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STEADY-STATE ANALYSIS OF CHOPPER-FED D.C MOTOR

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ABSTRACT

The paper describes a new chopper circuit. The steady-state performance of the new chopper-fed d.c. separately excited motor is studied. Three different models are given based upon the following particulars:
1) relatively small losses in the commutating circuit;
2) constant current during commutation; and
3) negligible commutation interval.The effect of source inductance and the influence of increased armature circuit resistance (due to pulsating armature voltage) on the motor performance are taken into consideration. The computed performance is verified experimentally where close agreement between the computed and test results is achieved.

LIST OF PRINCIPAL SYMBOLS

v_c, v_2	Instantaneous and maximum value of commutating capacitor voltages, (volts)
E_s	Supply voltage, (volts)
i_c	Instantaneous value of commutating capacitor current (A)
k	Machine back e.m.f coefficient volt/(rad/sec)
i_L	Instantaneous value of load current (Amps)
I_d	Current at end of duty interval (Amps)
I_c	Load current at end of commutation interval(Amps)
I_f	Load current at end of freewheeling interval I (Amps)
R	$=(R_a+R_e)$ Total Resistance of armature circuit(Ohms)
L_t	$=(L_s+L_e+L_a)$ Total armature circuit inductance,(Henry)

L_s	Source inductance	(Henry)
c	Commutation circuit capacitor	(μ.F)
τ, τ_t	Armature circuit time constants	(sec)
$k_d T$	Duty interval of chopper	(sec)
$k_c T$	Commutation interval of chopper	(sec)
$k_f T$	Freewheeling interval of chopper	(sec)
N_e	E_s/k	(r/min)
N/N_e	Speed factor	
I_e	Rated armature current	(Amps)
T_e	$(60\ k/2\pi)\ I_e$	(n.m)
T/T_e	Torque factor	

1. *INTRODUCTION*

In modern d.c. drives, the classical speed control of d.c. motors have been replaced by a thyristorized power converters, which provides faster response at a lower total cost.

The steady-state performance of a separately excited d.c. motor controlled by a chopper have been investigated [1,2]. Four different methods of analysis are developed [1], based on : 1) negligible commutation interval; 2) negligible ripple in the armature circuit; 3) constant current during commutation; and 4) direct solution of the governing differential equations. In the analyses the effect of source inductance has not been considered.

It is known from previous experience that, the presence of the source inductance has a considerable influence on the machine performance as well as on the commutation interval. The commutating capacitor sometimes charges to a voltage greater than the supply voltage and discharges partially during the freewheeling interval.

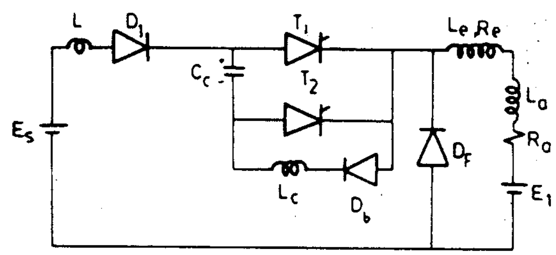

Fig 1 New chopper circuit

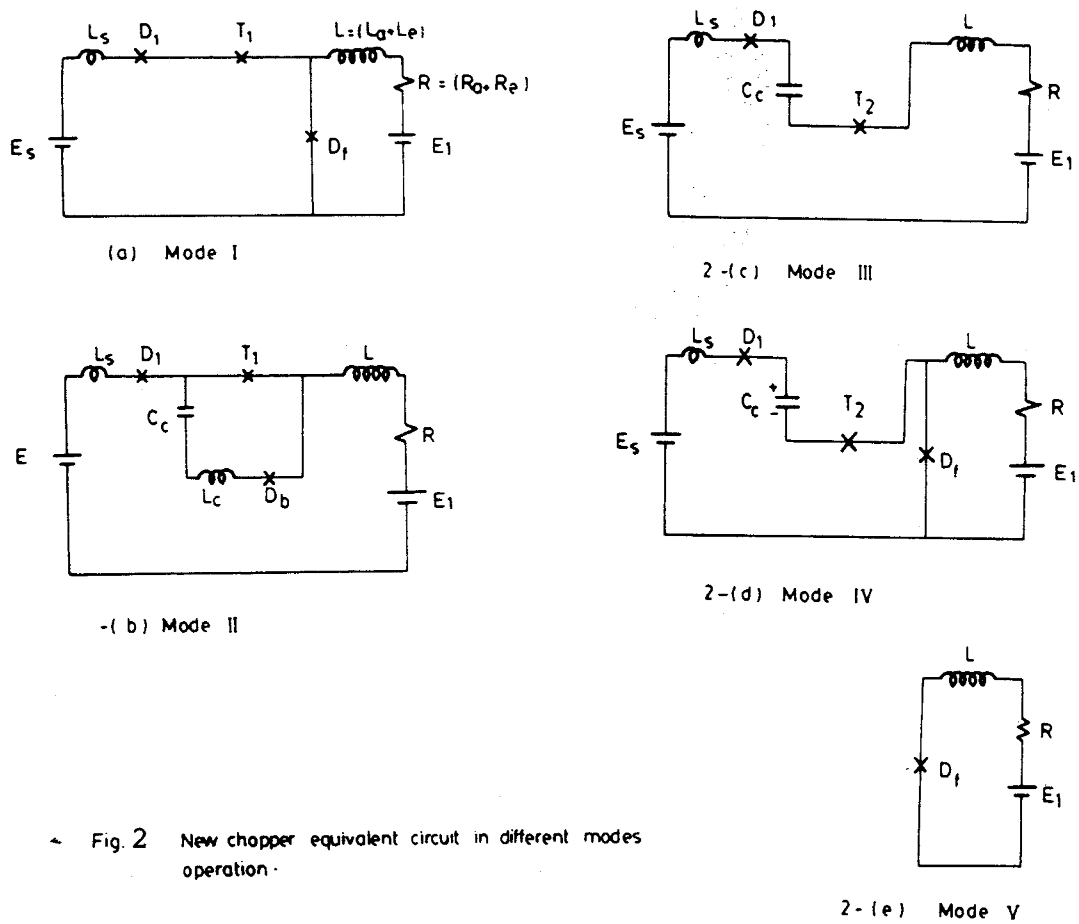

Fig. 2 New chopper equivalent circuit in different modes operation.

