

ELECTROMAGNETIC UNBALANCE
IN
UNTRANSPOSED OVERHEAD TRANSMISSION LINES
BY

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ABSTRACT:

This paper presents a complete analysis for the unbalance problem in O.H.T.L. which arises due to T.L untransposition.

Derivations for the electromagnetic unbalance factors by the application of symmetrical component analysis are introduced.

The main factors affecting electromagnetic unbalance factors are discussed with special regard to double circuit T.L.

Numerical application is made to clarify the degree of effectiveness for each factor.

1. INTRODUCTION:

For untransposed transmission lines, the electromagnetic effects may be so large that affecting the balance of T.L. phase impedances which when become so-out-of balance, create unbalanced voltages, unbalanced currents at the receiving-end leading to additional heating in terminal equipments. For synchronous machines, and induction motors, the negative sequence stator currents cause a field of double frequency and opposite direction to be set up w.r.t. the rotor. This field causes circulating rotor currents which produces additional rotor heating. The degree of this heating is greatest for synchronous machines and least for induction motors. So, The study of the electromagnetic unbalance in untransposed lines and factors affecting it, is so important.

2. ANALYSIS:

In untransposed multi-circuit lines with common buses, the induced voltages cause unbalanced current that are not be in phase with each others. The in-phase portions lead to the overall net through current unbalance causing additional heating in terminal equipments. The out-of-phase portions lead to circulating currents flowing down one circuit and returning through the others causing additional transmission losses and may cause false tripping in a line circuit-breakers.

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2.1. Definitions:

For untransposed overhead transmission lines, we have two electromagnetic unbalance factors:-

a) Negative sequence unbalance factor (m_2):-

It is the ratio between negative, and positive - sequence components of the unbalance current

$$m_2 = \frac{I_2}{I_1} = \frac{-Z_{21}}{Z_{22}} = \frac{-Z_{21}}{Z_{11}} \dots\dots\dots(1)$$

b) Zero sequence unbalance factor (m_0):-

It is the ratio between zero-, and positive - sequence components of the unbalance current.

$$m_0 = \frac{I_0}{I_1} = \frac{-Z_{01}}{Z_{00}} \dots\dots\dots(2)$$

where:-

I_0, I_2, I_1 are the symmetrical components of current I.

2.2. Transmission line impedance matrix:-

The inductive characteristics of a multi-conductor T.L. can be defined by its series impedance matrix Z per unit length.

The voltage-current relation in matrix form for the series impedance of T.L. is:

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \dots\dots\dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots\dots\dots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1} & Z_{n2} & \dots\dots\dots & Z_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \dots\dots\dots(3)$$

Or, $\begin{bmatrix} \Delta V \end{bmatrix} = \begin{bmatrix} Z \end{bmatrix} \cdot \begin{bmatrix} I \end{bmatrix} \dots\dots\dots(4)$

where, $\begin{bmatrix} V \end{bmatrix}$ is the series voltage drop along the line.
 $\begin{bmatrix} I \end{bmatrix}$ is the line current.
 $\begin{bmatrix} Z \end{bmatrix}$ is the T.l. impedance including the earth effect.

$$[Z] = [r] + [\Delta r] + j \{ [x] + [\Delta x] \} \dots\dots(5)$$

$[r]$ = diagonal matrix of conductor resistance.

$[x]$ = square matrix of conductor reactance.

$[\Delta r]$, $[\Delta x]$ = square matrices calculated from Carson's earth correction formulae.

The elements of the impedance matrix $[Z]$ for conductors arrangement shown in Fig. (1).

$$Z_{ii} = r_{ii} + \Delta r_{ii} + j(x_{ii} + \Delta x_{ii}) \dots\dots\dots(6)$$

$$Z_{ik} = \Delta r_{ik} + j(x_{ik} + \Delta x_{ik}) \dots\dots\dots(7)$$

r_{ii} = resistance of conductor at system frequency.

