Mansoura Bulletin Vol. 8, No. 1, June 1983.

CATASTROPHE AND HOPF BIFURCATION THEOREMS FOR SYNCHRONOUS MACHINE STABILITY STUDIES

A.A.SALLAM
Dept. of Elect. Eng.
Univ. of Suez-Canal
Port-Said , Egypt

J.L.DINELEY
Dept of Elect. Eng.
Univ. of Newcastle
Newcastle Upon Tyne, U.K.

ABSTRACT

Both Catastrophe theory and the Hopf bifurcation theorem can be applied to the study of the stability of synchronous generators connected to power systems, when subjected to small disturbances.

The Hopf bifurcation theorem can be used to determine the stability of the produced orbits of oscillation which indicate the stability state of the synchronous machine, here treated first as a second order problem and then by including the governor response.

Catastrophe theory usefully visualises the stability problem by deriving the approp riate catastrophe manifold. In this paper this is shown to be a "butterfly" manifold, after taking into account the described conditions.

INTRODUCTION

The possibility of using catastrophe theory for synchronous generator stability prediction under the effects of small disturbances (the steady state) has been discussed in a previous paper by the authors.[1]

In addition to the visualisation effects so attractive to the power system engineer, catastrophe theory offers a useful qualitative analysis. Hopf bifurcation theorem usefully extends the qualitative analysis and give indications of the stability of produced closed orbits of oscillation for nonlinear engineering systems. It is essentially a study of the dynamics of variation of the state variables and the control parameters under appropriate conditions, using analysis of bifurcation and the probability of occurence of orbits of flow induced oscillations.

One of the very useful results seen in bifurcation analysis is the role of positive or negative damping coefficient. Damping is taken as a bifurcation parameter, leading to determination of the stability boundary for given values of damping coefficient. The equilibrium point giving closed orbits of oscillation determines the system stability under small disturbance effects.

Whilst the theoretical background [2,3] cannot be simplified, the paper presents it in a form appropriate to synchronous power system studies and then proceeds to illustrate the effects by numerical examples.

Section 1 of the paper deals with the bifurc theorems (Centre-manifold and Hopf bifurcation th Their applications are illustrated in Section 2 w the stability of a synchronous generator first as a 2nd. order model, then takes it to a thirdorder representation by taking into account governor effects using one time lag. Section 3 illustrates the developement of the application of catastrophe theory to power system stability when it has been visualised by three dimensional plotting to produce a butterfly manifold together with its bifurcation set in the control space, instead of the simple cusp manifold described in the previous paper. It takes into account the effects of input power, synchronous and asynchronous power , and saliency.

In sections 2 & 3 a numerical example has been given for each case. The final conclu-sions are presented in Section 4.

1. BIFURCATION THEOREMS

It is important to understand the dynamic stability problem concerningthebifurcation of equilibrium states. For that purpose, it is useful to ensure that the bifurcated equilibria lies on invariant manifold of low dimensions for the full dynamical problem. Then the questions of stability become questions on the invariant manifold M (centre manifold).

1.1 The Centre Manifold Theorem.[4,5]

This is also called,"The Reduction Theorem" as it reduces the problem to a finite dimensional problem, the statement then is :

Let ψ be a mapping of a neighbourhood of zero in a Banach space z into z. We assume that ψ is c^{k+1}, $k \geq 1$ and that $\psi(0)=0$. We further assume that $D\psi(0)$ has spectral radius 1 and that the spectrum of $D\psi(0)$ splits into a part on the unit circle, the remainder which is at a non-zero distance from the unit circle. Let Y denote the generalized eigen-space of $D\psi(0)$ belonging to the part of the spectrum on the unit circle; assume that Y has dimensions $d < \infty$.

Then there exists a neighbourhood V of 0 in z and a c^k submanifold M of V, of dimension d, passing through 0 and tangent to Y at 0, such that: a)"local invariance" if $x \in M$ and $\psi(x) \in V$, then $\psi(x) \in M$

b)"local attractivity" if $\psi^n(x) \in V$ for all $n=0,1,2,\ldots$, then, as $n \longrightarrow \infty$, the distance from $\psi^n(x)$ to M goes to zero.

This reduction to a centre manifold M without any loss of information concerning stability permits the application of the Hopf bifurcation theorem to determine the stability of the produced orbits of oscillation under the effect of small disturbances at the equilibrium point.

1.2:A: The Hopf Bifurcation Theorem In R[6]

Let X_μ be a c^k $(k \geq 4)$ vector field on R^2 such that $X_\mu(0)=0$ for all μ and $X=(X_\mu, 0)$ is also c^k Let $dX_\mu(0,0)$ have two distinct, complex conjugate eigenvalues $\lambda(\mu)$ and $\overline{\lambda(\mu)}$ such that for $\mu>0$, $\mathrm{Re}\,\lambda(\mu) > 0$. Also, Let $\frac{d(Re\lambda(\mu))}{d\mu}\Big|_{\mu=0} > 0$. Then

a) there is a C^{k-2} function $\mu:(-\varepsilon,\varepsilon) \longrightarrow R$ such that $(x_1, 0, \mu(x_1))$ is on a closed orbit of period $\approx 2\pi/|\lambda(0)|$ and radius growing like $\sqrt{\mu}$ of x for $x_1 \neq 0$ and such that $\mu(0) = 0$.

b) there is a neighbourhood U of (0,0,0) in R^3 such that any closed orbit in U is one of those above. Further, if 0 is a "vague attractor" for x_0, then

c) $\mu(x_1) > 0$ for all $x_1 \neq 0$ and the orbits are attracting.

