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A STUDY OF THE EFFECT OF CHANGE IN MACH NUMBER ON VISCOUS SUPERSONIC FLOW PAST A FLAT PLATE

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دراسة تأثير تغيير عدد ماخ على الانسياب اللزج الفوق صوتي على سطح مستوى

خلاصة : يقدم هذا البحث دراسة نظرية عن تأثير اللزوجة في السرعات فوق الصوتية لهواء منساب على سطح مستوى ذو سمك معين ومقدمة حافة حادة ويقدم البحث تعريفا للمسألة الطبيعية المطلوب حلها وعمل الفروض المناسبة . واهم هذه الفروض ان السريان مستقر، ذو بعدين وان قوانين الغاز المثالي امكن تطبيقها لتبسيط المعادلات التي تحكم هذا البحث حتى يمكن حلها رياضيا. بالاضافة الى هذه الفروض فقد اعتبر انحدار الضغط وخاصة على الحافة الحادة بأنه يمكن اهماله. وقد درس التغيير في رقم ماخ.

وقد اظهرت نتائج هذا البحث ان رقم ماخ له التأثير الغالب على معامل الاحتكاك c_f، ودرجة حرارة السطح T_w، كذلك سمك الطبقات الجدارية المختلفة تتوقف اساسا على سرعة السريان.

اخيرا يمكن ان تقودنا هذه الدراسة الى تصور عن نوعية وخواص المواد المطلوب استخدامها لتجنب التأثير الحراري والاجهادات.

ABSTRACT - This paper presents a theoretical study of the viscous effects in supersonic air flow on a flat plate with finite thickness and sharp leading edge. The physical problem is given and the solution considered is based on suitable assumptions. These most important assumptions are that the flow is steady, two-dimensional, compressible and the perfect gas law is applied for simplifying the equations governing this problem to be solved analytically. Moreover, the effect of the pressuregradient due to the leading edge angle is neglected The change in the value of the Mach number is investigated. This investigation leads to a result that the Mach number has the major effect on the local skin friction coefficient c_f and the achieved wall temperature, T_w. Also, different boundary layer thicknesses depend mainly on the flow velocity.

Finally this study can lead to suggession about the expected type and properties of the material to endure the thermal and shear stresses resulted as effect of supersonic flow on the flat plate.

A computer program is constructed for the numerical analysis of this problem in BASIC languge.

NOMENCLATURE

a	speed of sound,	(m/s)
C	constant depending on the size of the molecule	

$\bar{c}$	velocity at the outer edge of boundary layer,	(m/s)
c_∞	free stream velocity,	(m/s)
c_f	local skin friction coefficient, $\tau_w / (1/2. \rho \bar{c}^{2})$	
c_x	velocity of the fluid inside the boundary layer in x-direction,	(m/s)
c_y	velocity component inside the boundary layer in y-direction,	(m/s)
c_p	specific heat at constant pressure	(J/kg..K)
c_v	specific heat at constant volume,	(J/kg..K)
i	local enthalpy,	(J/kg)
k	thermal conductivity,	(W/m..K)
p_∞	free stream static pressure,	(N/m²)
M	Mach number	
r_f	recovery factor	
T	temperature,	(K)
x, y	cartesian coordinates	

GREEK LETTERS

α	angle of attack,	
γ	ratio of specific heats, (c_p / c_v)	
δ	boundary layer thickness,	(m)
δ^*	displacement thickness, $\int_0^\infty (1 - c_x / \bar{c}).\, dy$	(m)
δ^{**}	boundary layer momentum thickness, $\int_0^\infty c_x/\bar{c}\, (1 - c_x / \bar{c})\, dy$	(m)
θ	leading edge semi-vertex-angle,	(degree)
β	angle of Mach line	(degree)
μ	dynamic viscosity,	(kg/.s.m)
ν	kinematic viscosity,	(m²/s)
ρ	mass density,	(kg/m³)
τ	shear stress,	(N/m²)
Φ	dissipation function	
$\bar{\theta}$	function of η	

DIMENSIONLESS PARAMETERS

$c_x / \bar{c}$	$= f(\eta)$
j	$= i/i_n$
m	$= u/u_o$
r	$= \rho/\rho_n$
$Re_{\delta^{**}}$	momentum thickness Reynolds number, $\bar{c}.\delta^{**}/\nu$
Pr	Prandtl number, $[\nu/a$ thermal diffusivity m²/s]

SUBSCRIPTS

∞	upstream of the plate
n	regions 1.....4
o	condition at 0 °C
w	wall

INTRODUCTION

The problem of the viscous effects in supersonic flow (air) past a flat plate with finite thickness has gained a significant importance in recent years. The study of the previous work in the field shows that all practical and theoretical studies do not consider the effects of the viscosity on both sides of the flat plate in their analysis.

The problem under investigation may be reduced to the problem of finding the steady flow over a flat plate. A general method for solving the laminar and turbulent boundary layer equations for three dimensional flow in cartesian coordinates is introducd by Douglas Cebci [1]. The accuracy of the results depends on the accuracy of the numerical method and the accuracy of the model for the Reynolds stresses.

The results of a detailed experimental study of velocity and temperature profiles, and also the skin friction in a test section of a low turbulence wind tunnel are presented by Kohn Kin and Shapovalov [2]. The experiments were carried out with values of Reynolds number, $Re_{\delta**}$, ranged from $0.5*10^4$ to $7.7*10^4$ and Mach number equal to 2, 3 and 4. A miniature balance was utilized to measure the skin friction in laminar and turbulent flows on a flat plate at M equal to 2 and 3 and different Reynolds number.

S.L. Gai [3] showed that for a flat plate of finite chord inclined to an incoming supersonic stream, the separation point on leeward side depends on the Mach number, Reynolds number and angle of attack.

A theoretical study for temperature distribution across the boundary assuming a linear absolute viscosity temperature relationship, constant specific heat and constant but arbitrary Prandtl number was suggested by Radkiewicz and Skiepko [4].

The main objective of this study is to analyze and evaluate the viscous effects in supersonic flow past a flat plate which has a sharp leading edge at different values of Mach number. The change in the flow parameters occuring due to the chock wave at the points of intersection of two planes has been investigated.

The study is based on the equation of steady motion, two-dimensional, compressible and laminar flow. On these assumptions, a mathematical model has been formulated to solve the introduced problem without affecting the main line of the solution.

2-ANALYSIS

2.1- The Physical Model

To have a clear imagination about the analysis of the problem, a schematic sketch of the flat plate is presented in figure 1. From the geometry of the leading edge of the flat plate at the points of conjuction between the inclined and upper surface, there is critical zone. A discontinuity of the flow parameters happens at that point. An assumption is introduced to overcome this obstacle. This assumption considers the inclined portion ab of the leading edge as an extension to the upper surface of the flat plate, which equals to:

$$L_1 = \frac{\text{plate thickness}}{\sin\theta}$$

It is clear from Fig. 1 that the lower surface (region no. 4) has no point of discontinuity because it is completely flat. The change in the flow parameters on this surface is compared with the same parameters under the condition on the upper surface (region no. 3)

The evaluation of the flow parameters under investigation will take place for the 3rd and 4th regions at the surface of the plate only, i.e., at $\eta = 0$. This choice is based on the fact that the maximum values of c_f and T ($T = T_w$) happen at the plate surface. Moreover, the values of these parameters at $\eta = 0$ will affect the design of the flat plate, i.e., the material of the plate will be selected to endure the thermal and shear stresses as a result of the effects of supersonic flow.

So, the problem here is to investigate the change of these parameters with the change of the Mach number.