$$x_{ii} = 0.7411 \cdot 10^{-3} \cdot w \cdot \log_{10} \frac{1}{GMR_i}$$

$$x_{ik} = 0.7411 \cdot 10^{-3} \cdot w \cdot \log_{10} \frac{1}{D_{ik}}$$

$$r_{ii} = 0.2628 \cdot 10^{-3} \cdot w \cdot + 2.599 \cdot 10^{-7} \cdot w \cdot h_i \cdot \sqrt{f/p}$$

$$x_{ii} = 0.7411 \cdot 10^{-3} \cdot w \cdot \log_{10} 2162 \sqrt{p/f} + 2.599 \cdot 10^{-7} \cdot w \cdot h_i \sqrt{f/p} \dots\dots\dots(8)$$

$$r_{ik} = 0.2528 \cdot 10^{-3} \cdot w + 2.599 \cdot 10^{-7} \cdot \frac{D_{ik}}{2} \cos \theta_{ik} \cdot \sqrt{f/p} \cdot w. \dots\dots\dots(9)$$

$$x_{ik} = 0.7411 \cdot 10^{-3} \cdot w \cdot \log_{10} 2162 \sqrt{p/f} + 2.599 \cdot 10^{-7} \cdot \frac{D_{ik}}{2} \cos \theta_{ik} \sqrt{f/p} \cdot w. \dots\dots\dots(10)$$

Equation (4) may be written as:-

$$[I] = [Y] \cdot [\Delta V] \dots\dots\dots(11)$$

where

$[Y]$ is the series admittance matrix
 $= [Z]^{-1}$

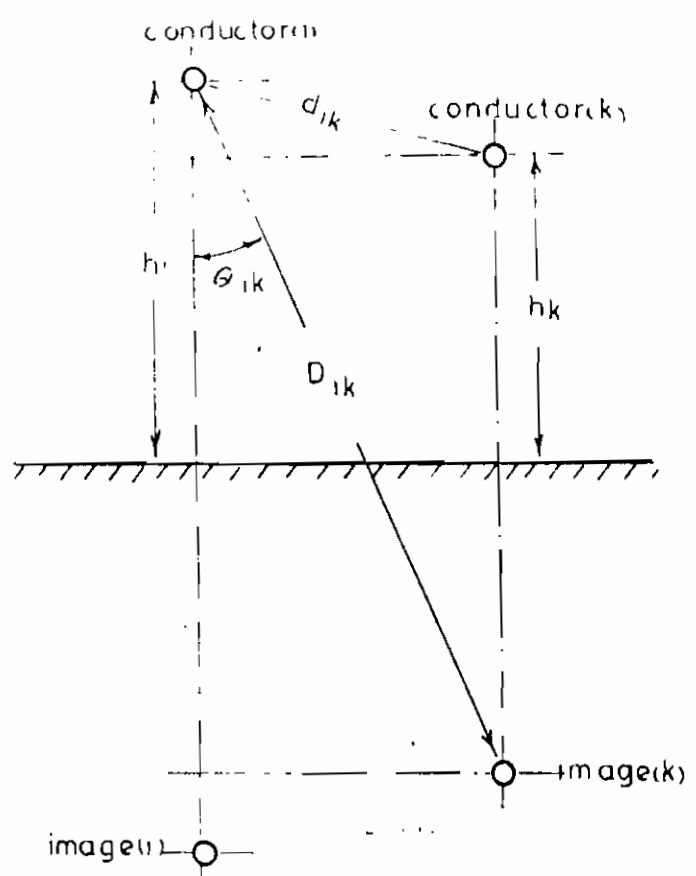


Fig.(1) SCHEMATIC ARRANGEMENT CONDUCTORS

2.3- Elimination of earthwires:-

For single-circuit O.H.T.L. with three conductors a,b,c and ground wire W, the voltage equations are:-

$$\begin{aligned} V_a &= I_a Z_{aa} + I_b Z_{ab} + I_c Z_{ac} + I_w Z_{aw} \\ V_b &= I_a Z_{ba} + I_b Z_{bb} + I_c Z_{bc} + I_w Z_{bw} \\ V_c &= I_a Z_{ca} + I_b Z_{cb} + I_c Z_{cc} + I_w Z_{cw} \\ 0 &= I_a Z_{wa} + I_b Z_{wb} + I_c Z_{wc} + I_w Z_{ww} \end{aligned} \dots\dots\dots(12)$$

from the last equation,

$$\therefore I_w = - \frac{1}{Z_{ww}} \sum_{i = a, b, c} I_i \cdot Z_{wi} \dots\dots\dots(13)$$

Substituting for I_w in the 1st three eq^{ns} of (12) and rearranging, we have:-

$$Z_{ij} \text{ (new)} = Z_{ij} - Z_{iw} \cdot Z_{jw} / Z_{ww} \dots\dots\dots(14)$$

For T.L. with multi-earthwires, eqⁿ (14) can be applied for many times as the number of earthwires.