Under regenerative braking, a general method of analysis for such a system has been presented [2], the analysis takes into account the effect of machine armature reaction and the source inductance.

The present paper describes a new chopper, in which an additional diode is used with the conventional chopper [1]. The diode prevents the discharge process and increases the commutation capability of the capacitor. A complete analysis, taking into account the effect of source inductance and the effect of increased armature circuit resistance on the performance of the motor has been investigated.

2. NEW CHOPPER CIRCUIT

The circuit diagram is shown in Fig.1. This chopper is essentially a modified version of the conventional chopper [1]. The main difference between this chopper circuit and the conventional chopper is the addition of diode D_1.The operation is being described from the initial instant of commutation for the main thyristor T_1.Just before this instant T_1 is conducting, while auxiliary thyristor T_2 is not conducting, and the commutating capacitor C is charged with a polarity such as shown in Fig.1. To commutate T_1 off, T_2 has to be gated on. Load current now is diverted from T_1 to T_2, charging C in the opposite sense. Source inductance L_s acts to charge C to a voltage level greater than the source voltage. This voltage level can be increased to a higher limit by increasing the load current. T_2 gets turned off naturally and the load current freewheels through the freewheeling diode D_f.

3. ANALYSIS OF THE CHOPPER

The following basic assumptions are to be taken into consideration :

1] The thyristors and diodes are considered as ideal switches.

2] The motor speed remains constant during steady state operation.

3] The source resistance is neglected .

4] The inductance of the armature circuit is assumed to be constant.

The machine back e.m.f. is taken as $E=k_c I_f N=kN$, where the field of the motor is excited from a constant current source.

3.1. Modes of Operation

The operation of the new chopper can be briefly explained by the following Five modes of operation.

Let initially freewheeling interval is in progress, therefore, T_1 and T_2 are not conducting, and chopper cycle starts with the turning on of T_1.

Mode I - Freewheeling Interval I $0 \le t \le k_f' T$

In this mode the interval starts as soon as T_1 is gated on. The rise time of the source current is affected by the source inductance. Therefore, the free-wheeling diode is still conducting until the source current is equal to the load current. Chopper equivalent circuit for this mode is shown in Fig.(2-a). The differential equations describing this mode are given by :

$$L_s \frac{di_s}{dt} = E_s \tag{1}$$

$$L \frac{di_L}{dt} + Ri_L + E_1 = 0 \tag{2}$$

and the initial conditions are

$$i_s(0) = 0 \quad , \; i_L(0) = I_f' \tag{3}$$

The solution of the above equations is

$$i_s(t) = \frac{V}{L_s} t \qquad (4)$$

$$i_L(t) = \frac{-E_1}{R} (1 - e^{-t/\tau}) + I_f' e^{-t/\tau} \qquad (5)$$

where

$$\tau = L/R_a \quad , \quad L = (L_a + L_e)$$

<u>Mode II - Duty Interval $0 \leqq t' \leqq k_d T$</u> $(t' = t - k_f' T)$

The interval starts when the current of the free-wheeling diode D_F becomes zero. The source voltage appears accross the load terminals and the commutating capacitor reverses its charge through the resonating circuit (L_c, C, D_b). This interval terminates as soon as T_2 is triggerd. The equivalent circuit of this mode is shown in Fig. (2-b), and the differential equations describing this mode are given by :

$$(L_s + L) \frac{di_L}{dt'} + Ri_L + E_1 = E_s \qquad (6)$$

$$L_c \frac{di_c}{dt} + \frac{1}{c} \int i_c \, dt = 0 \qquad (7)$$

and the initial conditions are

$$i_L(0) = I_f \quad , \quad i_c(0) = 0 \quad , \quad i_c'(0) = -V_2/L_c \qquad (8)$$

The solution of the above equations are :

$$i_L(t') = \frac{E_s - E_1}{R} (1 - e^{-t'/\tau_t}) + I_f e^{-t'/\tau_t} \qquad (9)$$

$$i_c(t) = - \frac{V_2}{\omega_r L_c} \sin \omega_r t_1 \qquad (10)$$

and

$$v_c(t) = -L_c \frac{di_c}{dt} = V_2 \cos \omega_r t_1 \qquad (11)$$

From equation (11)

$$v_c = -V_2 \quad \text{at} \quad \omega_r t_1 = \pi$$

$$\therefore \quad t_1 = \pi/\omega_r \qquad (12)$$

where : t_1=time required for the commutating capacitor to reverse its charge from V_2 to $-V_2$

$$\omega_r = 1/\sqrt{L_c C} \quad , \quad L_t = (L_s + L) \quad , \quad \tau_t = L_t/R$$

