Utilization of Turbulence Modeling for Predicting Convective Heat Transfer of Turbulent Boundary Layers

O. E. Abdellatif
Lecturer, Dept. Of Mechanical Eng., Zagazig Univ. (Shoubra)

استخدام نماذج الاضطراب للتنبؤ بانتقال الحرارة بالحمل لطبقات جدارية مضطربة

دكتور/ أسامة عزت عبد اللطيف
المدرس بكلية الهندسة بشبرا-فرع بنها- جامعة الزقازيق

الملخص العربي:

يمثل هذا البحث طريقة عددية جديدة لحساب السريان و الانتقال الحراري بالحمل للطبقات الجدارية التامة الاضطراب. و يتم الحساب اعتمادا على نموذج K - ε اللاخطى الذي اقترحه Speziale (١٩٨٧) . و يتم استخدام نموذج اضطرابي حراري ثنائي المعادلة يتكامل مع معادلة الطاقة لحساب الكميات الحرارية المختلفة. وتركز النتائج على الكميات ذات الأهمية الهندسية التطبيقية مثل معامل الاحتكاك (C_f) و رقم ستانتون (St). و قد أظهرت النتائج و المقارنات أنه يمكن التوصل للخصائص المختلفة المميزة للطبقة الجدارية تامة الاضطراب باستخدام نموذج حسابي خاص بدراسة السريان الاضطرابي كامل التطور فى القنوات المستقيمة الغير دائرية المقطع على أن يكون المقطع مستطيلا وله نسبة باع كبيرة (لا تقل عن ١٠:١).

Abstract

This paper presents a new numerical approach for computing flow and convective heat transfer of turbulent boundary layers. Turbulent flow is calculated using a nonlinear K-ε model (proposed by Speziale, 1987). The closure of the energy equation is presented at the two-equation level of turbulence modeling (proposed by Nagano and Kim, 1988). The results concentrate on the time-mean quantities such as skin friction coefficient (C_f) and Stanton-number (St). It is shown that turbulent boundary layer characteristics can be found using fully developed duct flow having a cross-section of a large aspect ratio.

Accepted April 26, 1999

Nomenclature

AR	Aspect ratio.
C_f	Skin friction coefficient $= \dfrac{\tau_w}{0.5\,\rho\,V_r^2}$.
L_r	Reference length.
Pr	Prandtl number.
q_w	Wall heat flux.
$\overline{q_w}$	Averaged wall heat flux.
Re	Reynolds number $= \dfrac{\rho\,V_r\,L_r}{\mu}$.
R_θ	Reynolds number based on momentum thickness $= \dfrac{\rho\,V_r\,\theta}{\mu}$.
St	Stanton number $= \dfrac{q_w}{\rho\,C_p\,V_r\,(T_w - T_r)}$.
T_r	Reference temperature.
T_w	Wall temperature.
V_r	Free stream velocity
μ	Dynamic viscosity.
ρ	Air density.
θ	Boundary layer momentum thickness.
τ_w	Wall shear stress.
$\overline{\tau_w}$	Averaged wall shear stress.

1. Introduction

The accurate prediction of the flow and thermal quantities of the boundary layers over a flat plate is a very important aspect in the design of gas turbine blading. The quality of these predictions can influence the aerodynamic efficiency and, through its effect on the cooling design, both the cycle efficiency and the hardware durability. Several attempts have been made to investigate flow and heat transfer for turbulent boundary layers as well as examining available turbulence models for applicability to this problem.