2.4- Symmetrical component analysis:-

Equation (12) can be written in matrix form after eliminating the earthwire as:-

$$\begin{bmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \dots\dots\dots(15)$$

$$\text{or } [I_\phi] = [Y_\phi] \cdot [\Delta V_\phi] \dots\dots\dots(16)$$

Applying symmetrical component analysis. So:-

$$\begin{aligned} [C_1] \cdot [I_s] &= [Y_\phi] \cdot [C_1] \cdot [\Delta V_s] \\ \therefore [I_s] &= \frac{1}{3} [C] \cdot [Y_\phi] \cdot [C_1] \cdot [\Delta V_s] \dots\dots\dots(17) \end{aligned}$$

$$= [Y_s] \cdot [\Delta V_s] \dots\dots\dots(18)$$

$$\& [Y_s] = \frac{1}{3} [C] \cdot [Y_\phi] \cdot [C_1] \dots\dots\dots(19)$$

where:-

$[Y_{\phi}]$ is the phase series admittance matrix

$[Y_s]$ is the sequence admittance matrix.

$$= \begin{bmatrix} Y_{00} & Y_{01} & Y_{02} \\ Y_{10} & Y_{11} & Y_{12} \\ Y_{20} & Y_{21} & Y_{22} \end{bmatrix} \dots\dots\dots(20)$$

$[C]$ is the spinor transform = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \dots\dots(21)$

$C_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \dots\dots\dots(22)$

$[C_1]^{-1} = \frac{1}{3} [C] \dots\dots\dots(23)$

$a = -\frac{1}{2} + j \frac{\sqrt{3}}{2}$; $a^2 = -\frac{1}{2} - j \frac{\sqrt{3}}{2}$

the suffix (S) indicate the symmetrical component values.
as

$\Delta V_s = \begin{bmatrix} \Delta V_0 \\ \Delta V_1 \\ \Delta V_2 \end{bmatrix}$ & $I_s = \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$

and suffix (ϕ) indicates the phase values.

Substituting for (C) , (Y_{ϕ}), and (C_1) in eqⁿ (19) then solving for (Y_s) in expanded form,

For a multicircuit line(with ^Nparallel circuits) as

$$\begin{bmatrix} [Y_s]_{11} & [Y_s]_{1N} \\ [Y_s]_{N1} & [Y_s]_{NN} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} [C] & [O] \\ [O] & [C] \end{bmatrix} \cdot \begin{bmatrix} [Y_{\phi}]_{11} & [Y_{\phi}]_{1N} \\ [Y_{\phi}]_{N1} & [Y_{\phi}]_{NN} \end{bmatrix} \cdot \begin{bmatrix} [C_1] & [O] \\ [O] & [C_1] \end{bmatrix} \dots\dots(24)$$

For a double-circuit line, the sequence admittance matrix will be:-

$$[Y_s] = \frac{1}{3} \begin{bmatrix} [C] & [0] \\ [0] & [C] \end{bmatrix} \begin{bmatrix} [Y_\phi]_{11} & [Y_\phi]_{12} \\ [Y_\phi]_{21} & [Y_\phi]_{22} \end{bmatrix} \begin{bmatrix} [c_1] & [0] \\ [0] & [c_1] \end{bmatrix} \dots\dots\dots(25)$$