1.2:B: The Hopf Bifurcation Theorem In R^n.

Let $X\mu$ be a $C^{k+1}, k \geqslant 4$, vector field on R^n with all the assumptions of the theorem in R^2 holding except for the assumption that the rest of the spectrum is distinct from the two assumed simple eigenvalues $\lambda(\mu)$, $\overline{\lambda(\mu)}$. Then conclusion (a) is true. Conclusion (b) is true if the rest of the spectrum remains in the left half plane as μ crosses zero. Conclusion (c) is true if, relative to $\lambda(\mu)$, $\overline{\lambda(\mu)}$, 0 is a "vague attractor" in the same sense as in the theorem in R^2 and if, when coordinates are chosen so that

$$dX_0(0) = \begin{bmatrix} 0 & |\lambda(0)| & d_3X^1(0) \\ -|\lambda(0)| & 0 & d_3X^2(0) \\ 0 & 0 & d_3X^3(0) \end{bmatrix}$$

and $\lambda(0) \notin \sigma(d_3X^3(0))$

For applications, the most interesting consequence of the theorems is the central significance of the eigenvalues on the imaginary axis where the nonlinear behaviour of the system is essentially unfolded through these eigenvalues Therefore, the determination of surfaces in the control space where one or more eigen - values cross the imaginary axis which may be obtained analytically is of large importance in studying the behaviour of nonlinear systems When more than two eigenvalues cross the imaginary axis at non-zero speed, the bifurcational behaviour analysis is still incomplete, but it is possible by using these theorems to study the effect of some system parameters on stability.

2. APPLICATION OF HOPF BIFURCATION TO THE SYNCHRONOUS MACHINE STABILITY.

2.1 The Synchronous Machine as a second order model. [7]

The equation of motion of the synchronous machine is then : $M\ddot{\delta} + D\dot{\delta} = P_i - P_m \sin\delta$

By expressing this equation as the vector field $X_a(\delta,\omega)$ on R^2 , we have $\dot{\delta}=\omega$ and $\dot{\omega}=c-b \sin +a\omega$

where, $c=P_1/M$, $a=-D/M$, $b=P_m/M > 0$
The system has two equilibrium points $(\delta_1, \omega=0)$ and $(\delta_2, \omega=0)$ where $\delta_1=\mathrm{Sin}^{-1}(c/b)$ and $\delta_2=\pi-\delta_1$
but in case of a power system, the operating point is normally at $0<\delta<\pi/2$, so we are concerned with $\delta_o = \sin^{-1}(c/b)$ at $\omega_o=0$ as the system equilibrium point. Taking a, which is a function of damping coefficient as a bifurcation parameter, then

$$X_a(\delta,\omega) = (\omega, c-b\,\mathrm{Sin}\delta + a\omega) \quad \text{and}$$

$$X_a(\mathrm{Sin}^{-1}(c/b),0) = (\omega, a\omega) = (0,0) \text{ for all a.}$$

At the equilibrium point;

$$dX_a(\delta_o,\omega_o) = dX_a(\mathrm{Sin}^{-1}(c/b),0) = \begin{bmatrix} 0 & 1 \\ -\sqrt{b^2-c^2} & a \end{bmatrix}$$

With eigenvalues

$$\lambda = \frac{a}{2} \pm i\,\frac{\sqrt{4N-a^2}}{2} \quad \text{where} \quad N=\sqrt{b^2-c^2} \text{ and must be positive.}$$

Also,

$$\left.\frac{d(\mathrm{Re}\lambda(a))}{da}\right|_{a=0} = \frac{1}{2}>0,$$ i.e. the eigenvalues cross the imaginary axis at non-zero speed,

and for

$-2\sqrt{N} < a < 0$	$\mathrm{Re}(\lambda a) < 0$
$a = 0$	$\mathrm{Re}(\lambda a) = 0$
$0 < a < 2\sqrt{N}$	$\mathrm{Re}(\lambda a) > 0$

Therefore, the Hopf bifurcation theorem is seen to apply and it is concluded that there is a one parameter family of closed orbits of $X=(\delta_a,0)$ in a neighbourhood $(\delta_o,0,0)$.

According to Hopf bifurcation theorem, in case of $a<0$ (the damping coefficient is posi-tive) the origin is attracting or as it is sometimes called asymptotically stable (sink point), and for $a=a_o=0$ (the damping is negligable) the bifurcation has occured. The closed orbits must be known if they are stable or not by calculating $V'''(0)$ as below (ref.6, section 4, pp.104). In the case of $a>0$ the origin is a source point and the system is unstable, fig.(1) .