Chambers and Wilcox (1977) made an examination of four types of two-equation turbulence closure models for boundary layers. They stated that two-equation models have a wider range of applicability than mixing length models. Kader (1981) discussed the different approaches to calculate temperature and concentration profiles in fully turbulent boundary layers. Sherma et al. (1982) investigated experimentally the development of boundary layers over the suction sides of two specific turbine airfoils. The obtained data were intended to be used to develop an improved turbulence model suitable for application to turbine airfoil design. Blair (1982) carried out experimental study to investigate the influence of freestream-turbulence on boundary layer transition in favorable pressure gradients. Schneider and Wasel (1985) studied numerically the breakdown of the boundary-layer approximation for mixed convection above a horizontal plate. They concluded that the boundary layer breakdown is due to the increase of adverse hydrostatic pressure accompanied with the increase of boundary-layer thickness. Laassibi et al. (1992) made an adaptation of the classical K-ε model for the simulation of a thermally stable stratified turbulent boundary layer. They proposed a new relation concerning the turbulent Prandtl number. Lindberg (1994) studied three-dimensional boundary layer with different K-ε near-wall models. He found that damping functions based on direct simulation data and experiments are better than those based on trial and error. Menter (1994) proposed two new two-equation eddy-viscosity turbulence models. The new modifications lead to major improvements in the prediction of adverse pressure gradient flows.

The principal motivation for the present work is the noticed similarity between the governing equations solved for both the fully developed flow through straight non-circular ducts and the turbulent boundary layers. These equations are "parabolic" in nature. Based on the author's experience with rectangular ducts (1990), he noticed that as the aspect ratio increases the behavior of the flow approaches that of a turbulent boundary layer. Thus, the idea of utilizing the nonlinear K-ε model which is used for rectangular ducts for solving turbulent boundary layers can be approached. This approach is validated by the results of an experimental investigation carried out by the author during his presence in London University (1989). Wherever possible, results of both flow and thermal fields are compared with previous experimental and theoretical data available in the literature.

2. Governing Equations

2.1. Nonlinear K-ε model

This model was proposed by Speziale (1987) and developed by the author (1990). Generally, a two-equation model uses the concept of eddy viscosity (γ_t) so as to describe the magnitude of turbulence intensity and its spatial extent. In the K-ε model, the velocity scale of turbulence is represented by $K^{0.5}$ determined from the turbulence kinetic energy (K). The length scale is given by the eddy length scale $L\varepsilon = K^{0.5}/\varepsilon$. the complete model is:

- Continuity Equation:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0 \tag{1}$$

- Momentum Equations:

$$\frac{\partial U^2}{\partial x}+\frac{\partial UV}{\partial y}+\frac{\partial UW}{\partial z}=-\frac{1}{\rho}\frac{\partial \bar{P}}{\partial x}+[\frac{\partial}{\partial x}(\nu\frac{\partial U}{\partial x})]+[\frac{\partial}{\partial y}(\nu\frac{\partial U}{\partial y})]-[\frac{\partial \overline{u^2}}{\partial x}+\frac{\partial \overline{uv}}{\partial y}]$$

$$\frac{\partial UV}{\partial x}+\frac{\partial V^2}{\partial y}+\frac{\partial VW}{\partial z}=-\frac{1}{\rho}\frac{\partial \bar{P}}{\partial y}+[\frac{\partial}{\partial x}(\nu\frac{\partial V}{\partial x})]+[\frac{\partial}{\partial y}(\nu\frac{\partial V}{\partial y})]-[\frac{\partial \overline{uv}}{\partial x}+\frac{\partial \overline{v^2}}{\partial y}]$$

$$\frac{\partial UW}{\partial x}+\frac{\partial VW}{\partial y}+\frac{\partial W^2}{\partial z}=-\frac{1}{\rho}\frac{\partial \bar{P}}{\partial z}+[\frac{\partial}{\partial x}(\nu\frac{\partial W}{\partial x})]+[\frac{\partial}{\partial y}(\nu\frac{\partial W}{\partial y})]-[\frac{\partial \overline{uw}}{\partial x}+\frac{\partial \overline{vw}}{\partial y}] \tag{2}$$

- K-equation:

$$U_i\frac{\partial K}{\partial x_i} = \frac{\partial}{\partial x_i}[(\frac{\nu_t}{\sigma_k})\frac{\partial K}{\partial x_i}] - \varepsilon + \frac{1}{\rho}G \tag{3}$$

- ε-equation:

$$U_i \frac{\partial \varepsilon}{\partial x_i} = \frac{\partial}{\partial x_i}[(\frac{\nu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial x_i}] + \frac{\varepsilon}{K}[C_{\varepsilon 1}\frac{G}{\rho} - C_{\varepsilon 2}\,\varepsilon] \qquad (4)$$

Where $G = -\rho\,\overline{u_i u_j}\,(\frac{\partial U_i}{\partial x_j})$ is a turbulence generation term.