So, eqⁿ (18) in expanded form (considering balanced voltage drops) will be:-

I_0	Y_{00}	Y_{01}	Y_{02}	$Y_{00'}$	$Y_{01'}$	$Y_{02'}$	0
I_1	Y_{10}	Y_{11}	Y_{12}	$Y_{10'}$	$Y_{11'}$	$Y_{12'}$	ΔV_1
I_2	Y_{20}	Y_{21}	Y_{22}	$Y_{20'}$	$Y_{21'}$	$Y_{22'}$	0
I_0'	$Y_{00'}$	$Y_{01'}$	$Y_{02'}$	$Y_{00''}$	$Y_{01''}$	$Y_{02''}$	0
I_1'	$Y_{10'}$	$Y_{11'}$	$Y_{12'}$	$Y_{10''}$	$Y_{11''}$	$Y_{12''}$	ΔV_1
I_2'	$Y_{20'}$	$Y_{21'}$	$Y_{22'}$	$Y_{20''}$	$Y_{21''}$	$Y_{22''}$	0

.....(26)

N.B. dash referred to the second circuit
Therefore,

$$\begin{aligned}
 I_0 &= (Y_{01} + Y_{01'}) \cdot \Delta V_1 \\
 I_1 &= (Y_{11} + Y_{11'}) \cdot \Delta V_1 \\
 I_2 &= (Y_{21} + Y_{21'}) \cdot \Delta V_1 \\
 I_0' &= (Y_{01'} + Y_{01''}) \cdot \Delta V_1 \\
 I_1' &= (Y_{11'} + Y_{11''}) \cdot \Delta V_1 \\
 I_2' &= (Y_{21'} + Y_{21''}) \cdot \Delta V_1
 \end{aligned}
 \dots\dots\dots(27)$$

& The net-through symmetrical component currents are:-

$$\begin{aligned}
 (I_0 + I_0') &= (Y_{01} + Y_{01'} + Y_{01'} + Y_{01''}) \cdot \Delta V_1 \\
 (I_1 + I_1') &= (Y_{11} + Y_{11'} + Y_{11'} + Y_{11''}) \cdot \Delta V_1 \\
 (I_2 + I_2') &= (Y_{21} + Y_{21'} + Y_{21'} + Y_{21''}) \cdot \Delta V_1
 \end{aligned}
 \dots\dots\dots(28)$$

So, The circulating symmetrical component current are:-

$$\begin{aligned} (I_0 - I_0') &= (Y_{01} + Y_{01}' - Y_{01}' - Y_{01}'') \cdot \Delta V_1 \\ (I_1 + I_1') &= (Y_{11} + Y_{11}' + Y_{11}' + Y_{11}'') \cdot \Delta V_1 \quad \dots\dots\dots(29) \\ (I_2 - I_2') &= (Y_{21} + Y_{21}' - Y_{21}' - Y_{21}'') \cdot \Delta V_1 \end{aligned}$$

3- Numerical application:-

For the double circuit O.H.T.L. with admittance matrix given in Table (1), it is required to have:-

- 1) The individual circuit unbalance, The net through unbalance, The circulating current unbalance.
- 2) The effect of terminal impedances on the unbalance.
- 3) The effect of series capacitor compensation on the unbalance.
- 4) The effect of T.L. phase arrangements on the unbalance.

Results:-

3.1- Unbalance factors

Applying equations (26, 27, 28, 29) and by the use of computer programming the individual circuit unbalance, the net through unbalance, and the circulating current unbalance are computed and tabulated in Table (2).

3.2- Effect of terminal impedances:-

Consider a terminal impedance of,

$$Z_{00T} = Z_{11T} = j 0.5 \text{ ohm:}$$

In series with the double circuit O.H.T.L., recomputing the zero-, and negative-sequence unbalance factors, it is found that the net through unbalance factors variation is:
 percent m_0 is reduced from 1.3019 to 0.9531.
 percent m_2 is reduced from 4.6676 to 1.7926.

Whilst the terminal impedances has no effect on the circulating current unbalance factors.

