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NONLINEAR ANALYSIS OF FLAT SLABS WITH DROP PANELS AND CANTILEVERS

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1. INTRODUCTION

Flat-slab and flat-plate floors are characterized by the absence of the beams along the interior column lines. So, the reinforced concrete slabs are supported directly on columns. Flat-slab floors provide an adequate shear strength and better resistance of the negative moments over the columns by having drop panels or column capitals. Flat-plate system is used when the spans are small and the loads are not so heavy. For larger spans and heavier loads the flat-slab can be used.

Flat-slabs with cantilevers are very popular because they increase the allowable used space by distinct areas which are very essential for many cases.

The behaviour of the reinforced concrete flat-slabs with drop panels and cantilevers are analyzed by using the non-linear finite element analysis. This was illustrated by solving a numerical example for a flat-slab with different lengths of cantilever at different loading stages.

2.NONLINEAR ANALYSIS OF REINFORCED CONCRETE

The nonlinear behaviour of reinforced concrete structures are mainly attributed to the nonlinear stress-strain relationship, cracking and crushing of concrete. The material properties of concrete and steel depend on the stress or strain state of the material. In this study the following material properties are adapted.

2.1. CONCRETE

The analytical model used in the present analysis was originally developed by Darwin and Pecknold[1]. This model has obtained a good match with the test results of kupfer and Nilson. In Darwin's model concrete is assumed to be an ortho tropic material in the two principal stress directions. The concrete is treated as an incrementally elastic material. At the end of each increment, material stiffness and stress are corrected to reflect the latest changes in deflection and strain. The curves selected for compressive loading are based on an equation suggested by Darwin and shown in Fig. 1

$$\sigma_i = \frac{\varepsilon_{iu} E_o}{1 + \left(\frac{E_o}{E_s} - 2\right)\frac{\varepsilon_{iu}}{\varepsilon_{ic}} + \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}}\right)^2} \quad \ldots\ldots\ldots (1)$$

where σ_i = the compressive stress.

E_o = the tangent modulus of elasticity at zero stress.

$E_s = \frac{\sigma_{ic}}{\varepsilon_{ic}}$ = the secant modulus at the point of maximum compressive stress σ_{ic}

ε_{ic} = the equivalent uniaxial strain at the maximum compressive stress.

ε = the equivalent uniaxial strain in the *ith* direction.

The tangent moduli in the principal directions for concrete in biaxial compression, can be obtained by differentiating Eq.(1) with respect to the equivalent uniaxial strain.

The values of the maximum stresses in the two principal directions, σ_{1c} and σ_{2c} are determined from the modified biaxial strength envelope of Kupfer and Gerstle[2]. They have suggested an analytical maximum strength envelope which is shown in Fig. 2. This criterion has been adapted in the present analysis.

2.2. STEEL

In reinforced concrete slabs, unlike beams, reinforcement is usually more uniformly distributed, and reinforcing bars tend to be smaller in size. In this study, the reinforcement is assumed to be uniformly distributed over the element. Thus each layer of reinforcement can be replaced by an equivalent distributed steel layer. The equivalent thickness of the layer is determined such that the corresponding area of the reinforcement in the element remains unchanged.

$$t_s = \frac{A_s}{b} = \mu \times d \quad \ldots\ldots\ldots\ldots \quad (2)$$

where A_s = the area of one reinforcing bar.

b = the spacing of the reinforcing bars.
μ = the reinforcement concrete ratio.
d = the effective depth of the slab.

Fig. 3 shows the stress-strain relation for steel which has been considered in the analysis.

3. LAYERED DISCRITIZATION

In order to account for the varied material properties within a finite element, the element is divided into imaginary concrete layers and steel layers (Fig. 4). According to the Kirchhoff's hypotheses the transverse normal stress is neglected. Thus any point in the element may be considered to be in a state of plane stress. The layered element approach, imagines every element to consist of a number of concrete and steel layers in plane state of stress so that the material property matrix can be written for any stress or cracked state. The entire element stiffness is obtained by summing up the stiffness of the layers.

3.1. EVALUATION OF THE ELEMENT STIFFNESS

The integration involving material properties can be integrated layer by layer. Let c and s denote the number of concrete layers and steel layers respectively for a typical layer finite element shown in Fig. 4.

Assuming the material properties are constant within each layer, the integration can be carried out as follows;

$$[D] = \int z^2 [D]\, dz = \sum_{i=1}^{c} \int_{Z_i}^{Z_{i+1}} z^2 [Dc]\, dz_i + \sum_{i=1}^{s} z_i^2 [Ds]\, t_i$$

$$= \sum_{i=1}^{c} \frac{1}{3} (z^3_{i+1} + z^3_i)\, [Dc]_i + \sum_{i=1}^{s} z_i^2 [Ds]\, t_i \quad \ldots \quad (3)$$

where $[Dc]_i$ is the material matrix of the i th concrete layer.

$[Ds]$ is the material matrix of the i th steel layer.

3.2.EVALUATION OF LAYER STRAINS AND STRESSES

Once the nodal displacements are known, the membrane strain on the reference plane $\{\varepsilon_o\}$ and the curvature $\{K\}$ can be obtained. The strains at the centers of layers can be computed.

$$\{\varepsilon_c\}_i = \{\varepsilon_o\} - \frac{1}{2}(z_{i+1} + z)_i \{K\} \quad (4)$$

$$\{\varepsilon_s\}_i = \{\varepsilon_o\} - (z)_i \{K\} \quad (5)$$

The layer stresses are computed as follows.

$$\{\sigma_c\}_i = [D_c]\{\varepsilon\}_{c\ i} \quad (6)$$

$$\{\sigma_s\}_i = [D_s]\{\varepsilon\}_{s\ i} \quad (7)$$

where $\{\varepsilon_c\}_i$ and $\{\sigma_c\}_i$ denote the strains and stresses at the center of the *ith* concrete layer, $\{\varepsilon_s\}_i$ and $\{\sigma_s\}_i$ denote those at the center of the *ith* steel layer.

4. CONCRETE CRACKING

Cracking in reinforced concrete is complicated by the presence of the steel reinforcement. When the concrete reaches the ultimate tensile strength, primary cracks form at finite intervals along the length. The total load is transferred across these cracks by the reinforcement, but the concrete between cracks is still capable of carrying stresses because of the bond between steel and concrete. This phenomenon is called the "tension stiffening effect".

Frank Vecchio[3] prepared a stress-strain relation for concrete in principal tensile direction, as shown in Fig. 5 which gives good results with the experimental works. The modulus of concrete E_p decreases gradually as the strain increases after cracking as follows;

$$E_p = \frac{f_t}{\varepsilon_i(1 + \sqrt{200\,\varepsilon_i})} \quad \text{................} \quad (8)$$

where f_t is the tensile cracking and ε_i is the strain at any stress level. This model is considered in the present study. So, For any layer, when any of the principal stresses exceeds the tensile strength, cracks will occur in a direction perpendicular to that principal stress. The modulus of concrete is reduced according Eq.(8).

5. THE FINITE ELEMENT PROGRAM

The computer program has been developed to implement the previous method of analysis. A layered quadrilateral isoparametric plate bending element is chosen in the program. An incremental procedure along with secant stiffness method is used.

The finite element program has been extended also to tak ing into account the variable thickness of the slabs to consider the effect of the drop panels and the placement of the steel reinforcement.