2.2- Postulate Assumptions

The postulate assumptions suggested for this physical problem are:

i) The maximum deflection angle of the flow over the body together with the minimum value of Mach number are chosen to give a shock wave on the body.

ii) The boundary layer equations are assumed to be applicable up to the point of separation.

iii) The air flow is assumed steady, compressible and perfect with constant specific heats.

iv) For air flow, Prandtl number "Pr" was found to vary within small limits around the value of 0.7 for the temperature range between -20 to 1000°C [5 , 6]. So Prandtl number is taken equal 0.7.

v) In the case of air, as shown by E. R. Van Driest [7], it is possible to use an interpolation formula based on D. M. Sutherlands theory of viscosity. This can be written as follows:-

$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{3/2} \cdot \frac{C + T_o}{C + T}$$

where μ denotes the viscosity at the reference temperature T_o, and C is a constant which for air assumes the value 114 K.

vi) The velocity and temperature profiles at different values of x are similar to that $c_x/\bar{c} = f(\eta)$ and $\partial i/\partial x = 0$.

2.3- Mathematical Description

2.3.1- Basic Equations

The governing differential equations consisting of mass, momentum, energy and equation of state are described as :

Mass conservation

$$\frac{\partial(\rho \cdot c_x)}{\partial x} + \frac{\partial(\rho \cdot c_y)}{\partial y} = 0 \qquad \ldots (1)$$

Conservation of momentum

$$\rho \cdot c_x \frac{\partial c_x}{\partial x} + \rho \cdot c_y \frac{\partial c_x}{\partial y} = - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \cdot \frac{\partial c_x}{\partial y}\right) \qquad \ldots (2)$$

Conservation of energy

$$\rho \cdot c_x \frac{\partial i}{\partial x} + \rho \cdot c_y \frac{\partial i}{\partial y} = c_x \frac{\partial p}{\partial x} + \frac{1}{Pr} \frac{\partial}{\partial y}\left(\mu \cdot c_p \cdot \frac{\partial T}{\partial y}\right) + \mu \left(\frac{\partial c_x}{\partial y}\right)^2 \qquad \ldots (3)$$

Equation of state

$$P = \rho \cdot R \cdot T \qquad \ldots (4)$$

L.Crocco; [9] ; discovered quite early a transformation which simplifies the task of integrating the equations for the cases when either (1) Pr = 1, and the viscosity function μ (T) is arbitrary, or (2) when the Prandtl number has an arbitrary value but μ/T = constant.

In the special cases of an adiabatic wall with Pr = 1 Crocco's transformation reduces the compressible boundary-layer equations to almost the same form as that valid for incompressible flow.

Using this transformation in basic equations leeds to :

Conservation of mass

$$\frac{\partial y}{\partial c_x} \cdot \frac{\partial(\rho \cdot c_x)}{\partial x} + \frac{\partial y}{\partial x} \cdot \frac{\partial(\rho \cdot c_x)}{\partial c_x} + \frac{\partial(\rho \cdot c_y)}{\partial c_x} = 0 \qquad \ldots (4)$$

Conservation of momentum

$$-\rho \cdot c_x \frac{\partial y}{\partial x} + \rho \cdot c_y = \frac{\partial \tau}{\partial c_x} \qquad \ldots (5)$$

Conservation of energy

$$\frac{\partial \tau}{\partial c_x} \cdot \frac{\partial i}{\partial c_x}(1 - Pr) + \left(Pr + \frac{\partial^2 i}{\partial c_x^2}\right)\tau^2 - Pr.\rho \cdot c_x \frac{\partial i}{\partial x} \cdot \frac{\partial y}{\partial c_x} = 0 \qquad \ldots (6)$$

To eliminate $\rho.c_y$ from equation (4) and (5) differentiate equation (5) according to c_x and equate $\partial(\rho.c_y)/\partial c_x$ from both. Introducing $\frac{\partial y}{\partial c_x} = \frac{\mu}{\tau}$, then:

$$\frac{\partial^2 \tau}{\partial c_x^2} + c_x \cdot \frac{\partial(\rho\mu/\tau)}{\partial x} \quad 0 \qquad \ldots (7)$$

and

$$(1 - Pr)\frac{\partial i}{\partial c_x} \cdot \frac{\partial \tau}{\partial c_x} \cdot \tau + \left(Pr + \frac{\partial^2 i}{\partial c_x^2}\right)\tau^2 - Pr \cdot \rho \cdot \mu \cdot c_x \frac{\partial i}{\partial x} = 0 \qquad \ldots (8)$$

Equation (7) and (8) represent the differential equations of the boundary layer in the reduced form. From these equations it is clear that the unknowns are reduced to 'i' and 'τ'. It is to be remarked that all gas properties can be achieved once 'i' is known. The enthalpy i together with the pressure defined in a unique manner any gas property can be estimated [5].

Using the assumption stated before that $\frac{\partial i}{\partial x} = 0$, and assuming a solution in the form :

$$\tau = \phi(x) \cdot f(c_x)$$

Substitute in equations (7) and (8) , then:

$$\frac{\phi'}{\phi^3} = \frac{f'' \cdot f}{\rho\mu c_x} \qquad \ldots (9)$$

and

$$\phi^2 \cdot f\,[\,(1 - Pr)\; i' \cdot f' + (i'' + Pr)\, f\,] = 0 \qquad \ldots (10)$$

Each side of equation (9) is a function of the independent variable ϕ and f , so

$$\frac{\dot{\Phi}}{\Phi^3} = \text{constant} \text{ , with } \Phi = \frac{\text{constant}}{\sqrt{x}} .$$

There is no loss of generality if it is assumed $\tau = f(x)/\sqrt{2x}$, and substituting in equations (9) and (10) it yields ,

$$f.f'' = c_x \cdot \rho \cdot \mu , \qquad \ldots (11.a)$$

and

$$(1 - P_r)\, i' .\, f' + (i'' + P_r) \,.\, f = 0 \qquad \ldots (11.b)$$

Introducing the dimensionless relations :-

$$\eta = \frac{c_x}{c_n} , \; j = \frac{i}{i_n} , \; r = \frac{\rho}{\rho_n} , \; m = \frac{\mu}{\mu_n} ,$$

$F(\eta) = \sqrt{\dfrac{2}{\bar{\rho}\,\bar{\mu}\,\bar{c}^2}} \,.\, f(c_x)$, and the relation for ideal gas $\dfrac{C_n^2}{i} = (\gamma - 1) \,.\, M_n^2$,

where n = 1, 2, 3, 4, the equations (11.a) and (11.b) take the form :

$$F.F'' + 2\eta .r.m = 0 \qquad \ldots (12.a)$$

and

$$(1 - P_r)\, j' \,.\, F' + [j'' + P_r .(\gamma - 1)\, M_n^2]\, F = 0 \qquad \ldots (12.b)$$

subject to the boundary conditions

at $\eta = 0 \quad F' = 0 \quad j = J_w$

$\eta = 1 \quad F = 0 \quad j = 1$

With these relations the shear stress at the wall ($\eta = 0$) can be written as follows:

$$\tau_w = \frac{1}{2} \bar{\rho} \bar{c}^2 \frac{F(0)}{\sqrt{Re_x}}$$

or

$$c_f = \frac{\tau_w}{\frac{1}{2} \bar{\rho}_n \bar{c}_n^2} = \frac{F(0)}{\sqrt{Re_x}} \qquad \ldots (13)$$

Introducing the viscosity-temperature relation , $[m = \mu / \mu_n = (T / T_n)^w]$, and ideal gas condition inside the boundary layer in equation (12.a) , it yields

$$F \,.\, F'' + 2\eta \,.\, j^{(w-1)} = 0 \qquad \ldots (14.a)$$

or

$$F \,.\, F'' + h(\eta) = 0 \qquad \ldots (14.b)$$

where : $h(\eta) = 2\eta r m = 2\eta\, j^{(w-1)}$

The iterative method for the solution of such equation runs as follows.