Generator damping is represented in the usual way as a velocity damping factor replacing sub-transient modelling of the gen- erator.Negative damping arises when resist- ances, either of generator armature or transm- ission line, become sufficiently large when compared with reactances. Whilst negative damping is not common, it is important for a study such as this to consider what its effects will be.[8,9,10]

So, the conditions for Hopf bifurcation to occur at a=0 are fulfilled with $(0) = + i\sqrt{N}$, then

$$dx_{ao}(\delta_o,\omega_o) = \begin{bmatrix} 0 & 1 \\ -N & a_o \end{bmatrix}$$

The system coordinates must be changed to new coordinates which are chosen so that $dx_{ao}(\delta_o,\omega_o)$ is in the form :

$$dx_{ao}(\delta_o,\omega_o) = \begin{bmatrix} 0 & |\text{Im}\,\lambda(0)| \\ -|\text{Im}\,\lambda(0)| & 0 \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{N} \\ -\sqrt{N} & 0 \end{bmatrix}$$

It should be noted that if N=1 and of course a equal to zero, then

$$dx_{ao}(\delta_o,\omega_o) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

So, without changing the coordinates, the original coordinates are appropriate to the calculation. To determine if the produced closed orbits are stable or not, it is necessary to calculate the sign of $V'''(0)$ by calculating the partials of $X_{ao}(\delta_o,\omega_o)$ up to order three (the negative sign for stable state and the posit- ive sign for unstable state).

$$\frac{\partial^n x_1}{\partial\delta^j\partial\omega^{n-j}}(\delta_o,0)=0 \qquad \text{for all } n>1 \text{ since } x_1(\delta,\omega)=\omega$$

$$\frac{\partial^n x_2}{\partial\delta^j\partial\omega^{n-j}}(\delta_o,0) = 0 \quad \text{for } n=1,2,3 \text{ except}$$

$$\frac{\partial^2 x_2}{\partial\delta^2}(\delta_o,0) = c \text{ and } \frac{\partial^3 x_2}{\partial\delta^3}(\delta_o,0) = N = 1$$

thus, $V'''(0)=(3\pi/4)(c+1) > 0$, since $c \geqslant 0$

Therefore, it is seen that the orbits in this case (N=1) are unstable.

This proves that, if the operating point is very close to the point at which the max-imum power is delivered, the power system is tending to the state of instability under the effects of any small disturbance (steady state stability limit), because at N=1, it means that $\sqrt{b^2 - c^2} = 1$, i.e. $b^2=c^2+1$.

As the values of b&c are much greater than 1 as shown in the numerical example in the next item, so $b \approx c$ hence $P_1 \approx P_m$ which means that the operating point is in a neighbourhood which is very close to the limiting point of maximum power.

In case of N≠1, it is necessary to express the damping coefficient in terms of the rotor angle [11] to be able to apply Hopf bifurcation theorem, Otherwise, the value of $V'''(0)$ is always equal to zero, i.e. it is not possible to decide the stability.

2.2 Regulated Synchronous Machine Including Governor Representation By One Time Lag.

Considering the block diagram of the synchronous machine shown in fig.(2) includes the combined effects of both the turbine and speed governor system [12], the system dynamic equations are:

$$\dot{\delta} = \omega$$

$$\dot{\omega} = cP_i - b\sin\delta + a\omega$$

$$\dot{P}_i = q - \ell\omega - rP_i$$

where,

$a=-D/M$, $b=P_m/M$, $c=1/M$, $r=1/_\tau$

$\ell=kr$, $k=1/R$, $q/r=\uparrow q=$ initial input power

The system has the equilibrium point $(\delta_o, \omega_o, P_{io})$ where, $\delta_o = \sin^{-1}(cq/br)$, $\omega_o=0$, $P_{io}=q/r$,thus $X_a(\delta_o, 0, q/r) = 0$ for all values of a, and a, is the bifurcation parameter.

$$dX_a(\delta_o, 0, q/r) = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{r}\sqrt{b^2r^2-c^2q^2} & a & c \\ 0 & -\ell & -r \end{bmatrix}$$

The characteristic polynomial equation is

$$X^3+(r-a)x^2+(\ell c+\sigma/r-ra)x+\sigma=0 \quad (*) \text{ and}$$

$$\sigma=\sqrt{b^2r^2-c^2q^2}$$

This equation has two purely imaginary roots if and only if the product of the coefficients of x^2 and x terms is equal to the constant term, i.e.

$$(r-a_o)(\ell c+\sigma/r-ra_o)=\sigma$$

then

$$a_o = \frac{1}{2r}\{(\sigma/r+r^2+\ell c)\pm\sqrt{(\sigma/r+r^2+\ell c)^2-4\ell cr^2}\}$$

Assuming the characteristic polynomial be written

$$(x-\lambda)(x-\bar{\lambda})(x-\alpha)=0 \quad \text{where} \quad \lambda=\lambda_1\pm i\lambda_2, \text{ i.e.}$$

$$x^3-(2\lambda_1+\alpha)x^2+(|\lambda|^2+2\lambda_1\alpha)x-|\lambda|^2\alpha=0 \qquad (**)$$

Equating coefficients of like powers of x in (*) & (**), we get

$$-(r-a)=2\lambda_1+\alpha, \qquad -\sigma = |\lambda|^2\alpha$$

$$\ell c+\sigma/r-ra=|\lambda|^2+2\lambda_1\alpha$$

solving these three equations

$$\alpha=-(r-a+2\lambda_1) \quad \text{and} \quad \alpha(\ell c+\sigma/r-ra)=-\sigma+2\lambda_1\alpha^2$$

$$-(r-a+2\lambda_1)(\ell c+\sigma/r-ra)=-\sigma+2\lambda_1(r-a+2\lambda_1)^2$$

Differentiating w.r.t. a, setting $a=a_o$,and subistituting $\lambda_1(a_o)=0$, and for $\ell c+\sigma/r>2ra_o$,

$$\lambda_1'(a_o) = \frac{r^2+\ell c+\sigma/r-2ra_o}{2\{(\ell c+\sigma/r-ra_o)+(r-a_o)^2\}} > 0$$

Therefore, the eigenvalues cross the imaginary axis with nonzero speed, & hence a Hopf bifurcation occurs at a_o.