Mode III- Commutation Interval $0 \leqq t' \leqq k_c T$

The interval starts with the turning on of the auxiliary thyristor T_2. The capacitor voltage ($-V_2$) is then applied accross the anode-cathode terminals of T_1 as an inverse voltage, and the load current flowing through T_1 is transferred through the capacitor (c) and T_2. The commutating capacitor gets charged from the source through the load and changes its polarity. This interval is completed with the charging of the capacitor to a voltage V. The equivalent circuit of the mode under consideration is as shown in Fig.(2-c). The equations describing this mode may be given by :

$$L_t \frac{di_c}{dt''} + Ri_c + v_c = E_s - E_1 \tag{13}$$

$$e \frac{dv_c}{dt''} = i_c \tag{14}$$

and the initial conditions are

$$v_c(0) = -V_2, \quad i_c(0) = I_d \ , \quad V_c'(0) = I_d/C \tag{15}$$

The solution of the above equations may be given by

$$v_c(t'') = (E_s - E_1) + \{-(E_s - E_1 + V_2) \frac{\omega_o}{\omega} \sin(\omega t'' + \phi) + \frac{I_d}{\omega_c} \sin\omega t''\} e^{-\beta t''} \tag{16}$$

$$i_c(t'') = [(E_s - E_1 + V_2) \frac{1}{\omega L_t} \sin\omega t'' - I_d \frac{\omega_o}{\omega} \sin(\omega t - \phi)] e^{-\beta t''} \tag{17}$$

where :

$$\omega_o = 1/\sqrt{L_t C}$$

$$\beta = R/2L_t$$

$$\omega = \sqrt{\omega_o^2 - \beta^2}$$

$$\phi = \text{arc tan } \frac{\omega}{\beta}$$

Mode IV- Freewheeling Interval II $0 \leq t''' \leq K_c'T$

The interval starts when the commutating capacitor is charged to a voltage (V), and the diode D_f is forward biased. Due to the source inductance, the capacitor charges to a voltage V_2 greater than the supply voltage . The charging period ends when $i_c=0$. The equivalent circuit of this mode of operation is illustrated by Fig.(2-d). The equations describing this mode are given by :

$$L_s \frac{di_c}{dt'''} + v_c(0) + \frac{1}{c} \int i_c \, dt''' = E_s \tag{18}$$

$$L \frac{di_L}{dt'''} + Ri_L + E_1 = 0 \tag{19}$$

and initial conditions are

$$v_c(o) = V \quad , \; i_L(0) = I_c \quad \text{and } i_c(0) = I_c \tag{20}$$

The solution of the above equations are

$$i_c(t''') = I_c \cos Y_o t''' \tag{21}$$

$$i_L(t''') = \frac{-E_1}{R} (1 - e^{-t'''/\tau}) + I_c \, e^{-t'''/\tau} \tag{22}$$

and

$$v_c(t''') = V + \frac{1}{c} \int i_c \, dt''' = V + I_c \sqrt{L_s/c} \, \sin Y_o t''' \tag{23}$$

This interval ends when $i_c = 0$ and $t''' = k_c'T$

From equation (21) $i_c = 0$ at $Y_o \, k_c'T = \pi/2$

$$k_c'T = \pi/2 \, Y_o \tag{24}$$

Then, the maximum capacitor voltage is given by :

$$V_2 = V + I_c \sqrt{L_s/C} \tag{25}$$

Mode V- Freewheeling interval III $0 \leq t'''' \leq k_f T$

The interval starts when the capacitor current equals zero, the capacitor voltage is than equal to V_2 and