Devide equation (14.b) by $F(\eta)$ and integrate twice to get :

$$F(\eta) = \int_{\eta}^{1} \left[\int_{0}^{\eta} \frac{h(\eta)}{F(\eta)} \, d\eta \right] d\eta \qquad \ldots (15)$$

Starting with an arbitrary function F_1 for F and introducingit in the right han side of equation (15), a function F_2 which is another approximation for the function F. Using the geometric mean $\sqrt{F_1 F_2}$ or arithmatic mean $\frac{1}{2}(F_1 + F_2)$, a trial and so on.

A. D. Young [10] proposed to use.

$$F_1 \cong F(o) \cdot (1 - \eta^2)^{\frac{1}{2}} \qquad \ldots (16)$$

Hence this gives.

$$F(\eta) = \int_{\eta}^{1} \left(\int_{0}^{\eta} \frac{2\eta \cdot i^{(w-1)}}{F(o) \cdot (1 - \eta^2)^{0.5}} \, d\eta \right) d\eta \qquad \ldots (17)$$

In particular :

$$F(o) = \int_{0}^{1} \left(\int_{0}^{\eta} \frac{2\eta \cdot i^{(w-1)}}{(1 - \eta^2)^{0.5}} \, d\eta \right) d\eta \qquad \ldots (18)$$

Simpson,s method is applied to get the value of the first integration in the form of function of η, to be able to evaluate the value of F (o) as a numerical quantity.

Solving for the case : $w = 0.76$, $\gamma = 1.4$ and $P_r = 0.7$, so the first integration takes the form.

$$F(1) = \frac{\eta^2}{3} \left[\frac{2\left\{(1 + 0.1673 M_n^2 (1 - 0.25\eta^2)\right\}^{-0.24}}{(1 - 0.25\eta^2)^{0.5}} + \frac{\left\{1 + 0.1673 M_n^2 (1 - \eta^2)\right\}^{-0.24}}{(1 - \eta^2)^{0.5}} \right] \qquad \ldots (19)$$

So the value of F (o) is obtained numerically for different values of Mach number (M_n), corresponding to each value of α and Θ and the free stream Mach.-number (M_∞)

2.3.2 Calculation of Flow Parameters After the Shock Wave

A- Calculation of free stream parameters in the 3rd region

As shown in Fig. 1, an attached shock wave occurs. A shock wave will be developed at point "a" when the flow changes its direction from region No. (1) to region No. (2) and an expanded flow will follow during the change in its direction from region No. (2) to region No. (3).

Taking the values of the angle of attack α ranging from 2° to 6° and the values of the leading edge angle Θ between 4° to 10° and the values of the Mach number ranging from 1.5 to 3, then:

Inside this envelope of parameters, attached shock wave equations are applicable in all cases.

B- Calculation of free stream flow parameters in the 4^{th} region

Again, referring to Fig. 1, attached shock wave, compression shock at point "a" and the flow changes its direction for region (1) to region (4) (lower surface). The same change in angle of attack α is considered (2° to 6°) and same change in Mach number values (M_∞) ranging between 1.5 to 3.

C. Calculation of wall temperature T_w.

In such case, the heat generated by internal friction cannot be neglected and the energy equation (Eq. 3) can be used to calculate the wall temperature T_w. Energy Eq.

$$\rho . c_x \frac{\partial i}{\partial x} + \rho . c_y \frac{\partial i}{\partial x} = c_x \frac{\partial p}{\partial x} + \frac{1}{P_r} \frac{\partial}{\partial y} (\mu . c_p \frac{\partial T}{\partial y}) + \mu . (\frac{\partial c_x}{\partial y})^2 \quad \ldots (3)$$

Prandtl introduced the transformations given in [6] as follows:

$$\eta = \frac{1}{2} (\bar{c} / \nu . x)^{1/2} . y$$

and stream function :

$$\Psi = (\nu \bar{c} x)^{1/2} F(\eta)$$

which leads to the equation

$$f(\eta) . f''(\eta) + f'''(\eta) = 0 \quad \ldots (20)$$

And using the transformation

$$T = T_1 + \frac{1}{2} \frac{\bar{c}^2}{cp} \bar{\Theta}(\eta)$$

where Θ is a new function of η will lead to the differential equation

$$\bar{\Theta}'' + P_r . f(\eta). \bar{\Theta}' + 2P_r (f''(\eta))^2 = 0 \quad \ldots (21)$$

The solution of the equation (21) after introducing the boundary conditions : at $y = o$, $\bar{\Theta}' = o$ and $y = \infty$, $\bar{\Theta} = o$,

is

$$\bar{\Theta}(\eta) = 2 P_r \int_{\eta}^{\infty} (f''(\eta))^{P_r} . (\int_0^{\eta} (f''(\eta))^{2 - P_r} . d\eta) d\eta \quad \ldots (22)$$

The value of $\bar{\theta}$ at $\eta = o$ for different values of P_r is given by the approximate relation

$$\bar{\Theta}(o) \simeq 4 (P_r)^{1/2}$$

The recovery factor is defined as

$$re = \frac{T_w - T_1}{(\bar{c}^2 / 2 c_p)} \quad \ldots (23)$$

Hence the recovery factor is equal to $1/4\ \tilde{\Theta}(o)$.

Since $\bar{C}^2 / 2\ cp = (T_o - T_1)$, and

$$\frac{T_o}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

so,

$$\frac{T_w}{T_1} = 1 + re \frac{\gamma - 1}{2} M_1^2 \qquad \ldots (24)$$

For air $\gamma = 1.4$ and $re = 0.84$, so

$$\frac{T_w}{T_1} = 1 + 0.168\ M_1^2$$

for region No. (3) where free stream conditions outside the boundary layer are M_3 and T_3, then

$$\frac{T_w}{T_1} = \frac{T_3}{T_1}(1 + 0.168\ M_3^2) \qquad \ldots (25)$$

D. Boundary Layer Parameters

The convenient form to evaluate the different boundary layer parameters is the assumed velocity distribution given by :

$$\frac{c_x}{c_n} = f(\eta) = \frac{3}{2}\eta - \frac{1}{2}\eta^3 \text{ for } o \leq y \leq \delta \qquad \ldots (26)$$

which is based on the thickness δ being defined as that for which $c_x = 0.99\ c_n$.

This assumption for velocity distribution leads to results very close to the exact solution; and at the same time the calculation is quite simple.

For this distribution

$$\frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}} \qquad \ldots (27)$$

Generally, the dimensionless displacement thickness is defined as :

$$\delta^*/\delta = \int_{y/\delta = o}^{y/\delta = 1} (1 - c_x/c_n)\, d(y/\delta) \qquad \ldots (28\ a)$$

$$\delta^* = \alpha_1 \cdot \delta$$

The dimensionless momentum thickness is defined by a similar equation.

$$\frac{\delta^{**}}{\delta} = \int_0^1 f(\eta)\,[\,1 - f(\eta)\,]\,d\eta \qquad \ldots (29)$$

or

$$\delta^{**} = \alpha_2 \,.\, \delta$$

According to the assumed velocity distribution the results of the calculation of the boundary layer for the flat plate based on approximate theory, are

$$\alpha_1 = \frac{3}{8} \text{ and } \alpha_2 = \frac{39}{280}$$

3- RESULTS AND DISCUSSIONS

In order to be able to evaluate the viscous-effects of the supersonic flow due to the variation of different parameters, one parameter only will be investigated considering the other parametersto be constant .