Circuit I			Circuit II		
0	1	2	0	1	2
0.0972	0.0706	-0.0674	0.0481	0.019L	-0.0344
-0.7372	0.0341	0.0293	0.3791	0.1001	0.0949
0.0706	0.1059	0.0844	-0.0720	-0.0491	-0.0361
0.0341	0.0863	-1.5552	-0.0732	-0.0375	-0.0498
-0.0674	0.0844	-0.1173	0.0842	0.0413	0.0529
0.0295	-1.5552	0.1091	-0.0625	-0.0433	-0.0308
0.0481	-0.0720	0.0842	0.0975	0.0233	-0.0209
0.3791	-0.0732	-0.0625	-0.7372	-0.0693	-0.0742
0.0191	-0.0491	0.0413	0.0233	0.1160	0.0844
0.1001	-0.0375	-0.0433	-0.0690	0.0731	-1.5552
0.1891	-0.0361	0.0529	-0.0209	0.0844	-0.1237
0.0259	-0.0498	-0.0308	-0.0742	-1.5552	0.0555

Symmetrical component admittance matrix (mho-miles)
for untransposed double circuit T.L.

Table 1

	Circuit I	Circuit II	
Y_{01}	$0.0706 + j0.0341$	Y'_{01}	$-0.0725 + j0.0732$
Y_{01}'	$0.0191 + j0.1001$	Y_{01}	$0.0233 + j0.0690$
$I_0 / \Delta E_1$	$0.0897 + j0.1342$	$I_0' / \Delta E_1$	$-0.0487 - j0.1422$
Y_{21}	$0.1059 + j0.0863$	Y'_{21}	$-0.0491 - j0.0375$
Y_{21}'	$-0.0491 - j0.0375$	Y_{21}	$0.1160 + j0.0731$
$I_2 / \Delta E_1$	$0.0568 + j0.0488$	$I_2' / \Delta E_1$	$0.0669 + j0.0356$
Y_{11}	$0.0844 - j1.5552$	Y'_{11}	$-0.0361 - j0.0498$
Y_{11}'	$0.0413 - j0.0433$	Y_{11}	$0.0844 - j1.5552$
$I_1 / \Delta E_1$	$0.1257 - j1.5985$	$I_1' / \Delta E_1$	$0.0483 - j1.6050$
percent m_0	10.0658	percent m_0'	9.3609
percent m_2	4.6703	percent m_2'	4.7195
Net through unbalance			
$(I_0 + I_0') / \Delta E_1$	$0.0410 - j0.008$		
$(I_2 + I_2') / \Delta E_1$	$0.1237 + j0.0844$		
$(I_1 + I_1') / \Delta E_1$	$0.1740 - j3.2035$		
percent m_0	$1.3019 + 76.66$		
percent m_2	$4.6676 \quad 121.6$		
Circulating current unbalances			
$(I_0 - I_0') / \Delta E_1$	$0.1384 + j0.2764$		
$(I_2 - I_2') / \Delta E_1$	$-0.0101 + j0.0132$		
$(I_1 - I_1') / \Delta E_1$	$0.1740 - j3.2035$		
percent m_0	$9.6346 \quad 150.3$		
percent m_2	$0.5180 \quad -145.67$		