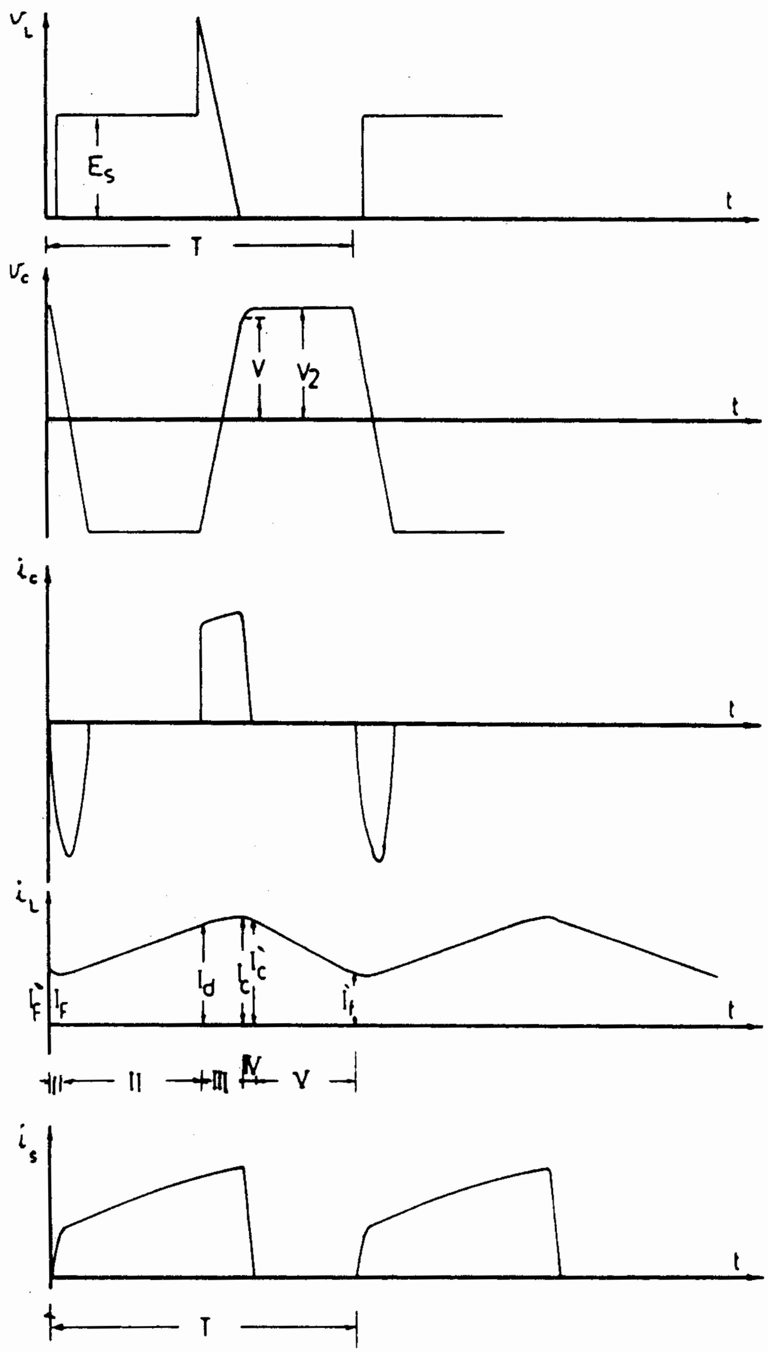

Fig 3 New chopper circuit waveforms

T_2 is turned off. The equivalent circuit of this mode is as shown in Fig.(2-e), the equation describing this mode may be given by :

$$L \frac{di_L}{dt''''} + Ri_L + E_1 = 0 \qquad (26)$$

and the initial condition is

$$i_L(0) = I_c' \qquad (27)$$

The solution of the above equation is

$$i_L(t'''') = -\frac{E_1}{R}(1 - e^{-t''''/\tau}) + I_c' \, e^{-t''''/\tau} \qquad (28)$$

The wave forms of V_L, V_c, i_c, i_s and i_L are shown in Fig. (3).

3.2. Speed-Torque Characteristics

To evaluate the performance characteristics of the motor, three different models are used :

Model (1) Relatively small losses in the commutating circuit;

Model (2) Constant current during commutation; and

Model (3) Negligible commutation interval.

Expressions for the current at the end of each interval and the average value of the current can now be determined using each of these models

Model (1): The losses in the commutation circuit is neglected, therefore, the capacitor charges to a voltage equal to the supply voltage at the end of the commutation interval.

From Eq. (25) $V = E_s$

$$V_2 = E_s + I_c \sqrt{L_s/C} \qquad (29)$$

and from Eq. (17) $\omega = \omega_o$, $\phi = \pi/2$

$$\therefore \; i_c(t'') = (2E_s - E_1 + I_c\sqrt{L_s/C})\frac{1}{\omega_o L_t}\sin(\omega_o t'')e^{-\beta t''} + I_d \cos \omega_o t'' \, e^{-\beta t''} \qquad (30)$$

From Eqs.(5, 9, 22, 28 and 30), the expressions for the current at the end of different modes of operation are:

$$I_f = -\frac{E_1}{R}\,(1 - e^{-k_f' T/\tau}) + I_f'\, e^{-k_f' T/\tau} \tag{31}$$

$$I_d = \left(\frac{E_s - E_1}{R}\right)(1 - e^{-k_d T/\tau_t}) + I_f\, e^{-k_d T/\tau_t} \tag{32}$$

$$I_c = (2E_s - E_1 + I_c\sqrt{L_s/C})\,\frac{1}{\omega_o L_t}\,\sin(\omega_o k_c T)\, e^{-\beta k_c T} + I_d\cos(\omega_o k_c T)\, e^{-\beta k_c T} \tag{33}$$

$$I_c' = -\frac{E_1}{R}\,(1 - e^{-k_c' T/\tau}) + I_c\, e^{-k_c' T/\tau} \tag{34}$$

and

$$I_f' = -\frac{E_1}{R}(1 - e^{-k_f T/\tau}) + I_c'\, e^{-k_f T/\tau} \tag{35}$$

The average value of the current is expressed by :

$$\begin{aligned} I_{av} = \Big[& \frac{E_s k_d - E_1(1-k_c)}{R} + \frac{E_s - E_1}{R}\cdot\frac{\tau_t}{T}\,(e^{-k_d T/\tau_t} - 1) \\ & + I_f\,\frac{\tau_t}{T}(1 - e^{-k_d T/\tau_t}) + \frac{2E_s C}{T} + \frac{1}{T(\omega_o^2+\beta^2)}\,[(\omega_o^2\sqrt{L_s C} - \beta)I_c + \beta I_d] \\ & - \frac{E_1}{R}\cdot\frac{\tau}{T}\,(e^{-k_c' T/\tau} + e^{-k_f' T/\tau} + e^{-k_f T/\tau} - 3) \\ & + I_c\cdot\frac{\tau}{T}\,(1 - e^{-k_c' T/\tau}) + I_c'\,\frac{\tau}{T}\,(1 - e^{-k_f T/\tau}) \\ & + I_f'\cdot\frac{\tau}{T}\,(1 - e^{-k_f' T/\tau})\Big] \end{aligned} \tag{36}$$

The average value of the torque developed by the motor is given by the following expression :

$$T = \frac{60\ k}{2\pi}\ I_{av} \tag{37}$$

The wave forms of V_L, V_c, i_L of model (1) are shown in Fig. (4).