At a_o : $|\lambda|^2=-\sigma/\alpha$ $\qquad \alpha=-(r-a_o)$

thus $|\lambda| = \sqrt{\sigma/(r-a_o)}$

the basis for R^3 must be computed in which

$$dX_{ao}(\delta_o, 0, q/r) = \begin{bmatrix} 0 & 1 & 0 \\ -\sigma/r & a_o & c \\ 0 & -\ell & -r \end{bmatrix}$$

becomes

$$\begin{bmatrix} 0 & \sqrt{\sigma/(r-a_o)} & 0 \\ -\sqrt{\sigma/(r-a_o)} & 0 & 0 \\ 0 & 0 & -(a_o - r) \end{bmatrix}$$

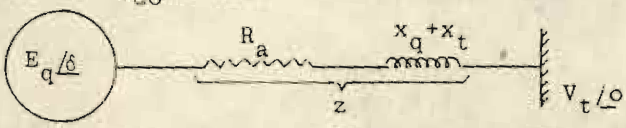
and obtaining the vector field of the system w.r.t. these bases to determine the sign of $V'''(0)$ which decides the system stability. The published computer program BIFOR2 [13] can be used for this purpose.

<u>NUMERICAL EXAMPLE</u> [14]: Considering one machine infinite bus system with the following data:

$V_t=1\angle 0^o$, $E_t=1.05$, $H=3$ MJ/MVA , $P_{io}=0.8$

$x_d=1.05$, $x_d'=0.29$, $R_a=0.07$, $x_q=x_q'=0.69$,

$x_t=0.36$, $\tau_{do}'=5$ sec.



The constants of the governor response are taken as

$R=0.05$, $k=20$, $\tau=0.5$ sec.

then by calculation :

$E_q=1.328\ \angle 39.2^o$, $P_m=1.265$, $M=0.02$

$c=50$, $a=-50\,D$ as a function of damping coefficient

$b=63.25$, $r=2$, $l=40$

$q=1.6$, $q/r=\tau q=0.8$

(all values are in per unit)

The system equilibrium point is (39.2^o , 0 , 0.8)

$\sigma=97.99$ &

$$dX_a(39.2^o, 0, 0.8) = \begin{bmatrix} 0 & 1 & 0 \\ -48.99 & a & 50 \\ 0 & -40 & -2 \end{bmatrix}$$

$a_o=1.95$, then $D=-0.04$, i.e. when the armature resistance is not negligable

$\lambda_1'(a_o)=0.5 > 0$, i.e. the eigenvalues cross the imaginary axis at nonzero speed.

At a_o, the two conjugate, pure imaginary eigen- values have the magnitude

$$|\lambda|=\sqrt{\sigma/(r-a_o)} = 45.2227$$

while the third eigenvalue is $\alpha = -0.0479$

Applying the program BIFOR2, the sign of the parameter $V'''(0)$

has been found to be negative i.e. the periodic orbits resulting from the Hopf bifurcation are stable.

Of course, if the gain constant or time constant of the governer response or any of the system operating point conditions(e.g.mech input power,...,etc.) has changed, then the damping bifurcational parameter, i.e. the bifurcation condition and the condition of stability will also change.

3. CATASTROPHE THEORY FOR DYNAMIC STABILITY OF THE SYNCHRONOUS MACHINES.[15,16]

Representing the motion of the synchronous machine by the following equation :

$$M\ddot{\delta}+D\dot{\delta}=P_i-\frac{1}{z^2}\{E_q^2R_a-E_qV_t(R_a\mathrm{Cos}\delta-x_q\mathrm{Sin}\delta)\} + V_t^2\frac{x_q-x_d}{2x_qx_d}\mathrm{Sin}2\delta$$

The equilibrium point of this equation is at $\dot{\delta}=0$ and when the right hand side vanishes.

A similar result is obtained when considering the energy balance of a power trans- mission system [17,18] where the system is stable at minimum potential energy function. Applying this principle, we get :

$$f=\int_o^{\bar{\delta}}\Big[P_i-\frac{1}{z^2}\{E_q^2R_a-E_qV_t(R_a\mathrm{Cos}\delta-x_q\mathrm{Sin}\delta)\} + V_t^2\frac{x_q-x_d}{2x_qx_d}\mathrm{Sin}2\delta\Big]d\delta=0$$

Assuming

$$A=V_t^2\frac{x_q-x_d}{2x_qx_d}, \qquad h=\frac{E_q^2R_a}{z^2}$$

$$\bar{\ell}=\frac{E_qV_t}{z^2}R_a \qquad b=\frac{E_qV_t}{z^2}x_q$$

Thus

$$f=(p_i-h)\bar{\delta}+\bar{\ell}\mathrm{Sin}\bar{\delta}+b\mathrm{Cos}\bar{\delta}-\frac{1}{2}A\,\mathrm{Cos}2\bar{\delta}+(\frac{1}{2}A-b)$$

where, $\bar{\delta}$ is the max angle of oscillations, i.e.