Table 2

Table (3) Ratios of circuit current components to ΔE_1

PHASE ARRANGEMENT	THROUGH CURRENTS			CIRCULATING CURRENTS		
	$(i_0 + i_0') / \Delta E_1$	$(I_1 + I_1') / \Delta E_1$	$(I_2 + I_2') / \Delta E_1$	$(i_0 - i_0') / \Delta E_1$	$(I_1 - I_1') / \Delta E_1$	$(I_2 - I_2') / \Delta E_1$
abc - abc	0.0410 -0.0080	1.071257 -1.5982	0.1237 0.0846	0.1384 0.2764	0.1257 -1.5982	-0.0101 0.0137
abc - bca	0.0043 -0.0914	0.0265 -1.5695	-0.0308 0.2725	0.2911 0.0264	0.0265 -1.5695	0.2276 0.0221
abc - cab	-0.0495 -0.0179	0.1010 -1.4979	0.2248 -0.0985	-0.0017 0.0191	0.1010 -1.4979	0.1002 0.2231
abc - bac	-0.0568 -0.0184	0.1727 -3.1789	0.0699 0.0373	0.1292 0.2764	0.1021 0.0065	0.0699 0.0373
abc - bac	0.0442 -0.0015	0.1725 -3.1646	0.0815 0.1420	0.2957 0.0344	-0.1104 -0.0066	-0.1104 -0.0066
abc - cba	0.0084 -0.0974	0.1612 -2.9877	0.1663 0.0796	0.0029 0.0111	0.0083 0.0001	0.1663 0.0796
acb - abc	0.0592 -0.0019	0.0352 -1.5922	0.0813 0.0959	-0.1553 0.2611	0.0353 -1.5922	0.2333 0.0363
acb - bca	0.0225 -0.0853	0.0769 -1.4942	-0.3546 0.1446	-0.0031 0.0111	0.0769 -1.4942	0.0043 0.0450
acb - cab	-0.0313 -0.0118	0.1410 -1.5792	-0.0786 0.0868	-0.2959 0.0038	0.1410 -1.5792	-0.1229 0.2460
acb - acb	-0.0386 -0.0123	0.1741 -3.2027	-0.0643 0.0781	-0.1650 0.2611	-0.0773 -0.0057	-0.0643 0.0781
acb - bac	0.0624 0.0046	0.1612 -2.9973	-0.1706 0.0787	0.0015 0.0191	-0.0413 -0.0017	-0.1706 0.0787
acb - cba	0.0266 -0.0913	0.1711 -3.1312	-0.1170 0.1705	-0.2913 -0.0042	0.0003 -0.0001	-0.1170 0.1705

Table (4) Net through, and circulating current unbalance factors for different phase arrangements of double circuit untransposed T.L.

PHASE ARRANGEMENT	THROUGH CURRENTS		CIRCULATING CURRENTS	
	M_0	M_2	M_0	M_2
ab - abc	0.0257	0.0936	0.1928	0.0106
abc - bca	0.0583	0.1747	0.1862	0.1457
abc - cab	0.0351	0.1635	0.0128	0.1629
abc - acb	0.0188	0.0249	0.0958	0.0249
abc - bac	0.0140	0.0516	0.0939	0.0516
abc - cba	0.0327	0.0616	0.0038	0.0616
acb - abc	0.0372	0.0739	0.1909	0.1483
acb - bca	0.0589	0.2550	0.0077	0.0302
acb - cab	0.0211	0.0738	0.1867	0.1734
acb - acb	0.0126	0.0315	0.0963	0.0315
acb - bac	0.0209	0.0626	0.0064	0.0626
acb - cba	0.0303	0.5112	0.0929	0.0659

3.3- Effect of series capacitor compensation:-

For 0.75 compensation, the compensation can be considered as an impedance in series with each circuit equal to:-

$$Z_{ooc} = Z_{11c} = -j (0.75) (Z_{11}) .2 = 0.4674 \angle -90$$

The through current unbalance of the compensated line is:-

$$\text{percent } m_0 = \frac{-Z_{01} .100}{Z_{00} + Z_{ooc}} = 1.326$$

$$\text{percent } m_2 = \frac{-Z_{21} .100}{Z_{11} + Z_{11c}} = 9.2424 \dots\dots(30)$$

So, the effect of series compensation is as:-

percent m_0 is increased from 1.3019 to 1.9172
 percent m_2 is increased from 4.6676 to 9.2541

3.4- Effect of T.L. phase arrangements:-

For a double circuit O.H.T.L., the electromagnetic unbalance factors are calculated for all possible different phase arrangements.

Table (3) indicates the computer results of through-, and circulating-currents for different T.L. arrangements.

Table (4) indicates the electromagnetic unbalance factors within and at the terminals of the double circuit line.

4. CONCLUSIONS:

The circulating currents in double circuit untransposed O.H.T.L. increase the line current in one circuit and decrease it in the other. The in-phase portion causes the net through unbalance at the terminals which leads to additional heating in terminal equipments. The out-of-phase portion creates additional transmission losses.

Series capacitor compensation greatly magnifies the net through unbalance.

Whilst the terminal impedances reduce the net through unbalance, it has no effect in the circulating current unbalance.

The optimum phase arrangement for minimizing through current unbalance will lead to maximum circulating current unbalance.

So, from the economical point of view, optimum phase arrangement should be carried out to minimize circulating current unbalance, Since the net through will decrease by terminal equipment impedances.

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