Model (2): Constant current during commutation intervals is considered i.e.

$$I_c' = I_c = I_d \tag{38}$$

$$k_c T = \frac{C(V + V_2)}{I_d} \tag{39}$$

From Eqs.(5,9,and 28) the expressions for the current at the end of different modes of operation are :

$$I_f = -\frac{E_1}{R}(1 - e^{-k_f' T/\tau}) + I_f' e^{-k_f' T/\tau} \tag{40}$$

$$I_d = \frac{E_s - E_1}{R}(1 - e^{-k_d T/\tau_t}) + I_f e^{-k_d T/\tau_t} \tag{41}$$

$$I_f' = -\frac{E_1}{R}(1 - e^{-k_f T/\tau}) + I_d e^{-k_f T/\tau} \tag{42}$$

The average value of the current is ;

$$I_{av} = [\frac{E_s k_d - E_1(1-k_c-k_c')}{R} + I_d(k_c+k_c') + I_d \frac{\tau}{T} \cdot (1 - e^{-k_f T/\tau}) + I_f' \frac{\tau}{T}(1 - e^{-k_f' T/\tau}) + I_f \cdot \frac{\tau_t}{T}(1 - e^{-k_d T/\tau_t}) + (\frac{E_s - E_1}{R})(\frac{\tau_t}{T}) \cdot (e^{-k_d T/\tau_t} - 1) - \frac{E_1}{R} \cdot \frac{\tau}{T}(e^{-k_f' T/\tau} + e^{-k_f T/\tau} - 2)] \tag{43}$$

Using Eq. 37, the average value of the torque can be obtained, and the waveforms of this model are as shown in Fig. (5).

<u>Model (3)</u>;The commutation interval ($k_c T$) is neglected (it is small compared with the chopper period). i.e.

$$k_c T = 0 \tag{44}$$

$$I_c = I_d \tag{45}$$

From Eqs.(5, 9, 22)and(28) the expressions for the current at the end of different modes of operation are:

$$I_f = -\frac{E_1}{R}(1 - e^{-k_f' T/\tau}) + I_f' e^{-k_f' T/\tau} \tag{46}$$

$$I_d = (\frac{E_s - E_1}{R})(1 - e^{-k_d T/\tau_t}) + I_f e^{-k_d T/\tau_t} \tag{47}$$

$$I_c' = -\frac{E_1}{R}(1 - e^{-k_c' T/\tau}) + I_d e^{-k_c' T/\tau} \tag{48}$$

$$I_f' = -\frac{E_1}{R}(1 - e^{-k_f T/\tau}) + I_c' e^{-k_f T/\tau} \tag{49}$$

The average value of the current is :

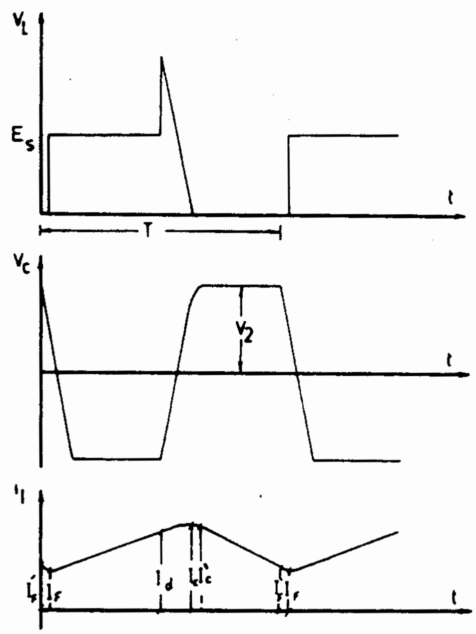

Fig. 4 Waveform of model (1)

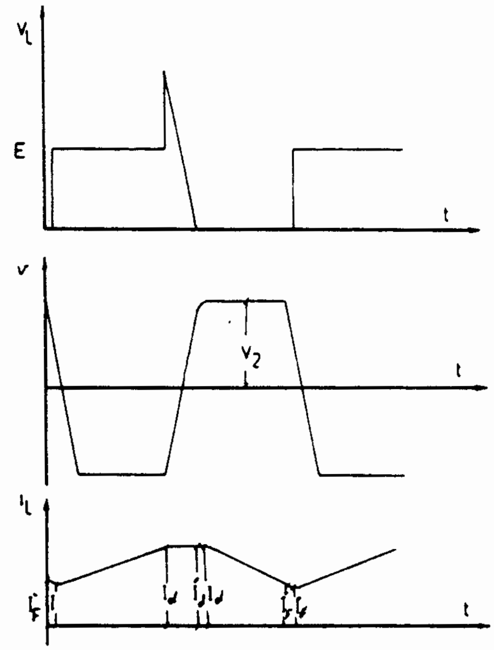

Fig. 5 Waveform of model (2)

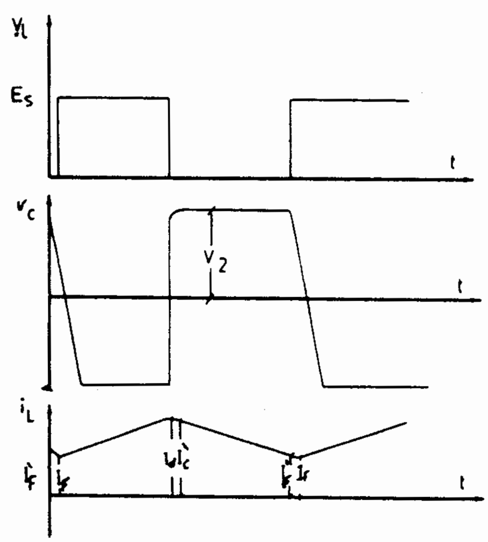

Fig. 6 Waveform of model (3)

$$I_{av} = [\frac{E_s\, k_d - E_1}{R} + (\frac{E_s - E_1}{R}) \cdot \frac{\tau_t}{T} (e^{-k_d T/\tau_t} - 1) +$$

$$I_f \cdot \frac{\tau_t}{T} (1 - e^{-k_d T/\tau_t}) + I_d \cdot \frac{\tau}{T} (1 - e^{-k_c' T/\tau}) +$$

$$I_c' \cdot \frac{\tau}{T} (1 - e^{-k_f T/\tau}) + I_f' \cdot \frac{\tau}{T} (1 - e^{-k_f' T/\tau}) -$$

$$\frac{E_1}{R} \frac{\tau}{T} (e^{-k_c' T/\tau} + e^{-k_f' T/\tau} + e^{-k_f T/\tau} - 3)] \qquad (50)$$

and the average torque can be obtained using Eq.(37). The waveforms of V_L, V_c, and i_L of model (3)are shown in Fig. (6).