The evaluation of this investigation will be as follows :

1- Variation of wall temperature T_w with the change of Mach number M, angle of attack α and leading edge angle θ .

2- Variation of local skin friction coefficient c_f with the same three parameters M, α and θ. All the discussionswill be for only in regions No. (3) and (4).

Fig. (2) illustrates the variation of wall temperature $\frac{T_w}{T_1}$ with the Mach number M_1. It is clear that as the value of M increases the value of the achieved wall temperature T_w/T_1 increases remarkabiely. It may also be concluded that the achieved wall temperature on both upper and lower surface are approximately the same for losmaller the angles of attack but Fig. (3) shows that there is a remarkable difference in the achieved wall temperature on both surfaces for higher values of the angle of attack.

From Fig. (2) shows the small change in the achieved wall temperature with the change of the leading edge angle θ for the same value of the angle of attack α. Moreover, comparing figures (2) and (3) it is clear that, as the value of the angle of attack α increases the achieved wall temperature increases.

Figures (4 to 7) indicate the variation in local skin friction coefficient c_f with the change of the dimesionless flat plate length x. Obviously as the value of x increases the value of c_f decreases and the maximum value of c_f is at the beginning of the horizontal part of the flat plate (region No. 3). These figures show also for the same geometric condition (certain and O) atacertain point on the flat plate, the value of the local skin friction coefficient c_f decreases with increasing Mach number of the flow.

Comparing figure (4) with figure (5) , that the change in the value of the leading edge angle has a remarkable change on the value of the local skin friction coefficient. As its value increases the resulting skin friction coefficient increases. A further comparison between figures (4) and (5) with figures (6) and (7) shows this change has a greater value at lower angle of attack.

Figure (8) shows that the change in the value of the angle of attack α has a slight change in the value of the gained c_f this change is inversely.

The variation of the dimensionless value of the δ/L , δ^*/L and δ^{**}/L over region no. 3 against the change of the dimensionless value $\bar{x}$ ($\bar{x} = x/L$) are illustrated in Fig. (9 to 14). It is remarkable from the figures that with increasing $\bar{x}$ the different boundary layer parameters increases.

From the same figures one may conclude also that for the same angle of attack α and the same leading edge angle O at certain point on the flat plate, the values δ/L , δ^*/L and δ^{**}/L decrease with increasing valuesof the Mach number.

The comparison made through this group of figures shows that the value of boundary layer parameters decrease as the value of the leading edge angle increases.

The different boundary layer parameter for the lower surface are also illustrated in figures (15 to 17).

CONCLUSIONS

The viscous effects of supersonic flow on a flat plate have been analyzed theoretically. A computer program has been constructed to perform the necessary calculations and to study the effect of the Mach-number on the viscous effects of the supersonic flow.

The results were obtained in the form of a series of curves showing the effects of the change in the value of the Mach-number governing the problem. Generally, speaking it was found that the local skin friction coefficient c_f and the achieved wall temperature T_w areconsiderably affected by the change in the value of the Mach-number M.

Also, it is found that the value of the boundary layer thickness δ, displacement boundary layer thickness δ^*, and the momentum boundary layer thickness δ^{**} are considerablly affected by the change in the value of the Mach-number.

Finally, this study can lead to a final count in deciding and chosing the type of material which can endure the achieved thermal and shear stresses created on the flat plate due to the resulted effects.

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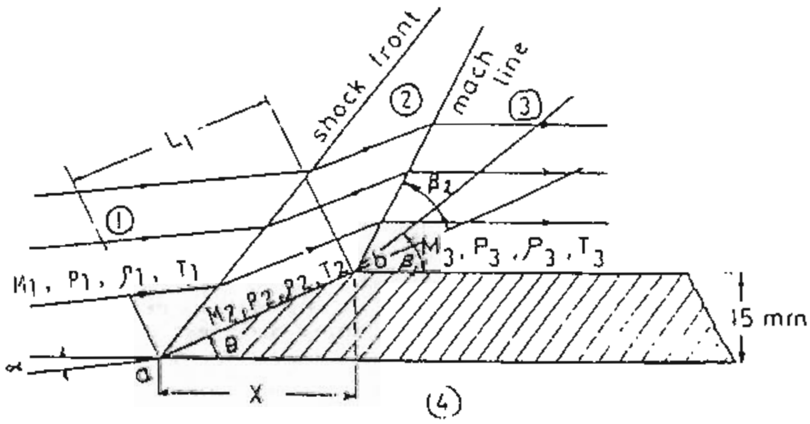