6.APPLICATION EXAMPLE

To study the effect of the cantilevers on the behaviour of flat-slab system with drop panels during different stages of loading, a numerical example for a roof of 18.0 x 18.0 m is presented. The design data are as follows;

- Spacings between columns = 6.00 m.
- Column dimensions = 0.40 x 0.40 m.
- Partition weight = 100 kg/m^2.
- Service live loads = 250 kg/m^2.
- Compressive strength of concrete C_{28}= 300 kg/cm^2.
- Tensile strength of concrete F_t= 30 kg/cm^2.
- Yield strength of reinforcement f_y= 4200 kg/cm^2.
- Thickness of the slab t_s= 0.16 m.
- Thickness of the drop panels t_D = 0.08 m.

The drop panels considered in this example extent from the center lines of the columns to 0.25 the span length in each direction.

The following cases for the reinforced concrete flat-slab with cantilever are studied;

Case 1 : L_{ca} = 0.0 (Without drop panel).
Case 2 : L_{ca} = 1.5 m. (L_{ca} / L = 0.25).
Case 3 : L_{ca} = 2.1 m. (L_{ca} / L = 0.35).
Case 4 : L_{ca} = 2.4 m. (L_{ca} / L = 0.40).
Case 5 : L_{ca} = 2.7 m. (L_{ca} / L = 0.45).
Case 6 : L_{ca} = 3.0 m. (L_{ca} / L = 0.50).

where L : panel length in the considered direction.
L_{ca}: length of the cantilever neighboringto L .

7. BEHAVIOUR OF FLAT-SLAB THROUGH LOADING

To show the changes in the behaviour of flat-slab with cantilevers during the history of loading, the following stages are considered;

Loading stage 1 : $W_1 = D + L$(working loads)

Loading stage 2 : $W_2 = 1.5\ W_1$(ultimate loads)

Loading stage 3 : $W_3 = 2.0\ W_1$

where W = Distributed load per unit area = W_1, or W_2, or W_3 according to the loading stage.

D = dead loads

L = live loads

The previous example is solved by the nonlinear F.E. program to analyze the behaviour of the slabs for each case such as the deflections at various locations and the distribution of moments at different stages of loading.

8. ANALYSIS OF THE RESULTS

Figures 6 to 17 illustrate the contour lines of the deflections for the flat-slab with different lengths of cantilevers for loading stage 1 and 2. Also, Figure 18 shows the values of the maximum deflections of the slabs. It is noticed that the general behaviour of the deflections is improved and decreased at any point within the flat-slab for $L_{ca} \leq 0.4$ L. The maximum deflection is decreased by about 33%. for L_{ca} = 0.4 L than that without cantilever. The deflection for $L_{ca} \geq$ 0.45 L increased< due to the heavy cracks at the top of the exterior column lines.

The distributions of the bending moments in the cantilever at the long directions of the slab for $L_{ca} = 0.4$ L are similar to the neighboring half column and field strips. Figure 20 (a,b) shows the distribution of the moments along axes (X2- X2), (X3-X3) and (X5-X5), (figure 19) and Figure 21 (a,b) shows the distribution of the moments along axes (X1-X1) and (X4-X4).

The cantilever slabs reduces the punching shear stresses. This is due to the reduction of the unbalanced moments transfer to the edge and corner columns. and the increase of the perimeters of the slabs which resist the punching shear stresses.

9. CONCLUSIONS

From the analysis of flat-slab with cantilevers by the finite element method at the different loading stages it can be concluded that;

- Whenever the length of the cantilever slabs increases the general behaviour of the flat-slab improves. It is recommended to limit the length of the cantilever to be equal 0.4 the adjacent span in the case of flat-slab with drop panel $t_D = 0.5\ t_S$

It is advised to make the cantilever slabs whenever possible to reduce the deflections and the punching shear stresses for the edge and the corner column.

- The distributions of the bending moments in the cantilever at the long directions of the slab must be considered. It is recommended to reinforce this direction similar to the neighboring half column and field strips respectively.

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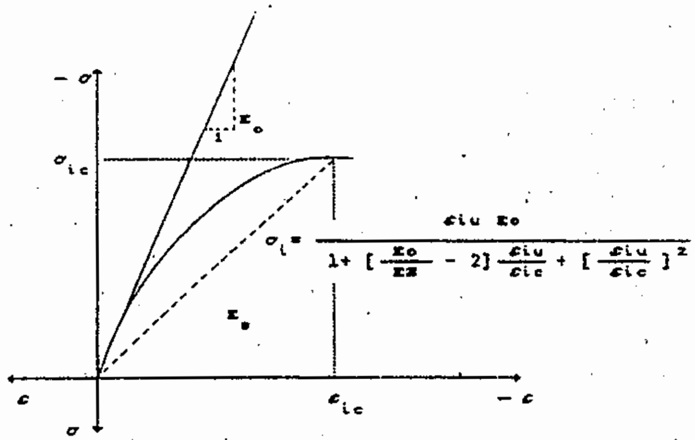
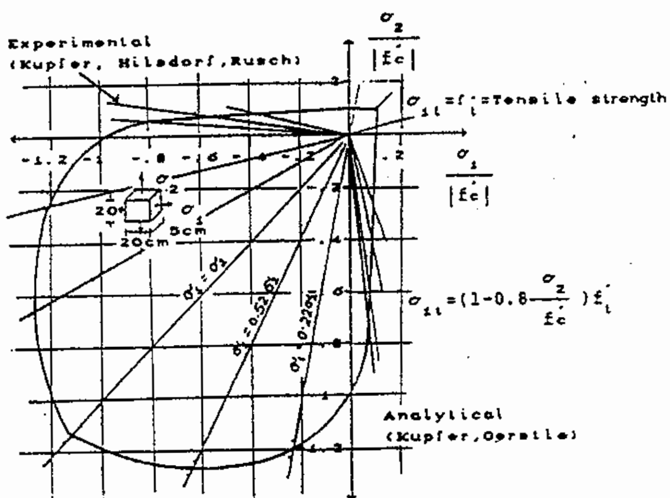


Equivalent uniaxial stress-strain curve.

Figure 1

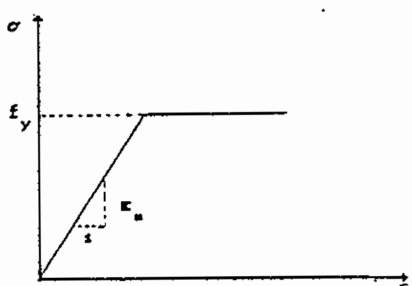


$$\left(\frac{\sigma_1}{f_c'} + \frac{\sigma_2}{f_c'}\right)^2 - \frac{\sigma_2}{f_c'} - 3.65\frac{\sigma_1}{f_c'} = 0$$

$$\text{or } \sigma_{2c} = \frac{1+3.65\,\alpha}{(1+\alpha)^2}\, f_c' \qquad \alpha = \frac{\sigma_1}{\sigma_2}$$

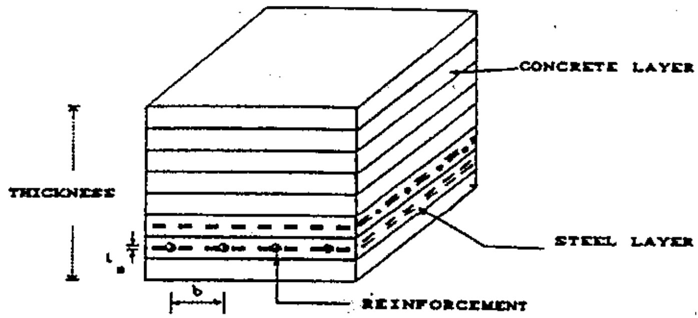
Biaxial strength envelopes.