$$f=(P_i-h)\bar{\delta}+\bar{\ell}(\bar{\delta}-\frac{\bar{\delta}^3}{6}+\frac{\bar{\delta}^5}{120}..)+b(1-\frac{\bar{\delta}^2}{2}+\frac{\bar{\delta}^4}{24}-\frac{\bar{\delta}^6}{720}...)$$
$$-\frac{1}{2}A(1-2\bar{\delta}^2+\frac{2\bar{\delta}^4}{3}-\frac{4\bar{\delta}^6}{45}..)+(\frac{1}{2}A-b)$$

the 6-jet of the total energy f is:

$$j^6f=(P_i+\bar{\ell}-h)+\bar{\ell}(-\frac{\bar{\delta}^3}{6}+\frac{\bar{\delta}^5}{120})-b(\frac{\bar{\delta}^2}{2}-\frac{\bar{\delta}^4}{24}+\frac{\bar{\delta}^6}{720})$$
$$+A(\bar{\delta}^2-\frac{\bar{\delta}^4}{3}+\frac{2\bar{\delta}^6}{45})$$

So, this unfolding is transversal, and hence equivalent to the standard butterfly, which can be written in the standard form as follows:

$$j^6 f=(P_i+\bar{\ell}-h)\bar{\delta} + (A-\frac{b}{2})\bar{\delta}^2 + (\frac{-\bar{\ell}}{6})\bar{\delta}^3$$
$$+(\frac{b}{24}-\frac{A}{3})\bar{\delta}^4 + (\frac{2A}{45}-\frac{b}{720})\bar{\delta}^6$$

Therefore,

$$j^6 f=\bar{\delta}^5+c_1+c_2\bar{\delta}+c_3\bar{\delta}^2+c_4\bar{\delta}^3 \qquad (***)$$

where,

$$c_1 = (P_i+\bar{\ell}-h)/6Q, \qquad c_2 = (A-\frac{b}{2})/3Q$$

$$c_3 = \bar{\ell}/\{12(\frac{b}{720}-\frac{2A}{45})\}, c_4 = \frac{2}{3}(\frac{b}{24}-\frac{A}{3}) \times \frac{1}{Q}$$

and,

$$Q = (\frac{2A}{45}-\frac{b}{720})$$

The equation (***) is the standard form of the butterfly manifold [19] shown in fig.(3) and more details are given in appendix(I).

Applying the butterfly manifold to the numerical example given in section 2, we can locate the stable zones for the machine working either as a generator or as a motor (zones I & II , respectively) by using the energy balance equation to determine the stable rotor angles for the coefficients of butterfly manifold equation, c_1 , c_2 ,and fixing c_3 , c_4 as shown in fig.(4). It has been found that the change of E_q does not appreciably change the values of c_3 & c_4 ,therefore we can make the assumption that the coefficients c_3 & c_4 are approximately constants, and consequently the bifurcation section in the control space is fixed and there is no swinging of the butterfly. On the other hand, if the transfer impedance is changed, it affects the values of all coefficients c_1 , c_2 , c_3 , c_4 , hence the butterfly manifold is swinging. It must be recalculated for its new coefficients, because the bifurcation set in the control space will change and take one of the forms which are illustrated in appindex (I), fig.(5). Also, it has been found that the input mechanical power P_i affects the value of the coefficient c_1 only. Then the effect of its change together with E_q is confined in a fixed bifurcation set in the control space, with a fixed butterfly manifold

Of course, if the change of E_q is too great, then it will considerably affect the values of $c_1, c_2, c_3 \& c_4$ both same effect as impedance changes.

4. CONCLUSIONS

It is seen that this paper has demonstrated the application of the Hopf bifurcation theorem to the study of the stability of closed orbits of oscillation produced at the equilibrium point of a synchronous generator, paralleled to a synchronous power system, when subjected to small disturbances.

This powerful mathematical method indicates the dependency of the properties of these orbits upon the system parameters, without resorting to the solution of swing equation by numerical integration. The important significance of the damping coefficient is also illustrated by the application of the theorem. The present limitation of the application to non-linear system is when not more than two eigenvalues cross the imaginary axis at non-zero speed.

Catastrophe theory has been used to visualise the stability problem and a "butterfly" manifold has been shown to be appropriate when the effects of input, synchronous& asynchronous powers and saliency effects are included. The authors have previously demonstrated the applicability of a "cusp" manifold [1] when only the effect of saliency is considered
The increasing complexity of the catastrophe manifold as more parameters are included is thus demonstrated.

When step-function changes of transfer impedance and E_q occur, the butterfly manifold must be recalculated, as both of these cause it to swing and change.