FIG. 1 SCHEMATIC MODEL OF FLAT PLATE

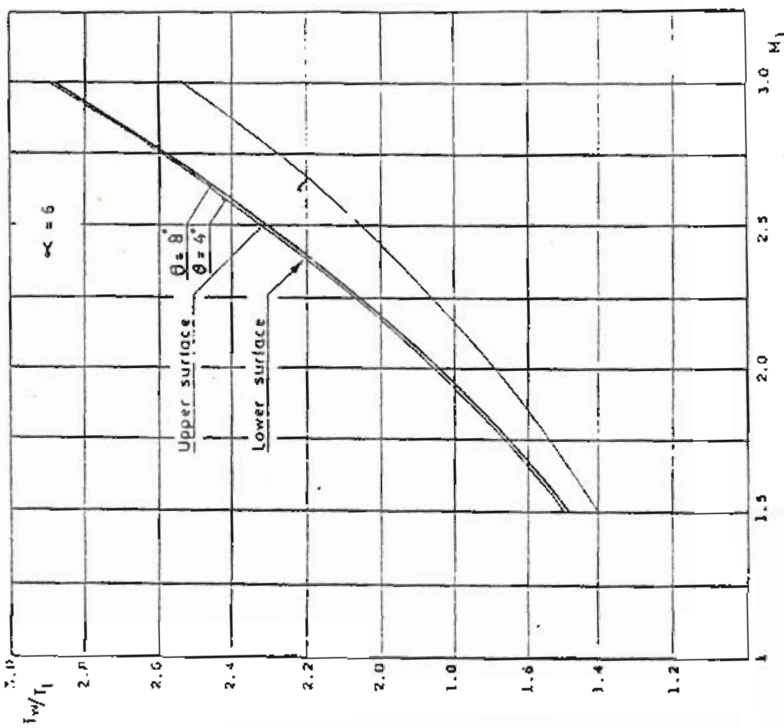

FIG. 3 CHANGE OF T_w/T_1 Versus M_1

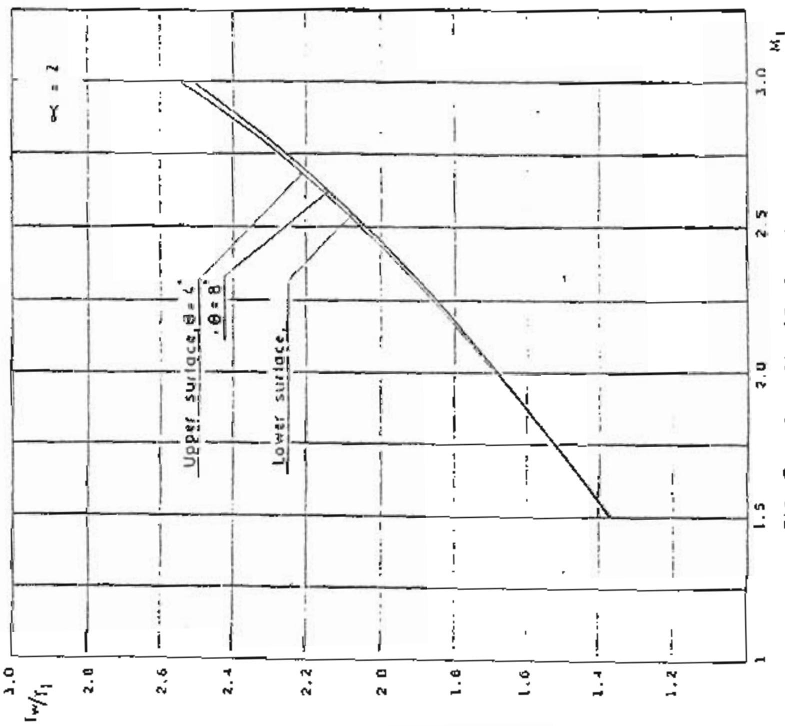

FIG. 2 CHANGE OF T_w/T_1 Versus M_1

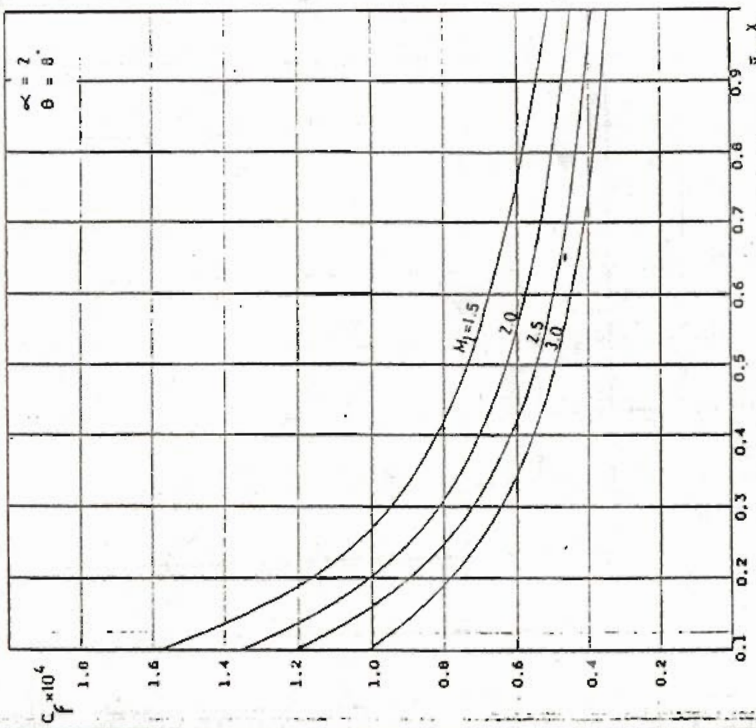

FIG. 5 CHANGE OF C_f Versus $\bar{x}$

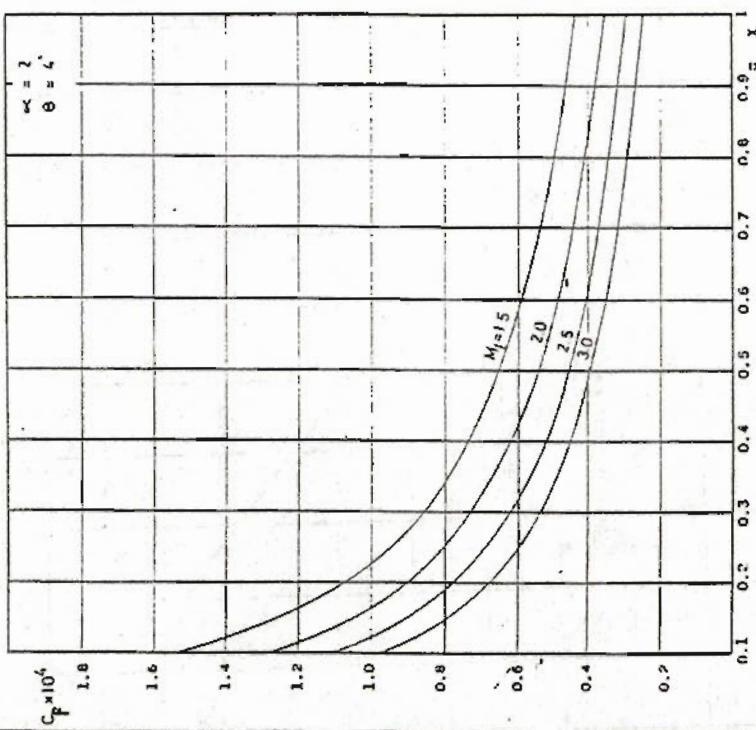

FIG. 4 CHANGE OF C_f Versus $\bar{x}$