Figure 2

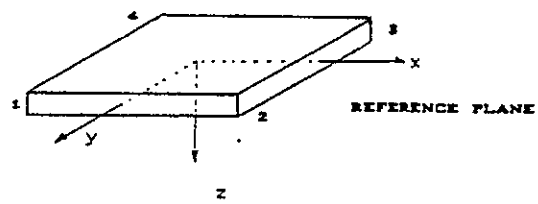


Stress-strain relation for steel.

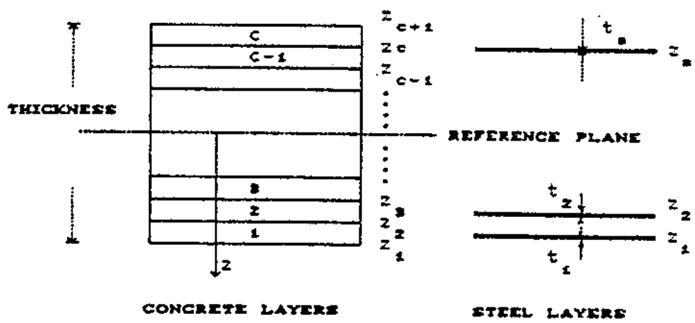
Figure 3



Layer system.

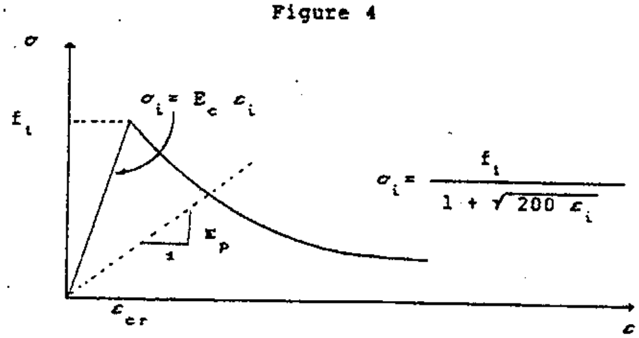


(b) Coordinate system



Construction of a typical
layer system.

Figure 4



Concrete in principal tensile direction

Figure 5

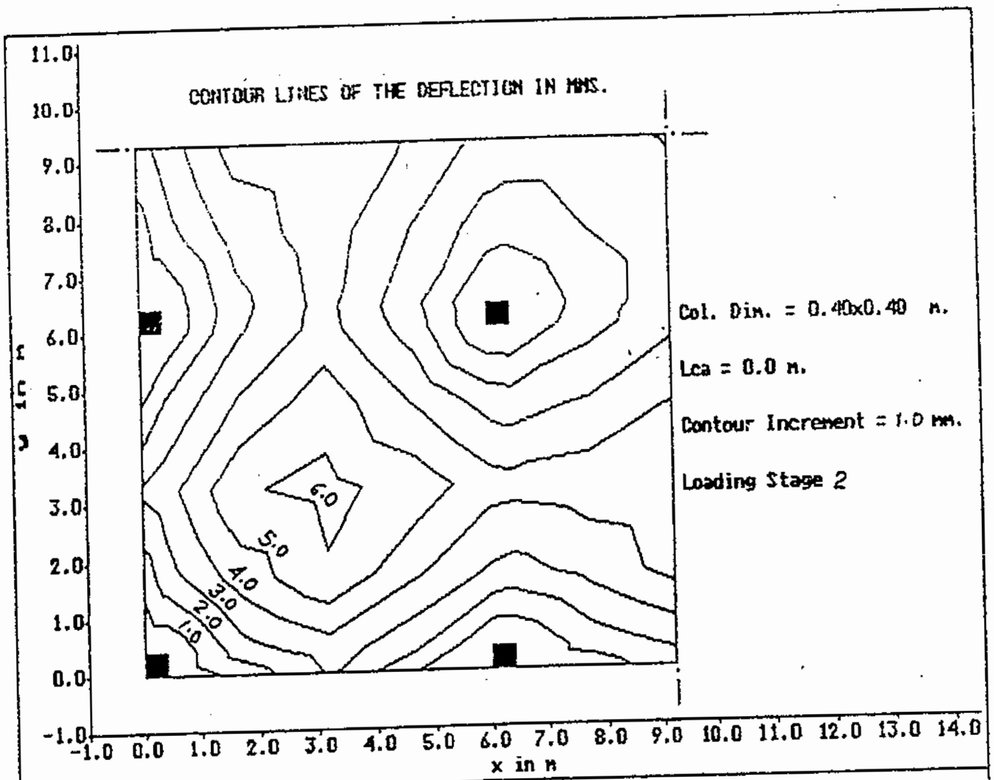


Figure (6)

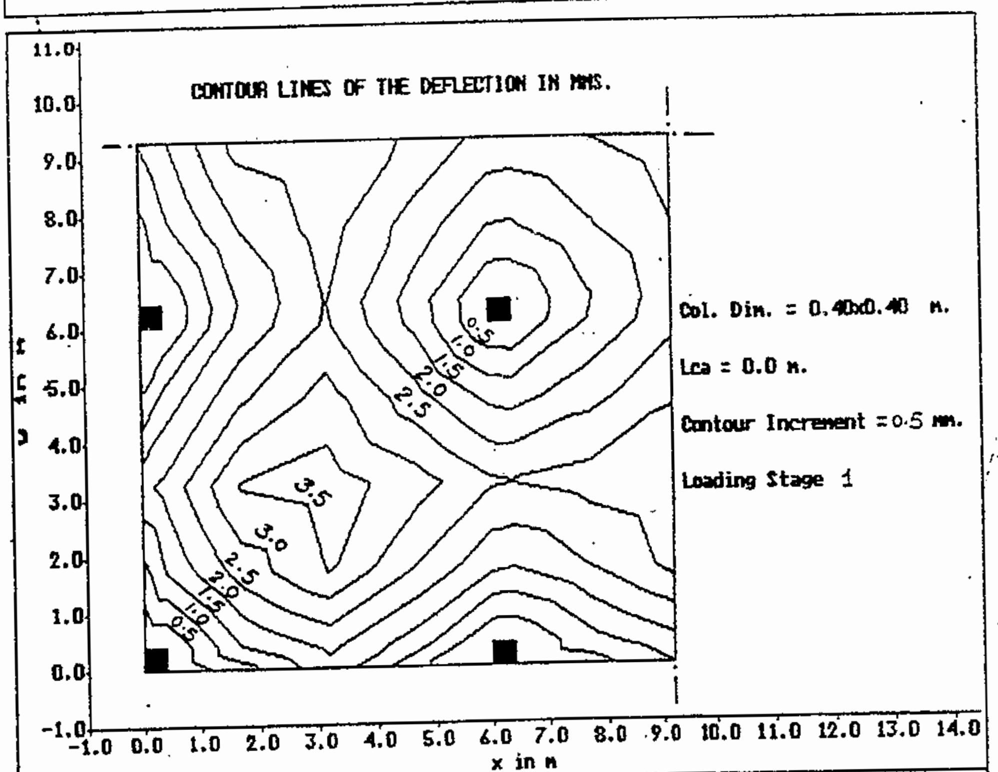


Figure (7)