- Turbulent viscosity equation:

$$\nu_t = C_\mu \frac{K^2}{\varepsilon} \qquad (5)$$

Where Cμ is a model constant.

-The nonlinear representation of Reynolds stresses ($-\overline{u_i u_j}$) is presented in detail in Abdellatif (1990). In the above set of equations, the values of the constants are: $\sigma_\varepsilon = 1.3$, $C_{\varepsilon 1} = 1.44$, $C_{\varepsilon 2} = 1.92$, $C_\mu = 0.09$, $K = 0.42$.

2.2 Two-Equation Model for Temperature Field

This model was proposed by Nagano and Kim (1988) and developed by Abdel Gawad (1998). When temperature is regarded as a passive scalar, the transport equations are expressed as:

- Energy equation:

$$U_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}(\alpha \frac{\partial T}{\partial x_j} - \overline{u_j t}) \qquad (6)$$

- Temperature variance ($\overline{t^2}$) equation:

$$U_j \frac{\partial \overline{t^2}}{\partial x_j} = \frac{\partial}{\partial x_j}[(\alpha + \frac{\alpha_t}{\sigma_h})\frac{\partial \overline{t^2}}{\partial x_j}] - 2\overline{u_j t}\frac{\partial T}{\partial x_j} - 2\varepsilon_t - 2\alpha(\frac{\partial \sqrt{\overline{t^2}}}{\partial x_j})^2 \qquad (7)$$

- Temperature variance dissipation rate (ε_t) equation:

$$U_j \frac{\partial \varepsilon_t}{\partial x_j} = \frac{\partial}{\partial x_j}[(\alpha + \frac{\alpha_t}{\sigma_\phi})\frac{\partial \varepsilon_t}{\partial x_j}] - C_{p1} f_{p1} (\frac{\varepsilon_t}{\overline{t^2}})\overline{u_j t}\frac{\partial T}{\partial x_j} - C_{p2} f_{p2}(\frac{\varepsilon_t}{K})\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}$$

$$-C_{D1} f_{D1}\frac{(\varepsilon_t)^2}{\overline{t^2}} - C_{D2} f_{D2}\,\varepsilon\frac{\varepsilon_t}{K} + \alpha\alpha_t\,(1 - f_\lambda)(\frac{\partial^2 T}{\partial x_j \partial x_k})^2 \qquad (8)$$

- Turbulent heat flux ($-\overline{u_j t}$) equation:

$$-\overline{u_j t} = \alpha_t\,(\frac{\partial T}{\partial x_j}) \qquad (9)$$

- Eddy diffusivity for heat (α_t) equation:

$$\frac{\alpha_t}{\nu} = C_\lambda\, f_\lambda\, Re_h \qquad (10)$$

Where $Re_h = K\left(\sqrt{\frac{K\,\overline{t^2}}{\varepsilon\ \varepsilon_t}}\right)/\nu$

In the above model, the values of the constants and the model functions which account for wall-proximity effects are, Abdel Gawad (1998):

$C_\lambda = 0.11$, $f_\lambda = (1 - \exp(-2\,(Pr)^{0.5}\ St\, y^+ / (30.5\, C_f)))^2$, $\sigma_\phi = 1.0$, $f_{p1} = 1.0$, $f_{p2} = 1.0$, $f_{D1} = 1.0$, $f_{D2} = 1.0$, $A = 30.5$, $C_{D1} = 2.2$, $C_{D2} = 0.8$, $C_{p1} = 1.8$, $C_{p2} = 0.72$.