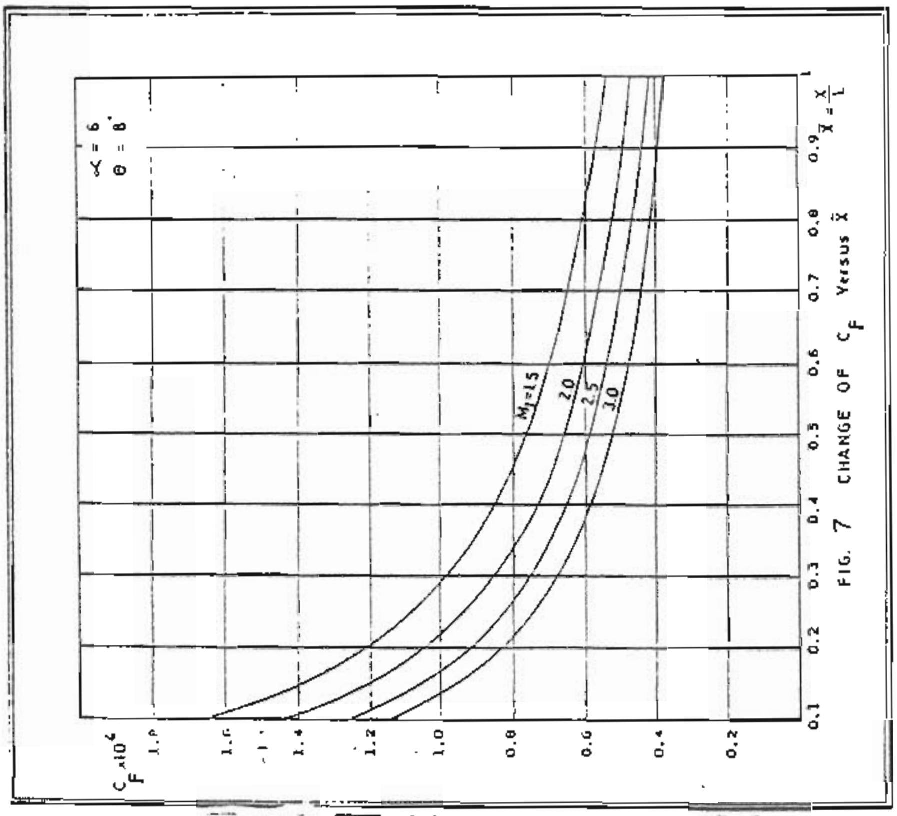

FIG. 7 CHANGE OF C_F Versus $\bar{X}$

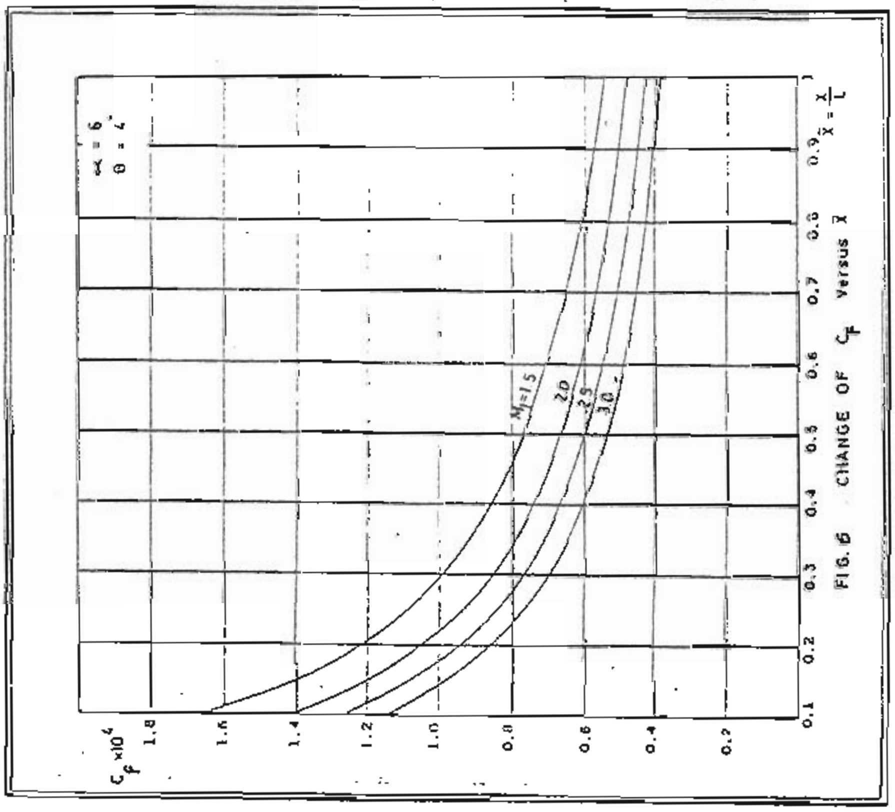

FIG. 6 CHANGE OF C_F Versus $\bar{X}$

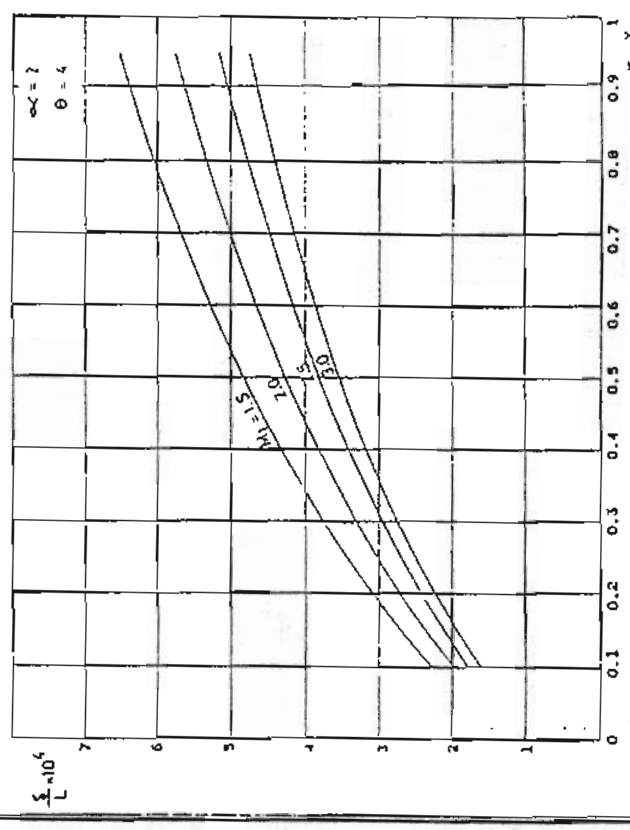

FIG. 9 CHANGE OF $\frac{\delta}{L}$ Versus $\bar{X}$

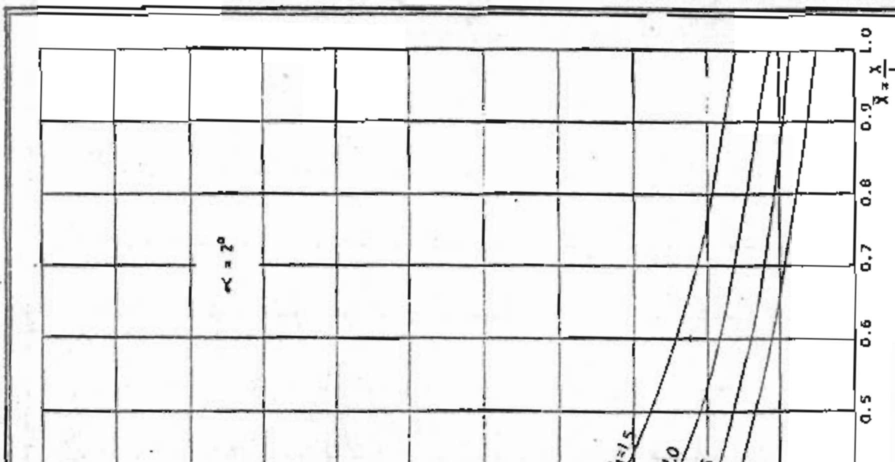

FIG. 8 CHANGE OF C_f Versus $\bar{X}$ ON LOWER SURFACE