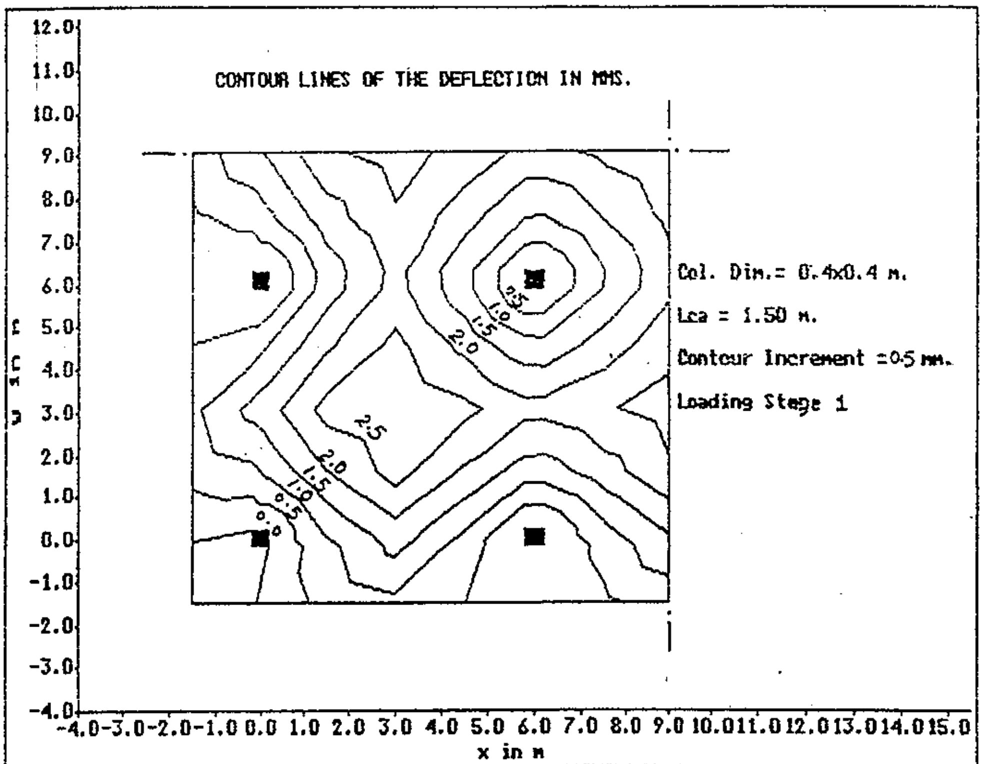


Figure (8)

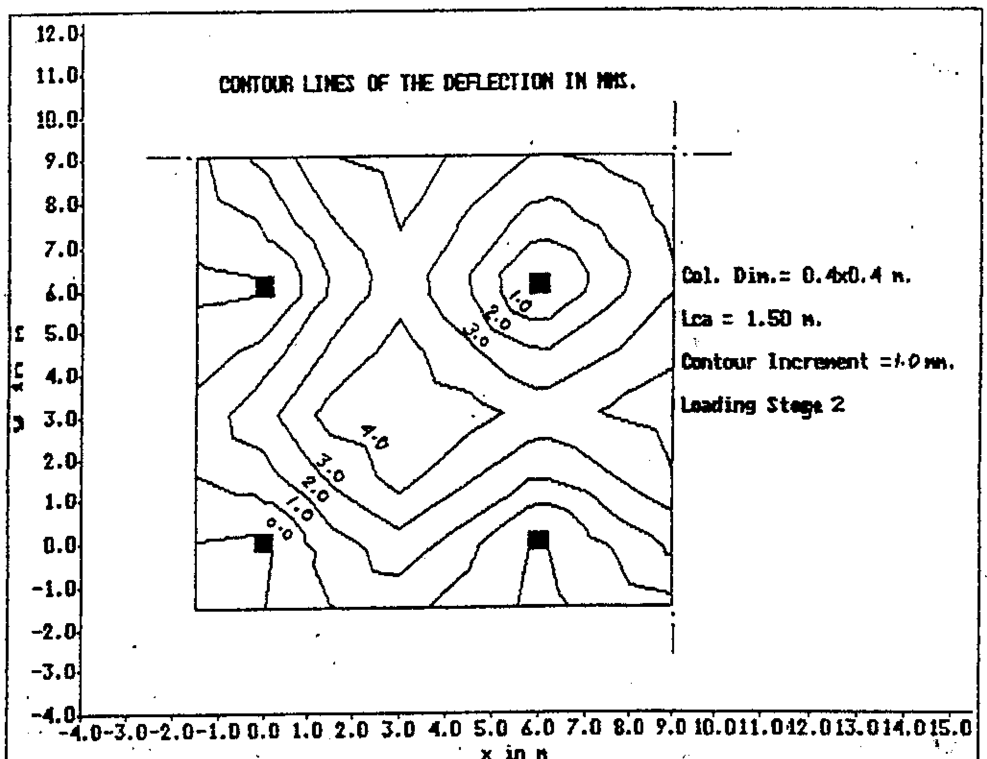


Figure (9)

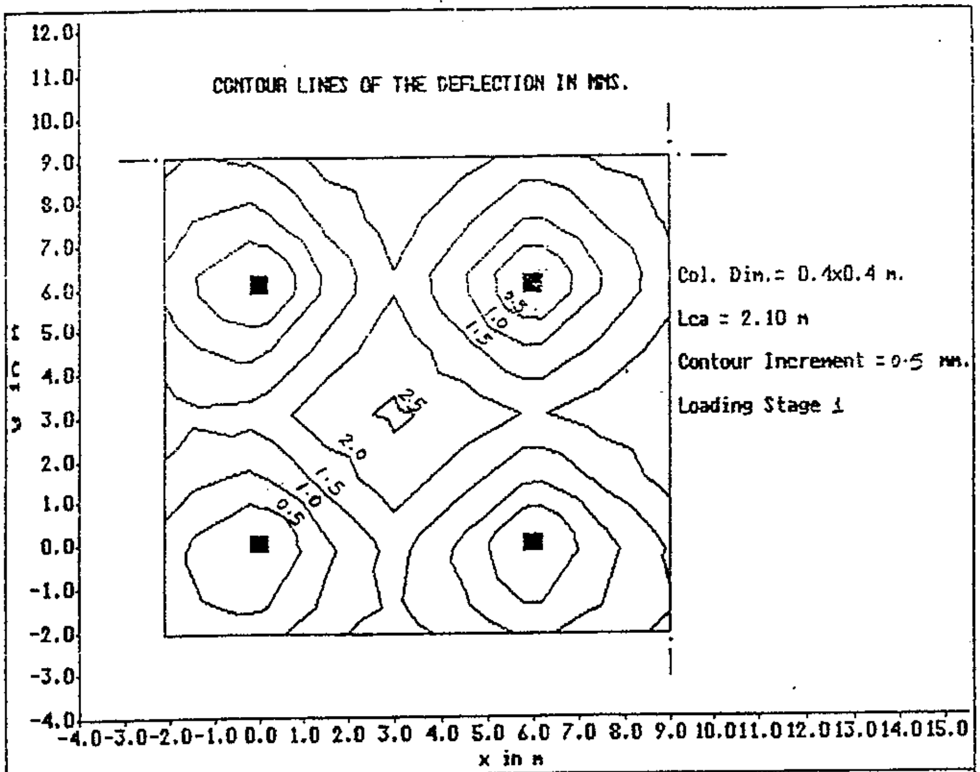


Figure (10)