3. Computational Details

The integration of the governing equations are carried out using a finite difference discretization on a staggered grid using a modification, Abdellatif (1990), of the SIMPLE technique proposed by Launder and Spaldings (1972). Convergence is measured in terms of the maximum residual in each iteration. The maximum residual allowed for the convergence check is set to 10^{-5}. Extensive study concerning the grid independence was performed.

4. Results and Discussion

Air is chosen as a test fluid with a molecular Prandtl number (Pr) of 0.71.

4.1. Preliminary Results

In this section preliminary results concerning the case of a straight rectangular duct with different aspect ratios are presented. This step is made to make sure that the numerical scheme and turbulence models behave in a good manner. Another benefit is to observe the dramatic change of flow and thermal quantities, with increasing the aspect ratio, towards the turbulent boundary layer behavior.

Figure (1) represents the normalized wall shear stress $(\frac{\tau_w}{\overline{\tau}_w})$ distribution along the long wall of the rectangular duct for different aspect ratios (AR) in comparison with the experimental data of Knight et al. (1985) at the corresponding Reynolds numbers (Re) of the experiments. These experimental results are the only available in the literature which cover a wide range of aspect ratios. Good agreements are noticed between the predictions and the experimental data.

Figure (2) shows the distribution of normalized wall heat flux $(\frac{q_w}{\overline{q}_w})$ along the long side of the rectangular duct for the same aspect ratios at a Reynolds number of 6.5×10^4. It can be seen that the increase of the aspect ratio forces the heat transfer to be more uniform in the middle region and increases its rate in this area (the maximum of $(\frac{q_w}{\overline{q}_w}) \approx 1.1$ for the square duct, whereas, the maximum of $(\frac{q_w}{\overline{q}_w}) \approx 1.3$ at aspect ratio (10)). Unfortunately, there are no available numerical or experimental data for the thermal field in rectangular ducts in the available literature to compare with.

Figure (3) shows the variation of skin friction coefficient (C_f) and Stanton number (St) with a wide range of Reynolds number. The presented values of (C_f) and (St) are averaged over the four walls of the duct cross section. Very good agreement is noticed with the empirical formulae (Cebeci and Bradshaw (1988), White (1991)) especially for higher Reynolds numbers. The Reynolds analogy is maintained. From the above discussion, we can conclude that the results of the present modeling approach are satisfactory. Thus, it is safe to move to the next step of investigating the turbulent boundary layer.

4.2 Turbulent Boundary Layer

By increasing the aspect ratio (AR) to 10:1, flow represents the case of turbulent boundary layer over a horizontal flat plate with zero pressure-gradient and no freestream-turbulence.

Figure (4) represents the predicted values of skin friction coefficient (C_f) at different Reynolds numbers (R_θ). R_θ is the flow Reynolds number based on the boundary layer momentum thickness (θ). The predictions are in a very good agreement with the empirical formulae of Coles (1962). This result is a very encouraging one.

Figure (5) shows the predicted results of the Stanton number (St) at the same Reynolds numbers (R_θ). Comparison is made with the experimental data found by the author. Generally, the same trend is well kept. The noticed difference between the predictions and the experimental data decreases with the increase in R_θ. At $R_\theta = 1667$, the difference is about 16.22%. Whereas, at $R_\theta = 5444$, the difference reduces to 4.6%. As reported by other investigators (Hirota et al., 1994), errors are expected in such kind of measurements. In their research, they estimated the error to reach ± 14% because of the difficulty of accurately measuring the mean temperature profile near a solid boundary to estimate the wall heat flux. It seems that measurements become more accurate as the Reynolds number (R_θ) increases which means an increase in the boundary layer thickness.

5. Conclusions

A turbulence-modeling scheme is used to model the flow and thermal fields of convective turbulent boundary layers. It has been proved that the approach of utilizing the numerical code of a fully developed rectangular duct flow to predict the turbulent boundary layer is a good idea. This means, if we are interested in turbulent boundary layers, that this approach of increasing the aspect ratio is suitable without the need to construct a new code for boundary layers only. Concentration was applied to the quantities that have certain importance in actual engineering applications.

