_o = 80$ volt.

The linear timing voltage characteristics

$A_1 = 5$ volt $T = 5$ m.s

Choper characteristics

$C = 4.74 \times 10^{-6}$ F , $E_i = 200$ volt

Steady state operating point

$i(t''_n) = I_o = 3.8$ A

$V_o = 3.8$ volt

$u = 7.9$ volt

Controller parameters

$K = 0.73$

$\tau = 0.00376$

The simulation results are shown in figure (8) which represient the current response due to step change in current reference.

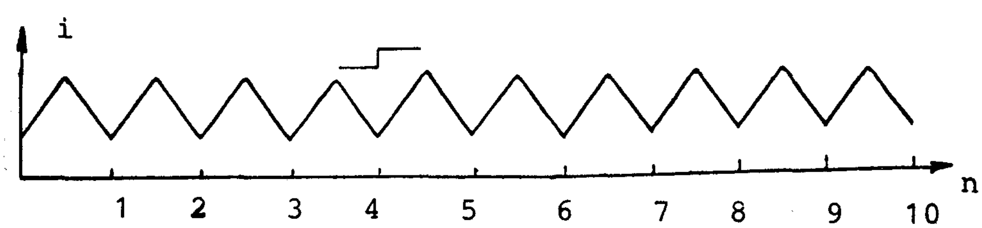


Figure (8) Simulation result

14. CONCLUSION

This paper deals with the application of a general modelling method for electrical machines fed by static converters. In this investigation the static converter is a D.C.-D.C. chopper. The use of recurrence equations and matricial joining conditions which joing the different modes of operation permits to deduce an over all linearised model. The characteristic equation of the system allows the determination of the controller parameters for the desired response. The poles of the current is choosen equal zero in the Z plane to obtain the dead beat response, The simulation results coincide with those which obtained theoreticaly.

15. APPENDIX

$$A_n = -\frac{A_1}{T} \quad , \quad t_a = t_o - t''_o \quad , \quad t_b = t'_o - t_0 \quad , \quad t_c = t''_o - t'_o$$

$$\phi_{11} = e^{-R/L\ t_a}$$

$$\phi_{12} = 0$$

$$\phi_{21} = \frac{L\ K_i}{R\tau}(\phi_{11} - 1) = \frac{A_{10}}{\tau} \quad , \quad \phi_{22} = 1$$

$$B = R/2\ L \quad , \quad \omega = \sqrt{(1/LC)-(R/2L)^2}$$

$$\psi_{11} = e^{-Bt_b}[\cos \omega t_b - (B \sin \omega t_b)/\omega]$$

$$\psi_{12} = -e^{-Bt_b} \sin \omega t_b / L\omega$$

$$\psi_{21} = e^{-Bt_b} \sin \omega t_b / C\omega$$

$$\psi_{22} = e^{-Bt_b}[\cos \omega t_b + (B \sin \omega t_b)/\omega]$$

$$\omega_1 = -K_i\ C\ \psi_{21} / \tau = K_{11} / \tau$$

$$\omega_2 = -K_i C\ (\psi_{22} - 1) = K_{12}/\tau$$

$$\phi'_{11} = e^{-Rt_c/L}$$

$$\phi'_{12} = 0$$

$$\phi'_{21} = \frac{L\,K_i}{R\tau}\,(\phi'_{11} - 1) = K_{13}/\tau$$

$$\phi'_{22} = 1$$

$$D = 1/[A_n - K_1 K_i\ (R\ i(t''_n) + E_o)/L]$$

$$A_{11} = \phi_{11}(1 + D\ E_i K_1 K_i/L)$$

$$A_{12} = \phi_{12} - \phi_{11}\ D\ E_i\ K_1/L$$

$$A_{21} = \phi_{21}\ (1 + D\ E_i\ K_1\ K_i/L)$$

$$A_{22} = \phi_{22} - \phi_{21}\ D\ E_i\ K_1/L$$

$$A_{13} = A_{11}\ \phi'_{11} + A_{12}\ \phi'_{21}$$

$$A_{14} = A_{11}\ \phi'_{12} + A_{12}\ \phi'_{22}$$

$$A_{23} = A_{21}\ \phi'_{11} + A_{22}\ \phi'_{21}$$

$$A_{24} = A_{21}\ \phi'_{12} + A_{22}\ \phi'_{22}$$

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