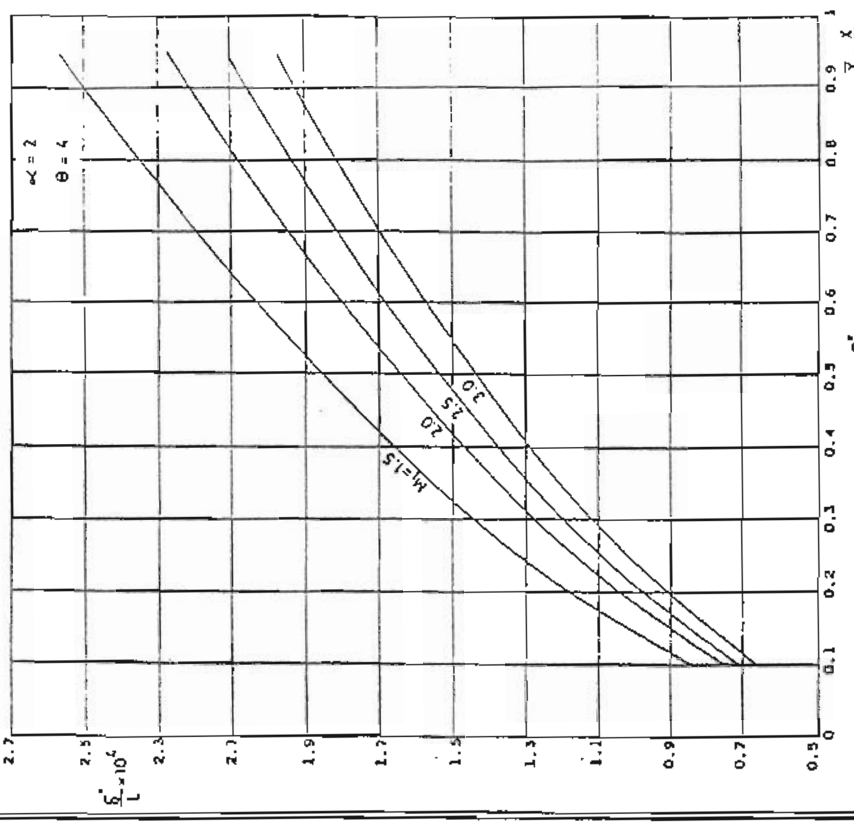

FIG. 11 CHANGE OF $\frac{\delta^*}{L}$ Versus $\bar{X}$

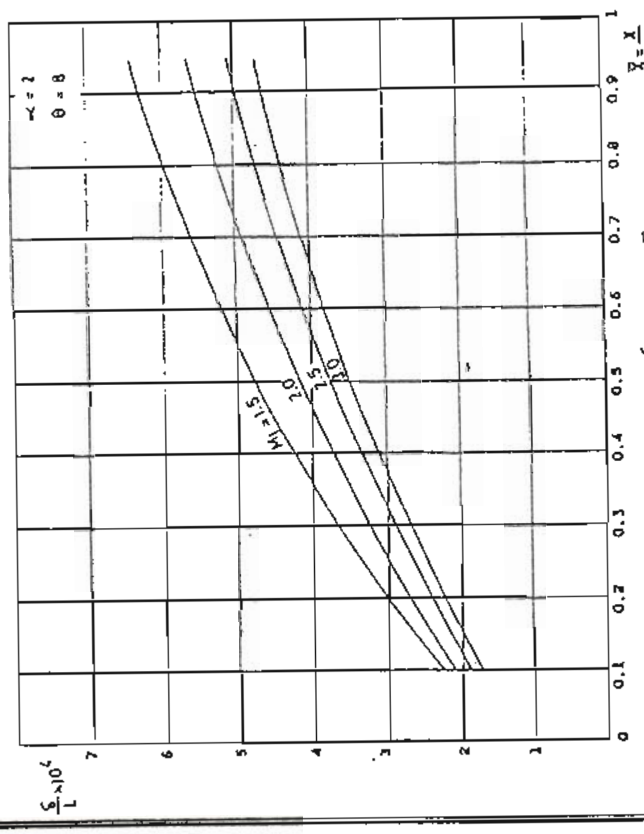

FIG. 10 CHANGE OF $\frac{\delta}{L}$ Versus $\bar{X}$

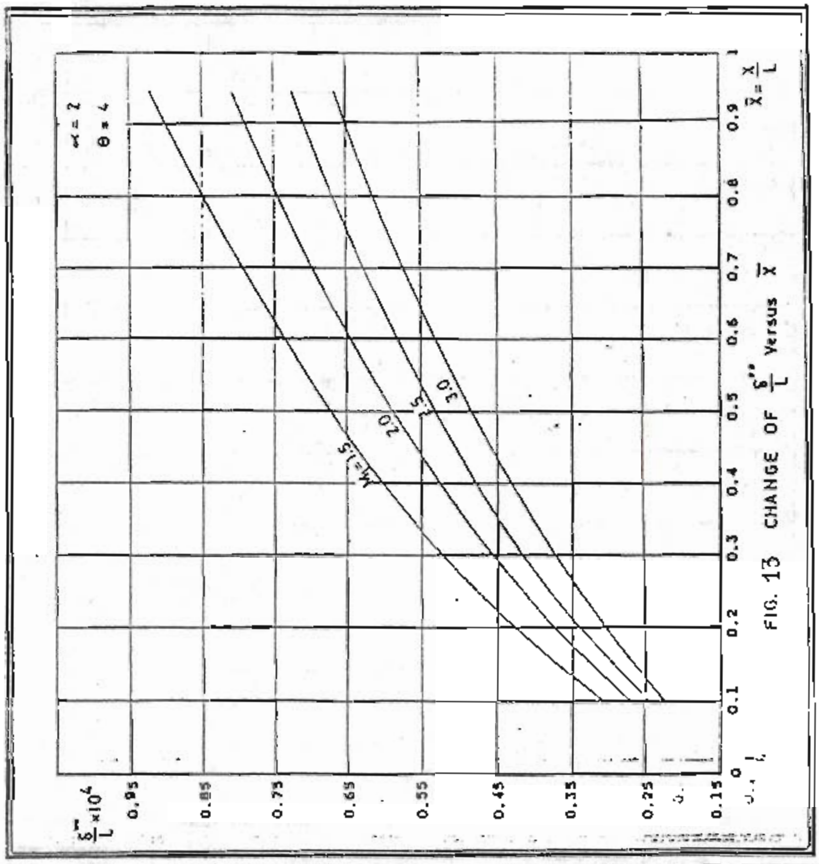

FIG. 13 CHANGE OF $\frac{\delta''}{L}$ Versus $\bar{X}$

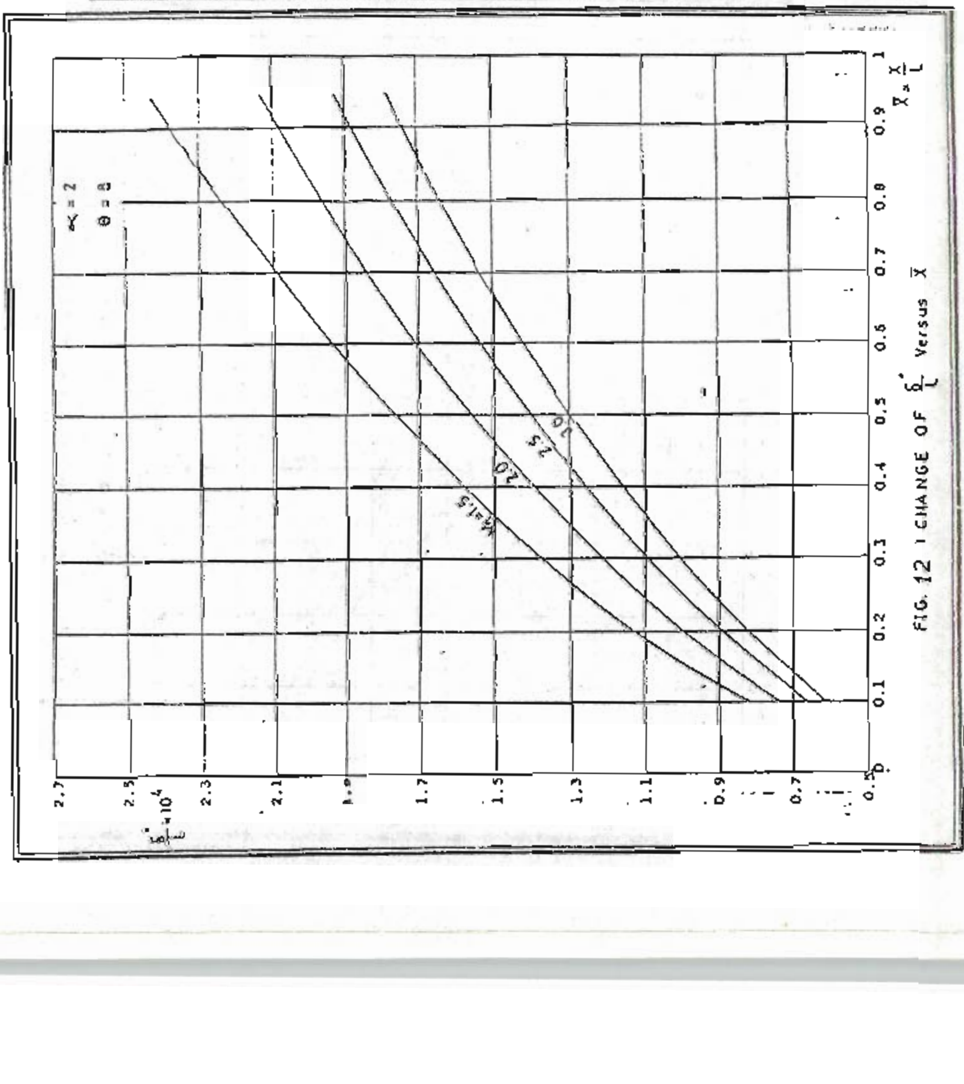

FIG. 12 CHANGE OF $\frac{\delta'}{L}$ Versus $\bar{X}$

