Menofia University **Faculty of Engineering Shebien El-kom** **Basic Engineering Sci. Department.** **First semester Examination, 2017-2018** **Date of Exam : 4 / 6 / 2018**	Minoufia University	**Subject: Partial Diff. Eqs.(1)** **Code: BES 615** **Year : Master** **Time Allowed : 3 hrs** **Total Marks: 100 Marks**

Answer the following questions ***Two Pages***

Question 1 **(40 MARKS)**

A) Show that a family of spheres $x^2 + y^2 + (z-c)^2 = r^2$ satisfies the first-order linear partial differential equation $y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$.

B) Obtain the general solution of the linear Euler equation $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$

C) Determine the integral surfaces of the equation

$$x(y^2+u)u_x - y(x^2+u)u_y = (x^2-y^2)u$$

With the data $x + y = 0$, $u = 1$.

D) Find the solution of the Burgers initial-value problem with physical significance for the discontinuous data $F(x) = A\delta(x)H(x)$,

Where A is a constant, $\delta(x)$ is the Dirac delta function, end $H(x)$ is the Heaviside unit step function.

Question 2 **(20 MARKS)**

Solve the following equation of motion of a vortex filament using the nonlinear Schrodinger Equation methods.

The motion of a very thin isolated vortex filament $\boldsymbol{X} = \boldsymbol{X}(x, t)$ of radius ϵ in an incompressible unbounded fluid by its own induction is described asymptotically by Hasimoto (1972) in the form

$$\frac{\partial \boldsymbol{X}}{\partial t} = G\,\kappa\,\boldsymbol{b}$$

where s is the length measured along the filament, t is the time, κ is the curvature, $\boldsymbol{b}$ is the unit vector along the binormal, and G is the coefficient of local induction

$$G = \left(\frac{\Gamma}{4\pi}\right)\left[\log \epsilon^{-1} + O(1)\right]$$

which is proportional to the circulation Γ of the filament and may be treated as constant if the slow variation of logarithm compared with that of its argument.

Question 3 (40 MARKS)

A) Solve the following equation motion of an electron fluid.

The basic equations of motion for the one-dimensional adiabatic motion of an electron fluid are:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{mn}\frac{\partial}{\partial x}(nT) - \frac{e}{m}\frac{\partial \phi}{\partial x} = 0$$

$$\frac{d}{dt}(n^{-2}\,T) = 0$$

$$-\frac{\partial^2 \phi}{\partial x^2} + 4\,\pi\, e(n - n_0) = 0$$

where n_0 and T_0 are the density and the flow velocity, respectively, ϕ is the electrostatic potential, and T is the electron temperature.

B) Solve the ion-acoustic wave's problem using the KdV Equation. The high temperature plasma is a fully ionized gas consisting of electrons and ions that are governed by the equations of continuity and momentum combined with the classical Maxwell equations. Using the subscripts e and i for quantities related to electrons and ions and neglecting dissipation due to collisions, we write the following equations of motion for plasma.

The equation of continuity is:

$$\frac{\partial n_j}{\partial t} + \nabla\cdot(n_j u_j) = 0$$

when $(j = e, i)$ is the density and u_j, is the flow velocity.

The equation of motion is:

$$m_j n_j[\frac{\partial u_j}{\partial t} + (u_j\,\nabla)u_j] = -\nabla p_j + n_j\, q_j\,[E + \frac{1}{c}(u_j \times B)].$$

The Maxwell equations are given by:

$$\nabla\cdot E = 4\,\pi\,(q_i\, n_i + q_e\, n_e)$$

$$\nabla\cdot B = 0$$

$$\frac{\partial B}{\partial t} + c\,(\nabla\times E) = 0$$

$$-\frac{\partial E}{\partial t} + c\,(\nabla\times B) = 4\pi(q_i n_i u_i + q_e\, n_e\, u_e)$$

The equation of state is given by: $P_j = n_j\, T_j$

Where E is the electric field, B is the magnetic field, T is the product of the Bolamann constant and the temperature. q and m are charge and mass, respectively, and c is the speed of light.

This exam measures the following ILOs												
Question Number	Q1-a	Q2-a			Q2-b	Q3-b			Q1-b	Q3-a	
Skills		b-i			b-i, b-iii						
	Knowledge &understanding skills				Intellectual Skills				Professional Skills		

With my best wishes

Associate Prof. Dr. Islam M. Eldesoky