4. *EXPERIMENTAL RESULTS*

For the experimental verification of the proposed models, tests were carried out on a d.c separtely excited motor(the motor and the new chopper parameters are listed in the Appendix). Four sets of experiments were conducted with values of k_d=0.2,0.31,0.5 and 0.7.

Fig.(7)shows the experimental and the calculated speed-torque characteristics of the motor using the three proposed models. From the characteristics, it is seen that, model(1) predicts the characteristics which are in a reasonable agreement with experimental values. For model (2), the computed characteristics agrees well with the experimental results for heavily loaded motor conditions. For model (3), the computed performance have a comparable error for lightly loaded motor conditions.

In general, using model(1),and taking the effect of increasing armature resistance (due to pulsating armature voltage), better agreement can be obtained as shown in Fig.(8). A close agreement is noted between these results and the test points over the entire range of loads. The armature resistance can be estimated to

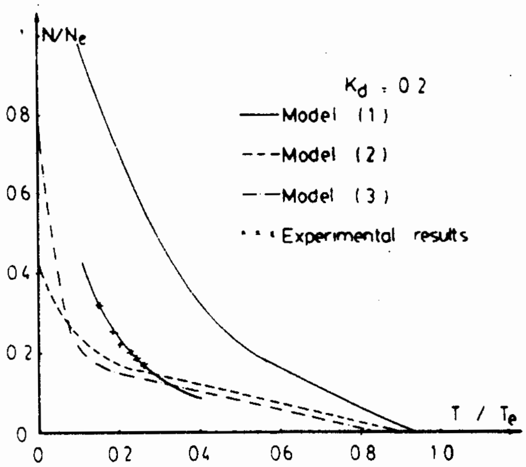

Fig 7-a Speed - Torque characteristics at K_d = 0.2

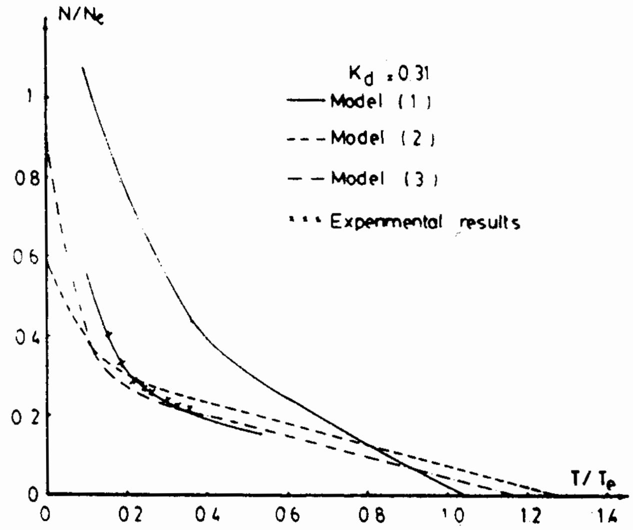

Fig 7-b Speed-Torque characteristics at K_d = 0 31

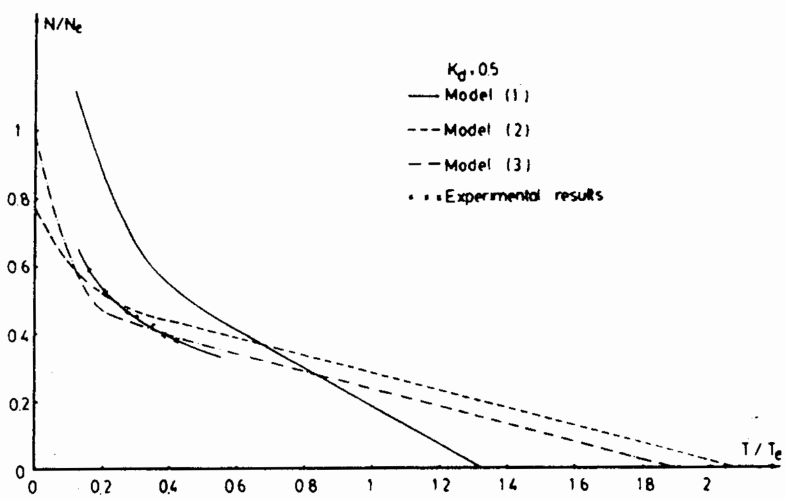