APPENDIX (I)
THE BUTTERFLY CATASTROPHE MANIFOLD

The standard function is

$$F_{c_1,c_2,c_3,c_4} = \frac{1}{6}x^6 + \frac{1}{4}c_4x^4 + \frac{1}{3}c_3x^3 + \frac{1}{2}c_2x^2 + c_1x$$

(x is defined in case of power system stability as δ)
The catastrophe manifold is defined by the first derivative of F & is given by

$$x^5+c_4x^3+c_3x^2+c_2x+c_1 = 0$$

and denoted by M_c. The following treatment must, in the interests of economy of space, assume the basic framework given in ref.[19], chapter 9, page 172.
Using (x,c_4,c_3,c_2) as a chart, mapped to M_c by

$$(x,c_4,c_3,c_2) \longrightarrow (x,c_4,c_3,c_2,c_1)$$

we have:

$$c_1 = -c_2x-c_3x^2-c_4x^3-x^5$$

The Taylor expansion is

$$F_{c_1,\ldots,c_4}(x+y) = \frac{1}{6}y^6 + xy^5 + (\frac{5}{2}x^2 + \frac{1}{4}c_4)y^4$$

$$+(\frac{10}{3}x^3+c_4x+\frac{1}{3}c_3)y^3+(\frac{5}{2}x^4+\frac{3}{2}c_4x^2+c_3x+\frac{1}{2}c_2)y^2+0y$$

$$-(\frac{1}{2}c_2x^2+\frac{2}{3}c_3x^3+\frac{3}{4}c_4x^4+\frac{5}{6}x^6)$$

By using the coefficients of Taylor series, and taking the coordinates as follows:

quadratic : $p(x,c_4,c_3,c_2)=\frac{5}{2}x^4+\frac{3}{2}c_4x^2+c_3x+c_2/2$

cubic : $q(x,c_4,c_3,c_2)=\frac{10}{3}x^3+c_4x+c_3/3$

quartic : $r(x,c_4,c_3,c_2)=\frac{5}{2}x^2+c_4/4$

quintic : $s(x,c_4,c_3,c_2)=x$ &

sextic : $t(x,c_4,c_3,c_2)=1/6$

Thus the hyperplane $t=1/6$ in (p,q,r,s,t)-space ,hence (p,q,r,s)-space can be used as a chart for M_c , and this relates to the (x,c_4,c_3,c_2)- chart as

$$\left.\begin{aligned} x(p,q,r,s) &= s \\ c_4(p,q,r,s) &= 4r-10s^2 \\ c_3(p,q,r,s) &= 3q-12rs+20s^3 \\ c_2(p,q,r,s) &= 2p-6qs+12rs^2-15s^4 \end{aligned}\right\} (****)$$

The quadratic term is degenerate iff $p=0$, which defines the (q,r,s) space. On this space the 3-jet is of type y^3 unless $q=0$ also. This defines the (r,s) plane; on this we have a 4-jet of type y^4 or $-y^4$,unless $r=0$. This defines the s-axis: on this we have a 5-jet of type y^5 ,unless $s=0$. Finally, at the origin we have the original unfolded 6-jet y^6 type of function. We then have a flag of subspaces

$$R^4 \supseteq R^3 \supseteq R^2 \supseteq R^1 \supseteq R^0$$

i.e. the butterfly point at the origin of type y^6 , on a line of swallowtails y^5 . This lies in a plane of cusps, with standard cusps y^4 on one side and dual cusps $-y^4$ on the other side. This plane lives in an R^3 of folds y^3 ; on one side of this we get Morse maxima and on the other side Morse minima.

Using equations (****) we obtain the same structure on the (x,c_4,c_3,c_2)-space, and this transfers to M . In fact M is given parametri- cally in terms of p,q,r,s as the set of points

$$(\underline{s}\ ,\ \underline{4r-10s^2},\ \underline{3q-12rs+20s^3},\ \underline{2p-6qs+12rs^2-15s^4}$$
$$\underline{-2ps+3qs^2-4rs^3-4s^5})$$

So, the singularity set is given by setting $p=0$, which gives

$$(\underline{s} \ , \underline{4r-10s^2}, \underline{3q-12rs+20s^3}, \underline{-6qs+12rs^2-15rs^4}, \underline{3qs^2-4rs^3-4s^5})$$

and the higher degeneracy $q=0$ occurs on

$$(\underline{s} \ , \underline{4r-10s^2} \ , \underline{-12rs+20s^3} \ , \underline{12rs^2-15s^4}, \underline{-4rs^3-4s^5}) \qquad (*x)$$

while the swallowtails line degeneracy $r=0$ occurs on

$$(\underline{s} \ , \underline{-10s^2} \ , \underline{20s^3} \ , \underline{-15s^4} \ , \underline{-4s^5})$$

and the butterfly point goes to the origin. Hence the bifurcation set is parameterized by q,r,s as the set of points

$$(c_4, c_3, c_2, c_1) = (\underline{4r-10s^2}, \underline{3q-12rs+20s^3}, \underline{-6qs+12rs^2-15s^4}, \underline{3qs^2-4rs^3-4s^5})$$

We expect to find a plane of cusps by projecting $(*x)$ into c, i.e. the set of points parameterized by r,s as

$$(c_4, c_3, c_2, c_1) = (\underline{4r-10s^2}, \underline{-12rs+20s^3}, \underline{12rs^2-15s^4} \ , \ \underline{-4rs^3-4s^5})$$

The bifurcation set is drawn as a two dimensional family of two dimensional cross-sections as shown in the fig.(5), where illustrates how sections corresponding to various values of c_4, c_3 are viewed in the (c_2, c_1)-plane. If $c_3=0$ & $c_4>0$, then the section looks like a cusp catastrophe bifurcation set with controls c_2 & c_1 .Varying c_3 has the effect of swinging the whole picture from side to side , the direction depending on the sign of c_3. Varying c_4 into the region c_4 negative introduces a much more complicated structure, with a new pocket. Now as well as swinging from side to side, varying c_3 causes one side or other of the pocket to collapse like a swallowtail and disappear, leaving a single cusp curve again.

