Mansoura University Faculty of Engineering Structural Engineering Dept.	**First Page 1/2**	Graduate Studies Theory of Plasticity 2012 - 2013
Time allowed: 3 hours		
Students are allowed to only have the textbook "Plasticity for Structural Engineers" by Chen & Han		
Any missing data may be assumed		

Problem (1)-20%:

Fig. P1 shows a bar with a variable cross section; the left part has a length a = 0.6L with cross sectional area A and the right part has a length b = 0.4L with cross sectional area 0.5A, where L is the bar length. The bar is subjected to an axial force P through two rigid plates which are attached to the bar ends, as shown in the figure. The The bar is made of an elastic-perfectly plastic matreial with a yield stress σ_o . The axial force is first increased from zero until plastic flow occurs; then, P is completely unloaded to zero, followed by a reloading in the reversed direction until plastic flow occurs.

i. Determine the elastic and plastic limit loads P_e and P_p during the loading.
ii. Find the residual stress and plastic strain in the bar when the axial load P is unloaded to zero.
iii. Determine the plastic limit load P_p during the reversed loading.

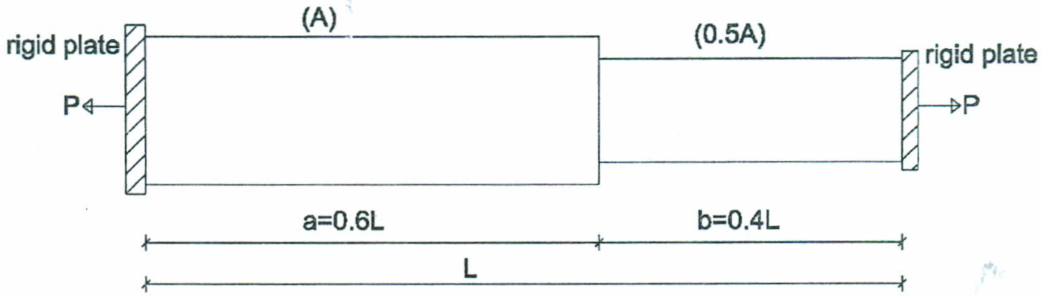

Fig. P1.

Problem (2)-25%:

The stress tensor at a point under the working load condition is given by

$$\sigma_{ij} = \begin{bmatrix} 15 & 12 & 0 \\ 12 & 20 & 0 \\ 0 & 0 & -8 \end{bmatrix} \text{MPa}$$

Based on the yield criterion $\tau_{oct} = 90$ MPa

i. Calculate the safety factor of the point against yield, if all the stresses are increased proportionally to reach the yield surface.
ii. Calculate the safety factor of the point against yield, if the stress σ_x is increased to the critical value of yielding, while the other stresses remain the same.
iii. Determine the yield stress in simple compression.
iv. List the general characteristics of the yield surface associated with such criterion.

Problem (3)-20%:

Consider a nonlinear elastic material based on the complementary strain energy density, Ω, given by

$$\Omega(I_1, J_2) = aI_1^2 + bJ_2^2$$

where a and b are material constants. The stress-strain relationship of the material in simple tension is given by

$$10^7 \varepsilon = 10^3 \sigma + \sigma^3$$

i. Determine the constants a and b.

ii. Predict the the corresponding components of strain ε_{ij} of an element of this material subjected to a loading history which produces the following stress state,

$$\sigma_{ij} = \begin{bmatrix} 20 & 15 & 0 \\ 15 & 24 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

iii. Show that if this material model satisfies Drucker's stability postulate $\sigma_{ij}\varepsilon_{ij} > 0$ for any uniaxial tension ($\sigma_x = \sigma \geq 0$) or uniaxial compression ($\sigma_x = \sigma \leq 0$) state of stress.

Problem (4)-14%:

A thin walled tube is subjected to a constant torsion and a variable axial tension. The torsion stress is $\tau = 0.5\sigma_o$. According to both the (i) von Mises criterion and (ii) Tesca criterion, find the magnitude of the axial stress σ_z such that the tube begins to yield. Also, find the ratio of the plastic strain increments $d\varepsilon_{ij}^p$ when the tube is yielded.

Problem (5)-5%:

Verify the upper bound to the limit load for the deformation mode shown in Fig. P5.

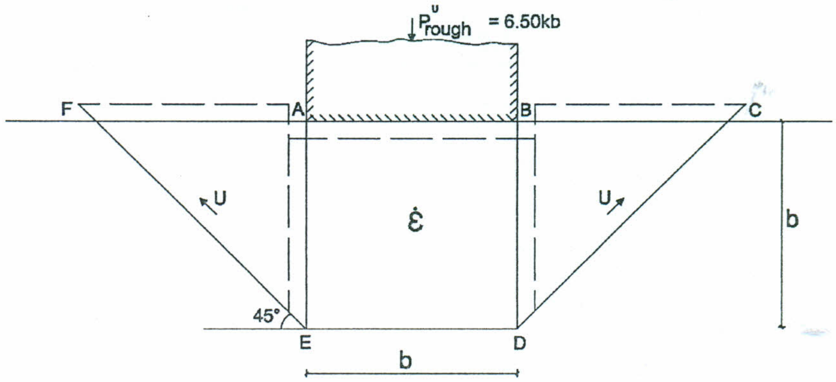

Fig. P5.

Problem (6)-16%:

For the beam and frame shown in Fig. P6, assuming that all members have the same plastic moment, M_p, find the collapse load P in terms of M_p.

(i)

(ii)

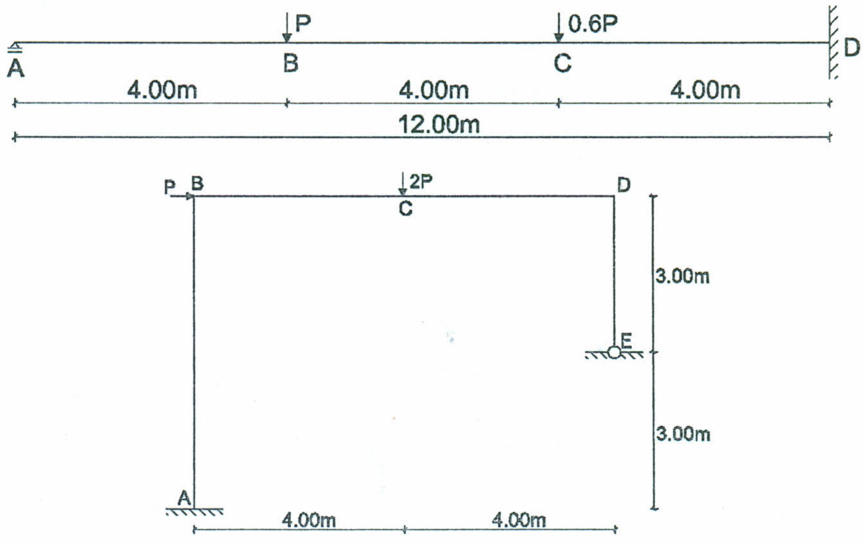

Fig. P6.