Fig. 7-c Speed-Torque characteristics at K_d =0.5

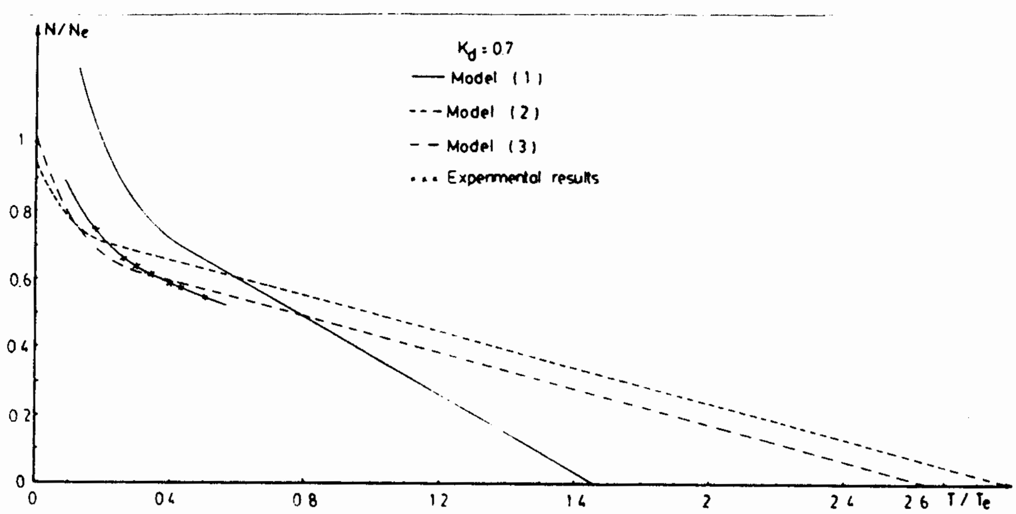

Fig 7-d Speed - Torque characteristics at K_d = 0.7

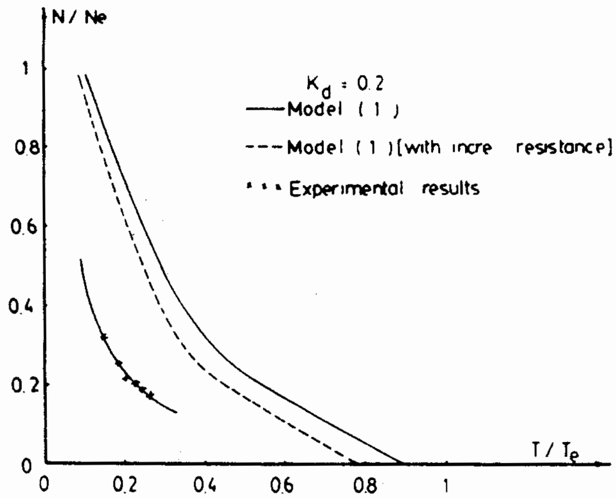

Fig.(8-a) Speed_Torque characteristics

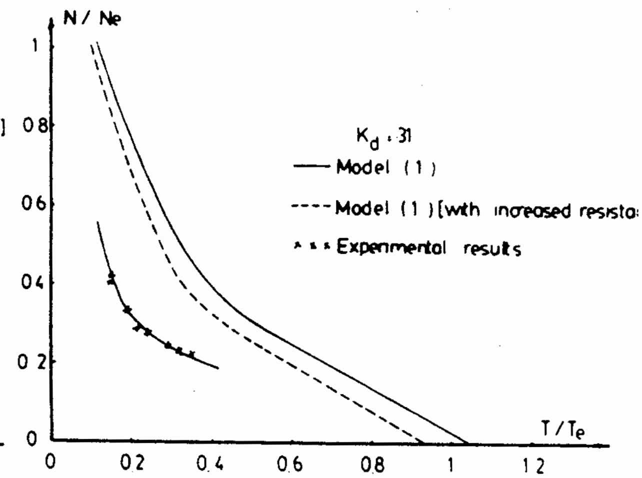

Fig 8–b Speed_Torque characteristics

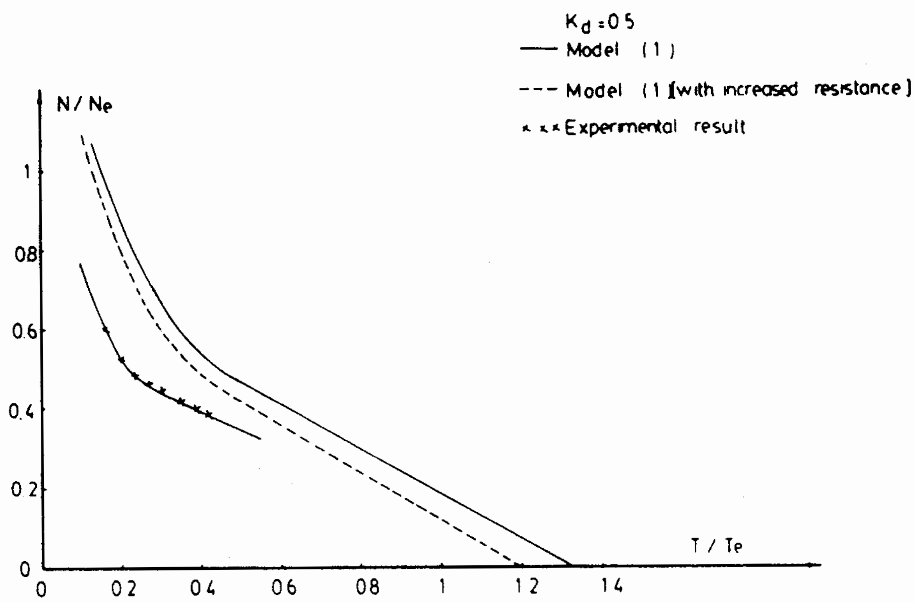

Fig 8–c Speed _ Torque characteristics

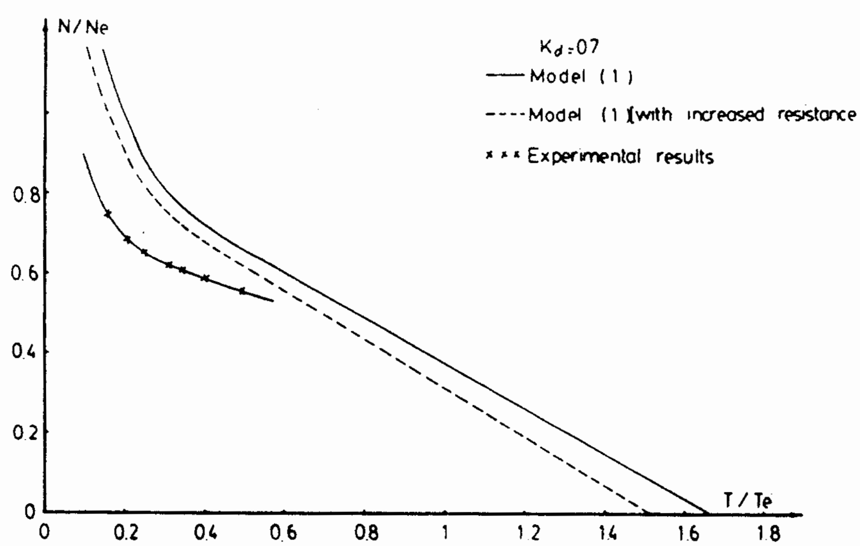

Fig 8–d Speed_Torque characterstics taking into account increased armature resistance

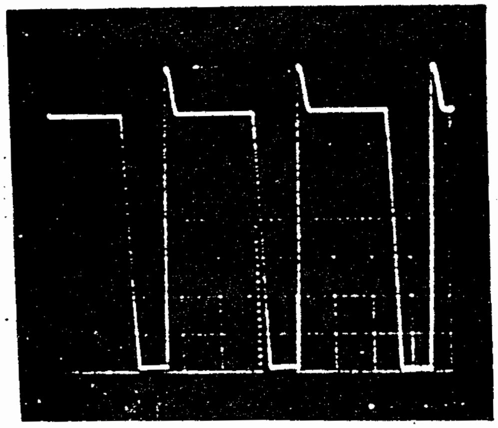

Fig.9. Capacitor voltage with conventional chopper

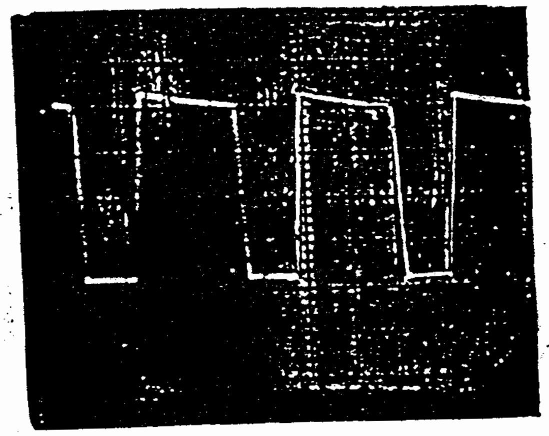

Fig.10. Capacitor voltage with new chopper

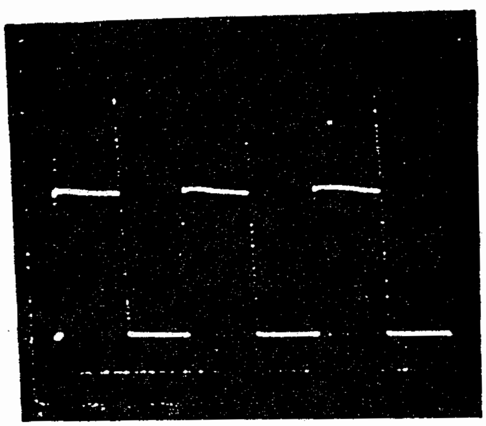

Fig.11. Load voltage

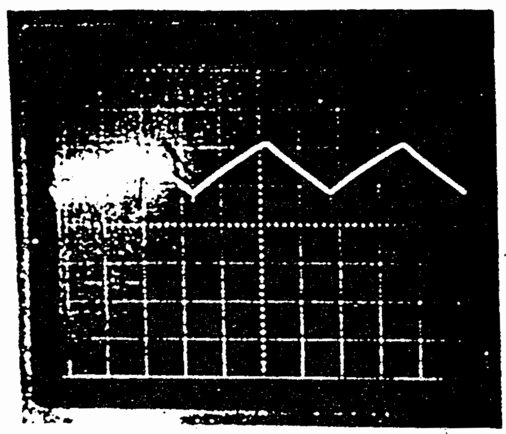

Fig.12. Load current.

be 1.3 times that of the d.c. value [1].

Photographs 9 and 10 show the capacitor voltage waveforms obtained with the conventional and the new chopper, respectively. It is clearly seen that the spike of the capacitor voltage is eliminated by using the new chopper circuit. The load voltage and current are shown in Figs. (11) and (12), respectively.

5. CONCLUSION

The steady-state performance of a separately excited d.c. motor fed by the new chopper circuit has been analysed. The analysis has been carried out taking the source inductance and the increase of armature resistance into consideration. Due to the presence of source inductance, the capacitor charges to a voltage greater than the supply voltage, which leads to restrict the commutation process. Using the new chopper circuit, the turn off time of the principal thyristor (T_1) is increased, and in turn the commutation process has been improved.

The speed-torque characteristics have been predicted by three different models. Over a wide range of load conditions, the method of analysis, which is based on the assumption of negligible losses in commutating circuit (Model 1), predicts acceptable characteristics which are considered to be in good agreement when compared with the experimental results.

APPENDIX

Experimental Set-up

The new chopper was built for a 1.5 kw, 220 V, 1500 r.p.m. d.c separately excited motor. The field current of the motor was maintained constant at its rated value of 0.42 A. A 300 m.H. smoothing inductance was used in the armature circuit of the motor. A 15 KW d.c. generator is used as the main source. The value of commutation circuit elements were L=15 m.H,C=10 μf. Chopping period was 17.5 m.sec. The motor resistance and inductance are respectively R_a=6 ohms and L_a=36 m.H. The source inductance is L_s = 3.14 m.H.

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