Figure (6) expands upon fig.(5) by showing the effects upon the projection in the control space (bifurcation section) for the corresponding butterfly manifolds a&b. The differences between these arise from the swinging of the manifolds, and lead to the changes in the bifurcation sections seen.

REFERENCES

1-A.A.Sallam & J.L.Dineley, "Catastrophe Theory as a Tool for Determining Synchronous Power System Dynamic Stability",IEEE Trans.on Power Apparatus and Systems Vol PAS-102,No.3,March 1983,pp.622-630.

2-D.R.J.Chillingworth, "Differential Topology with a View to Applications" , Research Notes in Math. No.9 ,Pitman Publishing Co., 1976.

3-R.Abraham & J.E.Marsden , "Foundations of Mechanics",2nd. Edition , The Benjamin/Cummings Publishing Co., 1978.

4-A.Kelly"TheStable , Centre, Centre-unstable, Unstable, Manifolds"J. Diff. Eqns.,3,1967

5-N.Kopell,"Bifurcations and Trajectories Joining Critical Points",Advances in Math. , 18, 1976.

6-J.E.Marsden & M.McCraken,"The Hopf Bifurcation and its Applications"Applied Mathematical Sciences,19,Springer-verlag, 1976

7-A.Arapostathis,S.Sastry & P.Varaiya ,"Global Analysis of Swing Dynamics" , IEEE Trans. on Circuits and Systems,Vol CAS-29, No. 10, Oct. 1982.

8-C.Concordia&G.K.Carter,"Negative Damping of Electrical Machinery" AIEE Transaction, Vol.60,March 1941,pp.116 - 119.

9-M.M.Liwschitz,"Positive and Negative Damping in Synchronous Machines",AIEE Transactions,Vol.60,May 1941, pp. 210-213.

10-C.F.Wagner,"Effect of Armature Resistance Upon Hunting of Synchronous Machines"AIEE, Vol.49,July 1930,pp. 1011 -1024.

11-E.Abed&P.Varaiya,"Nonlinear Oscillations in Power Systems", Electronics Research Laboratory,College of Eng.,Univ.of California,Berekely,Memorandum No. UCB/ERL M83/4, 3 Jan. 1983.

12-P.M.Anderson&A.A.Fouad,"Power System Control and Stability",Iowa State Univ.Press , Iowa, 1977,pp. 68.

13-B.D.Hassard,N.D.Kazarinof&Y-H.Wan,"Theory and Applications of Hopf Bifurcation" , London Math.Soc. Lect. Note,No.41,Cambridge Univ. Press, 1981 .

14-E.W.Kimbark,"Power System Stability"Vol.3, Dover Publications, New York 1968,pp.247-314.

15-E.C.Zeeman,"Catastrophe Theory - Selected Papers", Addison-Wesley Publishing Co.Inc., London, 1977.

16-T.Poston&I.Stewart,"Taylor Expansions and Catastrophes" ,Pitman Publishing Co.London, 1976.

17-C.J.Tavora,O.J.M.Smith"Stability Analysis of Power Systems",IEEE Summer Meeting and Int. Symp. on High Power Testing(Portland,OR),July 1971.

18-A.Arapostathis,S.Sastry&P.Varaiya,"Analysis of Power Flow Equation",Int. J. Elec. Power Energy Syst., Vol.3, July 1981, pp. 115-126.

19-T.Poston&I.Stewart,"Catastrophe Theory & its Applications", Pitman Publishing Co., London, 1987.

LIST OF SYMBOLS USED

δ =rotor angle
ω =rotor angular velocity
M =inertia constant
D =velocity damping constant
P_i =input mechanical power
P_m =maximum power limit
δ_o =rotor angle at equilibrium point

ω_o =change of rotor angular velocity, at the equilibrium point=ω ω_s =0, where $\omega=\omega_s$ (synchronous speed)
Re =real part of ...
Im =imaginary part of ...
τ =time constant
k =gain constant
E_q =internal machine voltage behind quad. axis reactance
V_t =terminal voltage , R_a =armature resis.

x_d =synchronous m/c react. in the direct axis
x_q =synchronous m/c react. in the quadrature axis
Z =the transfer impedance (internal imped. +system imped.)
x_t =system reactance (external reactance)

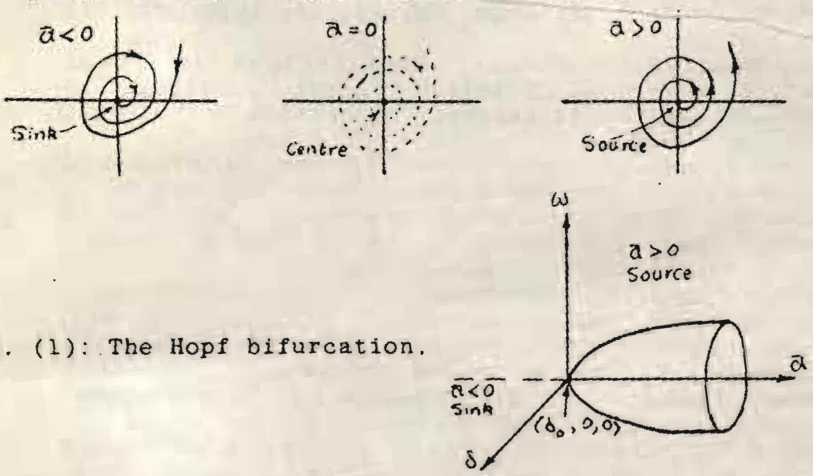


Fig. (1): The Hopf bifurcation.