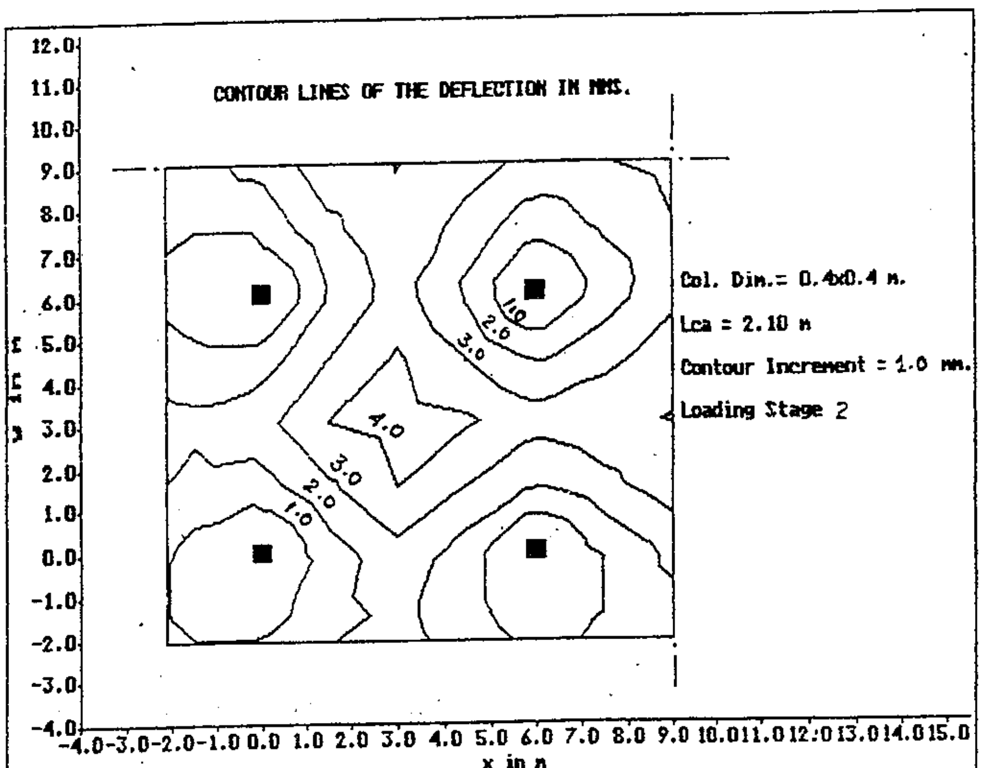
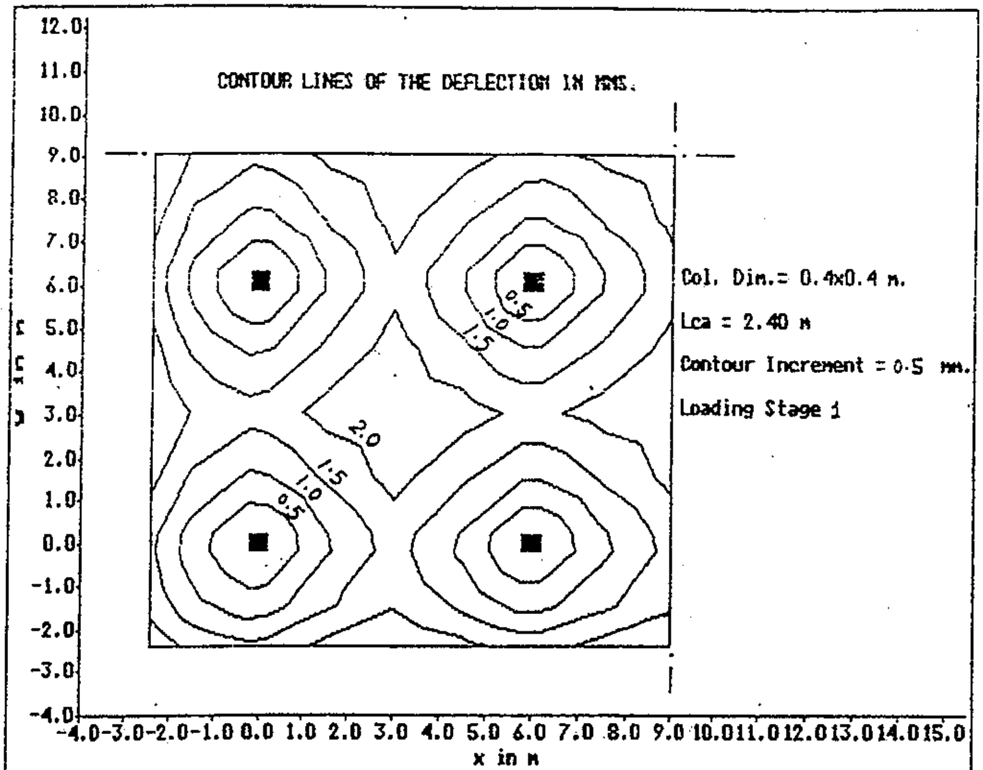


Figure (11)



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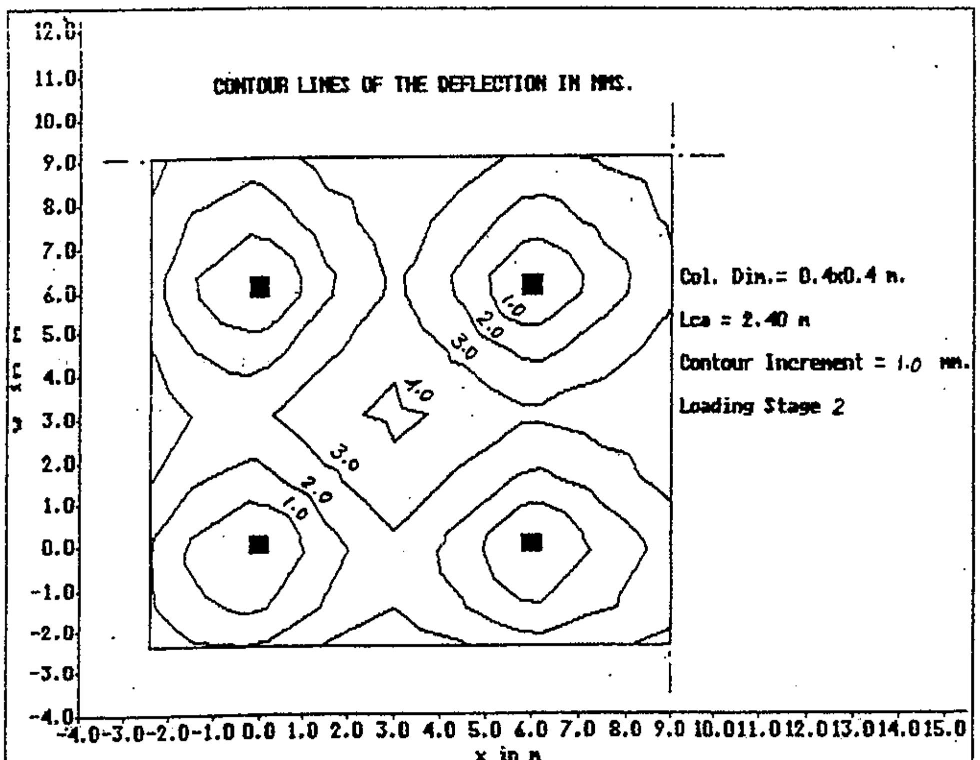