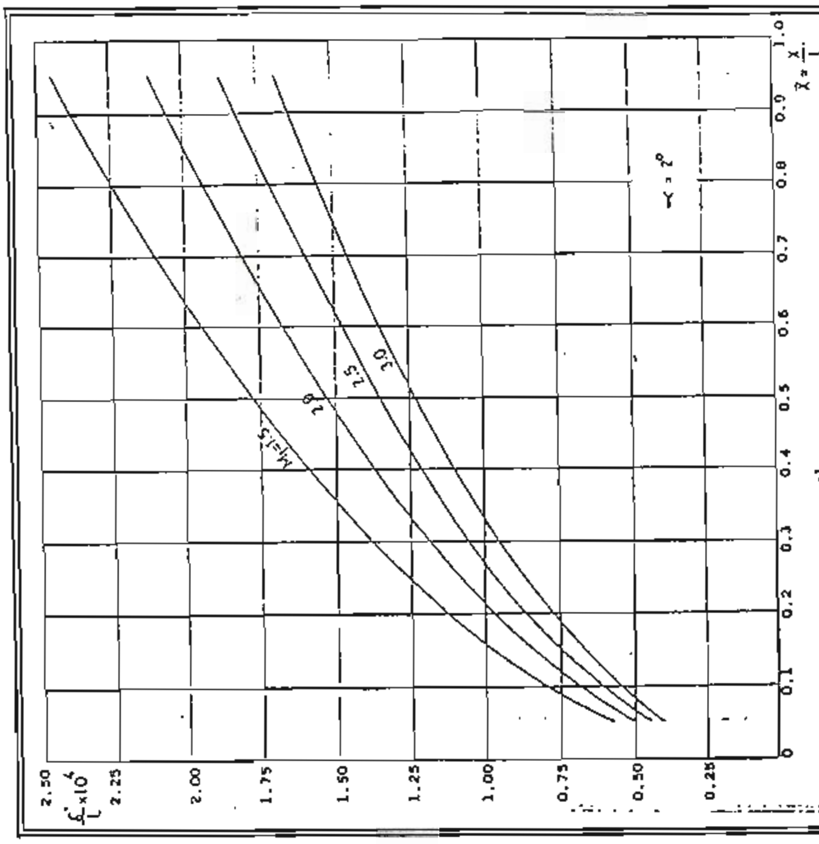

FIG. 15 CHANGE OF $\frac{\delta'}{L}$ Versus $\bar{X}$ ON LOWER SURFACE

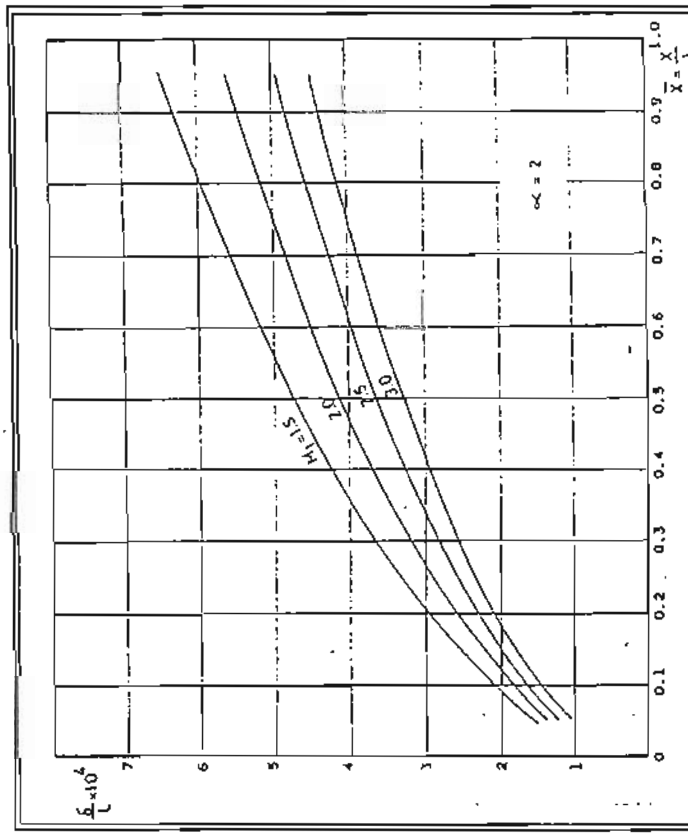

FIG. 14 CHANGE OF $\frac{\delta}{L}$ Versus $\bar{X}$ ON LOWER SURFACE

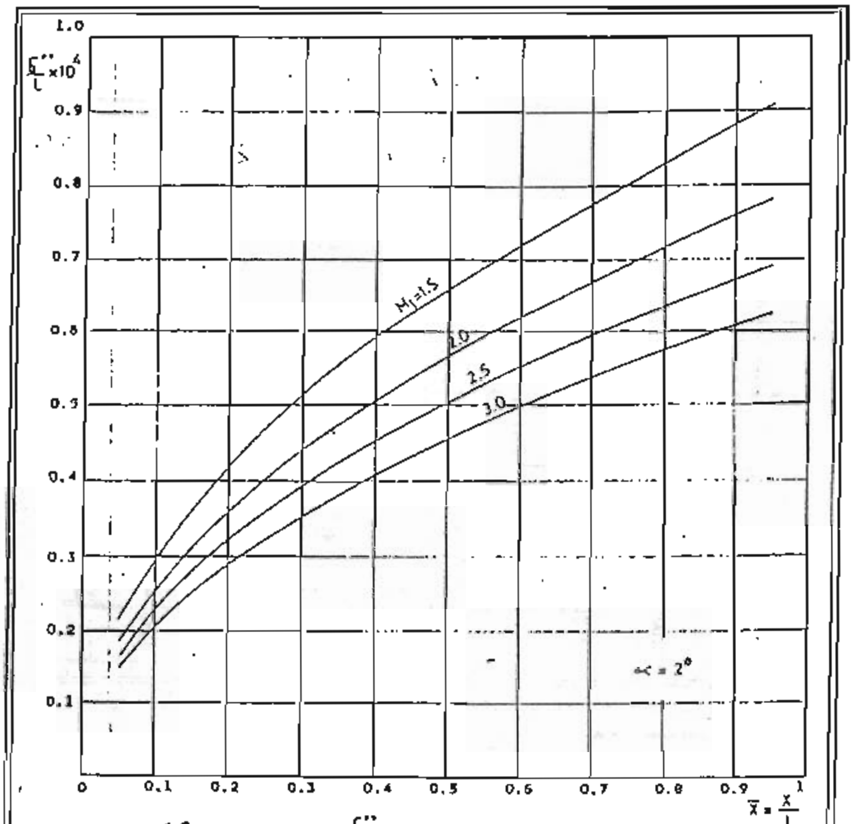

FIG. 16 CHANGE OF $\frac{\delta^{**}}{L}$ Versus $\bar{X}$ ON LOWER SURFACE