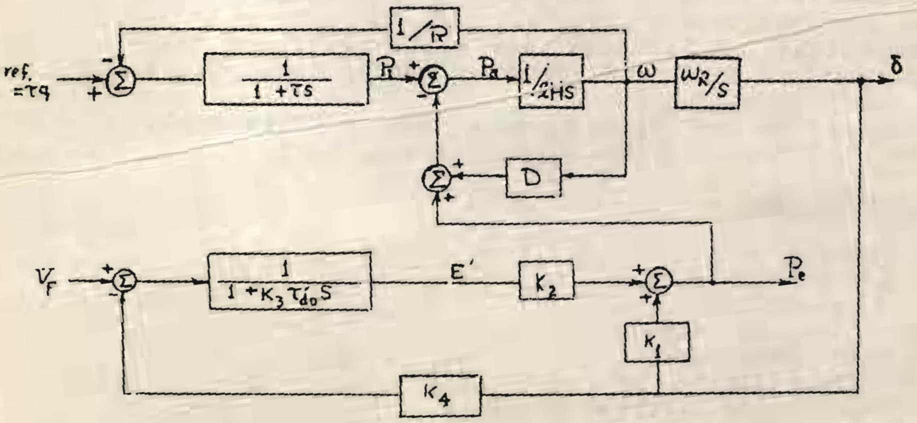


Fig. (2): Block diagram of the system including speed governor regulation effects.

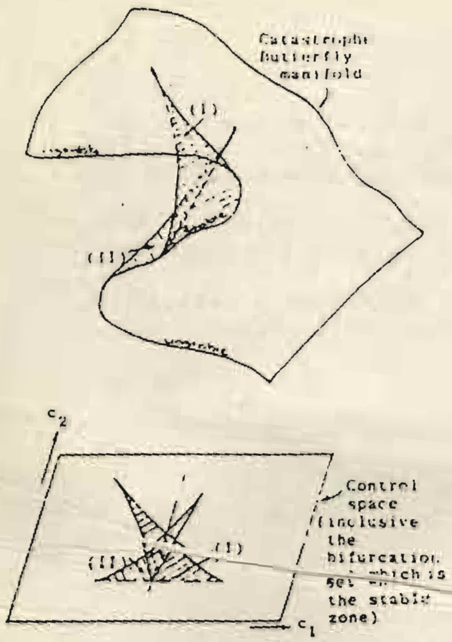


Fig (3): Butterfly manifold with location of stable and unstable zones.

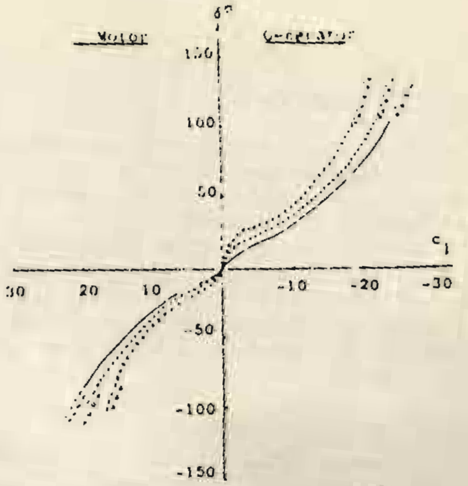


Fig. (4). Rotor angle δ vs. c_1 at different values of c_2, the dotted lines denoted by U are unstable points.

(—— c_2 = 36.6 , c_3 = 1.04 , c_4 = -8.8

... c_2 = 34.1 , c_3 = 0.95 , c_4 = -8.5

xxx c_2 = 30.5 , c_3 = 0.82 , c_4 = -8.1)

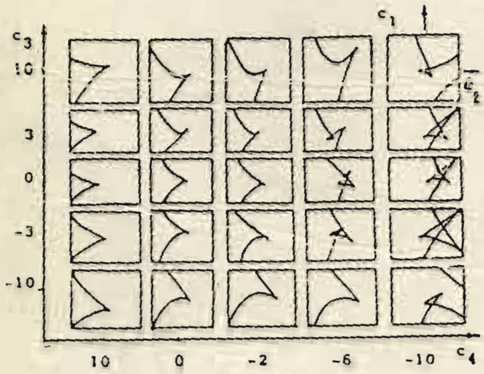


Fig. (5): The different shapes of bifurcation sections.

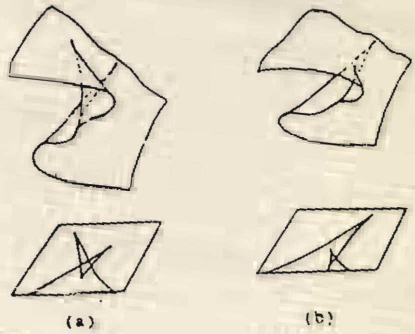


Fig. (6): Two butterfly manifolds with two different bifurcation sections.

Proposed Future Work

This is the theoritical basis of a more comprehensive power systems laboratory study. The next step will be presentation, a dynamic form, of the catastrophe manifold on a high-resolution graphic monitor.

The position on that manifold corresponding to the actual rotor angle will demonstrate visually the position relative to the stability boundary.

Finally, it is hoped to monitor a synchronous generator and identify the manifold under on-line conditions.