References

1. Abdel Gawad, A. F., 1998, "Computation of Turbulent Flow and Heat Transfer in Rotating Non-Circular Ducts," *Ph. D. Thesis, Mech. Power Dept., Faculty of Eng., Zagazig Univ.*
2. Abdellatif, O. E., 1990, "Numerical Simulation of Turbulence-Driven Secondary Flows in Non-Circular Ducts with and without Rotation," *Ph. D. Thesis, Zagazig Univ., Shoubra Faculty of Eng.*
3. Blair, M. F., 1982, "Influence of Free-Stream Turbulence on Boundary Layer Transition in Favorable Pressure Gradients," *J. Of Engineering for Power, SAME Transactions*, Vol. 104. pp. 743-750.

4. Cebeci, T., and Bradshaw, P., 1988, "Physical and Computational Aspects of Convective Heat Transfer," *Springer-Verlag.*
5. Chambers, T. L., and Wilcox, D. C., 1977, "Critical Examination of Two-Equation Turbulence Closure Models for Boundary Layers," *AIAA J.,* Vol. 15, No. 6, pp. 821-828.
6. Coles, D. E., 1962, "The Turbulent Boundary Layer in a Compressible Fluid," *RAND Corp. Rept. R-403-pr.*
7. Hirota, M., Fujita, H., and Yokosawa, H., 1994, "Experimental Study on Convective Heat Transfer for Turbulent Flow in a Square Duct with a Ribbed Rough Wall, Characteristics of Mean Temperature Field," *J. Heat Transfer,* Vol. 116, pp. 332-340.
8. Kader, B. A., 1981, "Temperature and Concentration Profiles in Fully Turbulent Boundary Layers," *J. Heat Mass Transfer,* Vol. 24, No. 9, pp. 1541-1544.
9. Knight, D.W., ASCE, M. and Patel, H. S.,1985 "Boundary Shear smooth Rectangular Duct," J. of Hydraulic Engineering, Vol. 111, No 1,pp. 29-47.
10. Laassibi, A., Morel, R., Jeandel, D., Brun, G., and Alcaraz, E., 1992, "Adaptation of the Classical K-ε Model for the Simulation of a Thermally Stable Stratified Turbulent Boundary Layer," *Fourth International Conference of Fluid Mechanics, Alexandria, Egypt, April 28-30, 1992.*
11. Launder, B. and Spalding, D. B.,1972 "Mathematical Models of Turbulence," *academic Press, London, U.K.*
12. Lindgerg, P. A., 1994, "Near-Wall Turbulence Models for 3D Boundary Layers," *Applied Scientific Research J.,* Vol. 53, pp. 139-162.
13. Menter, F. R., 1994, "Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications," *AIAA J.,* Vol. 32, No. 8, pp. 1598-1605.
14. Nagano, Y., and Kim, C., 1988,"A Two-Equation Model for Heat Transport in Wall Turbulent Shear Flows," *J. Heat Transfer, ASME Transactions,* Vol. 110, pp. 583-589.
15. Schneider, W., and Wasel, M. G., 1985, "Breakdown of the Boundary-Layer Approximation for Mixed Convection above a Horizontal Plate," *Int. J. Heat Mass Transfer,* Vol. 28, No. 12, pp. 2307-2313.
16. Sharma, O. P., Wells, R. A., Schlinker, R. H., and Bailey, D. A., 1982, "Boundary Layer Development on Turbine Airfoil Suction Surfaces," *J. Of Engineering for Power, ASME Transactions,* Vol. 104, pp. 698-705.
17. Speziale, C. G., 1987, "On Nonlinear K-l and K-ε Models of Turbulence," *J. Fluid Mechanics,* Vol. 178, pp. 459-475.
18. White, F. M., 1991, "Viscous Fluid Flow," *McGraw-Hill.*