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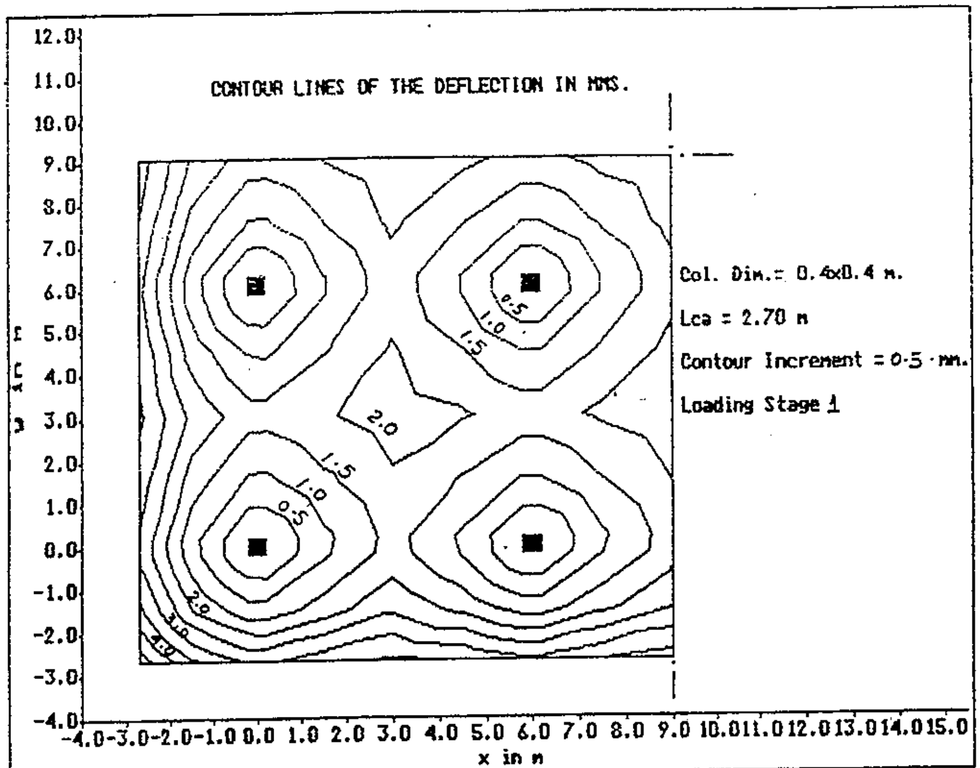

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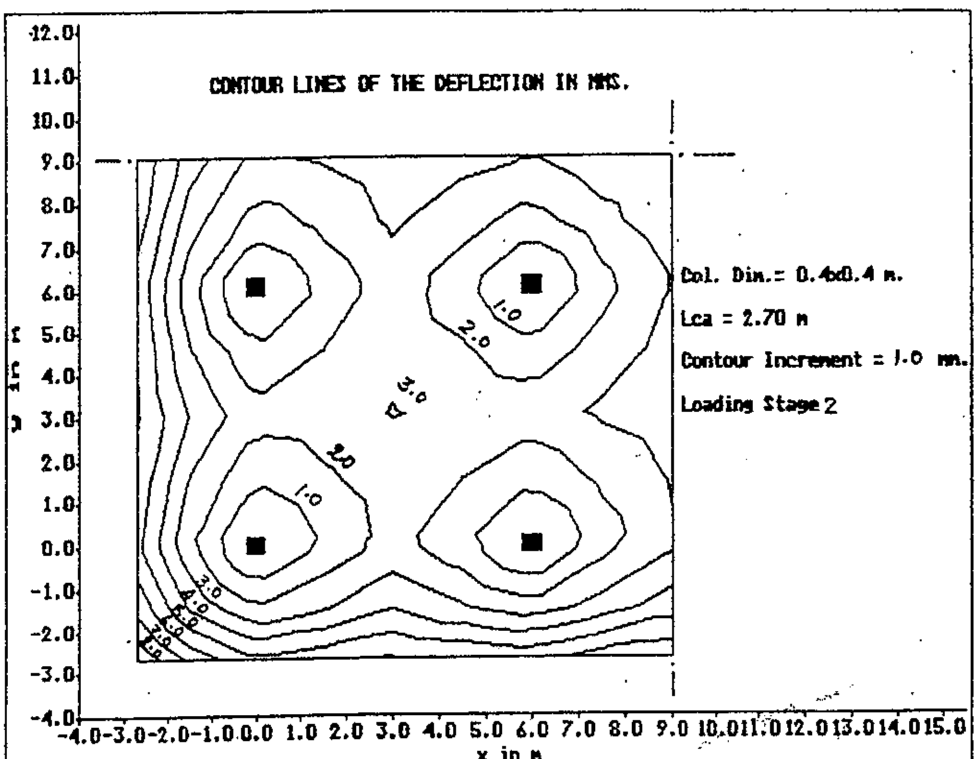

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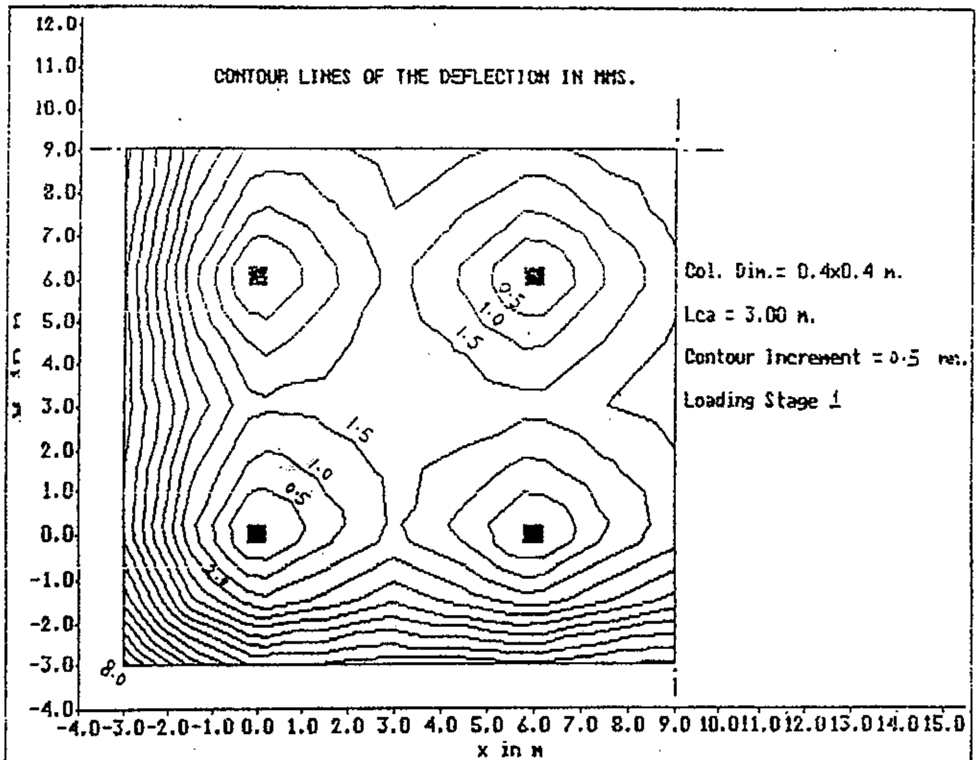

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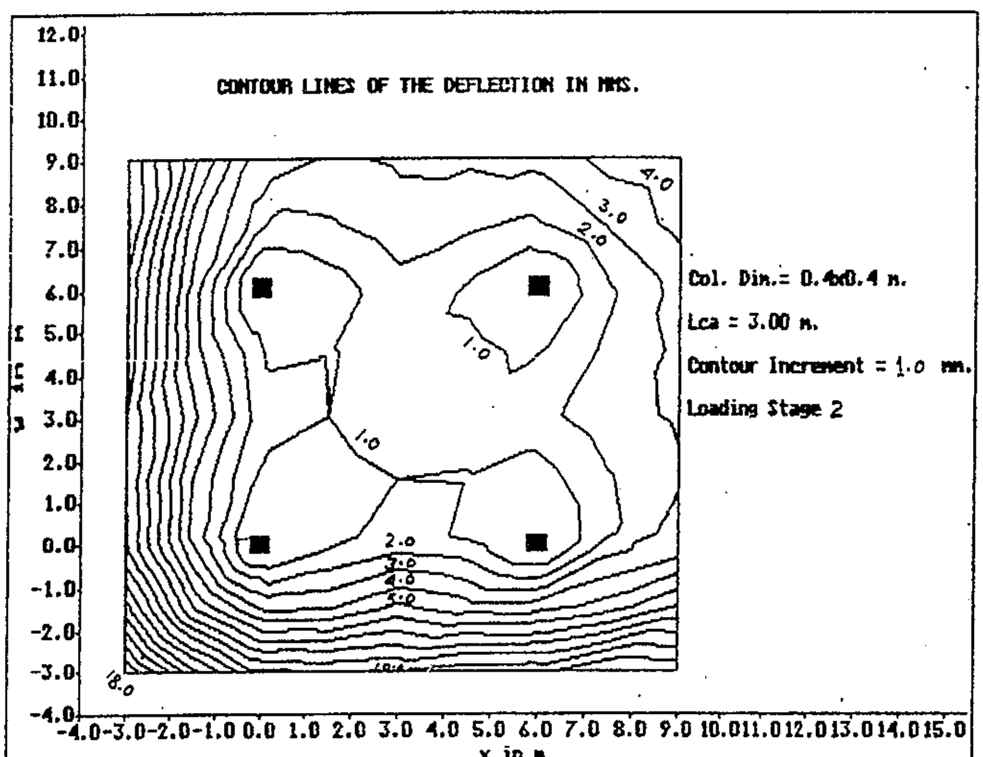
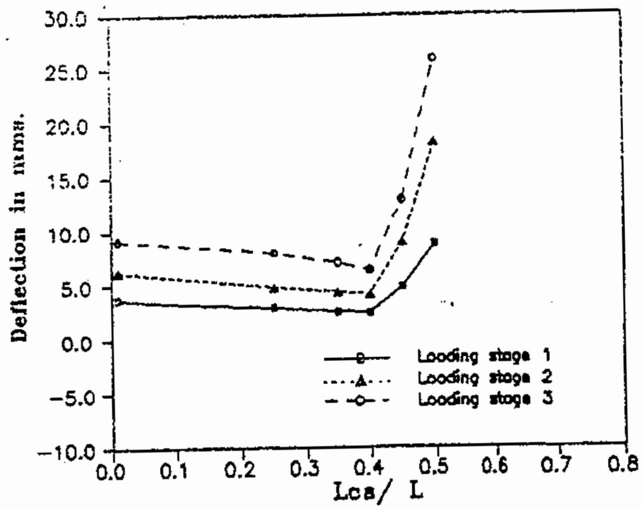


Figure (17)



Maximum deflection in the studied flat-slabs with drop panels and cantilevers.

Figure 18

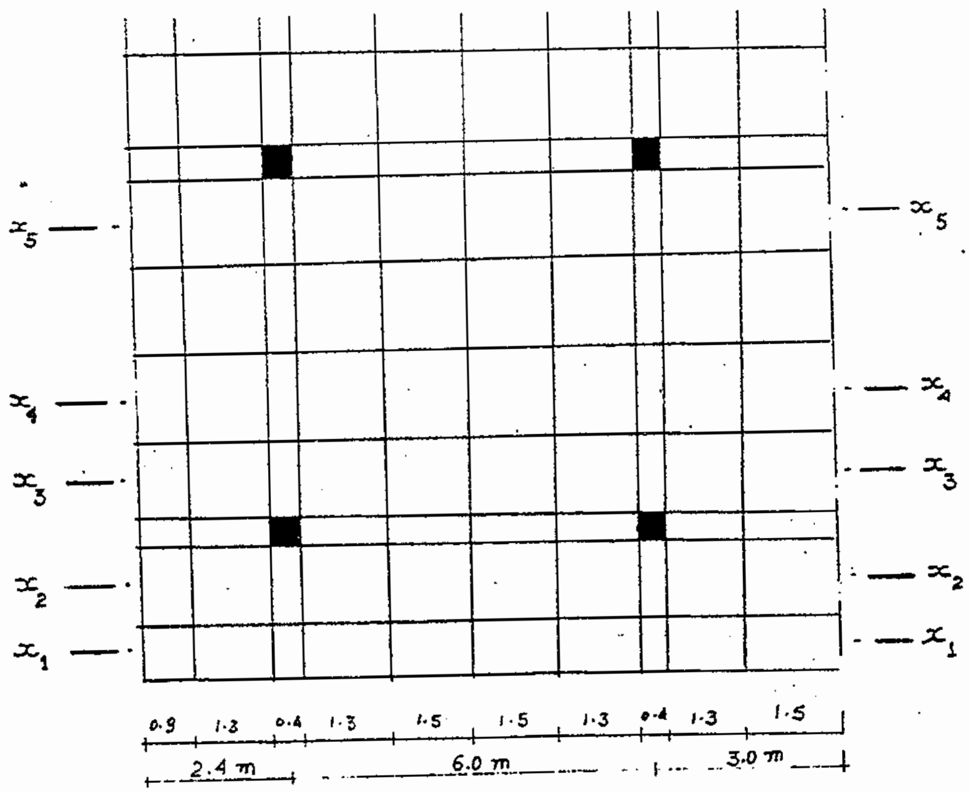
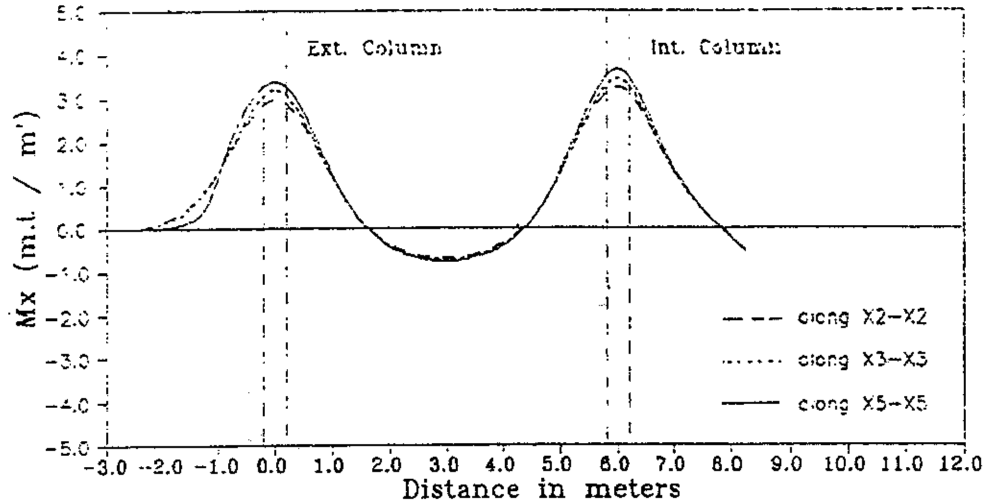
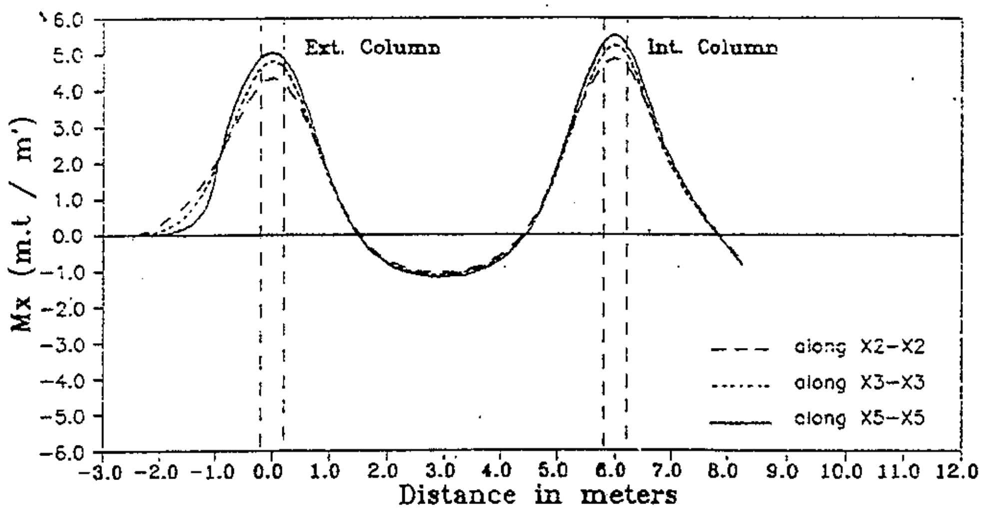


Figure 19



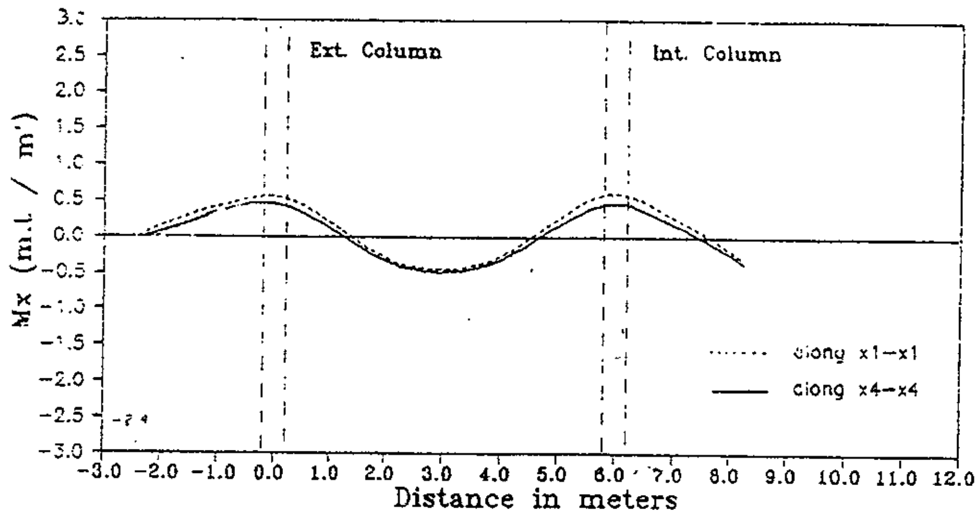
Moments Mx along axes (x2–x2), (x3–x3) and (x5–x5)
for loading stage 1

Figure 20 (a)



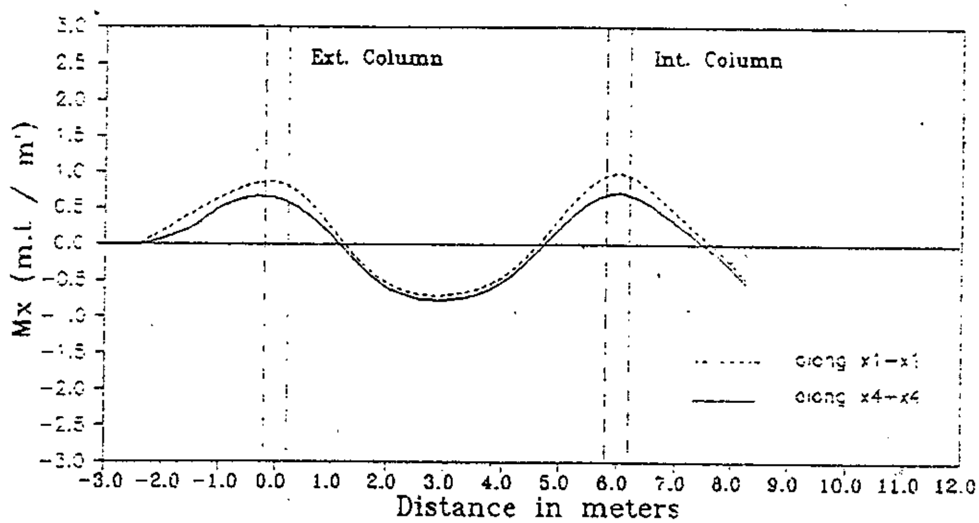
Moments Mx along axes (x2–x2), (x3–x3) and (x5–x5)
for loading stage 2

Figure 20 (b)



Moments Mx along axes (x1-x1), (x4-x4)
for loading stage 1

Figure 21 (a)



Moments Mx along axes (x1-x1), (x4-x4)
for loading stage 2

Figure 21 (b)

"التحليل اللاخطـــى للبلاطات المستويـــة ذات الكوابيل السمك الأكبر حول الأعمـــدة"

أ٠د/"منيـــر حسيـــن سليمـــان   و   م/ناجـــح نصيـــف مليكـــه

قسم الهندسه المدنية ــ كلية الهندسة بشبين الكوم ــ جامعـــة المنوفيـــة

الملخـــــــص العربـــــــى :ــ

تـــم استخـــــدام طريقـــة العناصـــر المحـــددة اللاخطيـــه لدراســـــــة ســـــلوك البلاطـــة المستويـــة ذات الكوابيـــل مختلفـــة الأطـــوال وكذلـــــك ذات سمـــك أكبـــر مختلـــف حـــول الأعمـــدة فى مراحـــل تحميـــل مختلفـــة لاستنتـــــاج الترخيـــــم وتوزيعـــات العـــزوم للحـــالات المختلفـــة ٠

وقـــد تـــم عـــرض مثـــال رقمـــى لتوضيـــح طريقـــة الحـــل والوصـــول إلـــى الاستنتاجـــات المطلوبـــه ٠