

# Proposed High-Performance Digital Distance Relay

## تصميم مقترح لمتمم مسافى رقمى على الأداء

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### ملخص

يقدم البحث تصميمًا جديدًا لمتمم مسافى رقمى على الأداء مزود بطريقة للحساب اللحظى لقيمة مركبة التيار المستمر المتناقص والتخلص منها خلال دورة واحدة و بذلك يمكن الحصول على تقدير دقيق للمركبات الأحادية اللازمة لعمل المتمم بتطبيق تحويل فورير السريع، كما يحتوى التصميم المقترح على طريقة جديدة لتصحيح الخطأ فى قيمة المعاوقة المقدره بين المتمم و مكان العطل الذى يحدث نتيجة قيمة مقاومة العطل الأرضى، تعتمد هذه الطريقة على افتراض متمم مسافى تخيلى عند النهاية الأخرى للخط حيث يتم التوصل الى صيغ رياضية جديدة تمكن من تقدير مقاومة العطل الأرضى و الخطأ الناتج عنها فى قيمة المعاوقة المقدره بين المتمم و مكان العطل بدقة كبيرة فيتم التحديد الصحيح لمكان العطل مما يساعد المتمم على اتخاذ القرار المناسب حتى فى حالات الأعطال ذات المقاومة العالية أو التى تحدث على الحدود المسافية بين مناطق الحماية الثلاث، و قد تم بناء نموذج ديناميكى كامل للمتمم المقترح مقترنا مع منظومة نقل قوى و دراسة الأداء عند ظروف عطل متنوعة حيث برهنت كفاءة المتمم المقترح، كما تم مقارنته بتصميمات أخرى للمتمم.

### Abstract

This paper proposes a new design for high-performance digital distance relay. It is equipped with a dedicated decaying dc component estimation algorithm and a new method for compensation of ground fault resistance. The proposed relay algorithm can estimate and eliminate the dc decaying component from fault current signals within one cycle from the fault instant. Then, it uses fast Fourier transform to extract accurate phasor quantities of fault currents. Also, an innovative algorithm for compensation of fault resistance is introduced. The later technique is based on supposing a virtual distance relay at the other end of the line. Using the preliminary estimation of the apparent relay-to-fault impedance at both sides, the unknown fault resistance and its related error are determined. Hence, this error can be compensated in computing the actual relay-to-fault impedance. Many test cases are performed under different fault locations and several values of ground fault resistance to confirm the efficacy of the proposed relay.

### 1. Introduction

When a fault occurs on a power system not only the fundamental component and harmonics are included, but also a decaying dc component is found. Protection system uses principally the fundamental component for fault discrimination. There should be a mean to extract only the fundamental component from current waveform [1]- [5]. The most preferable method to extract the fundamental component at a relatively fast response is the fast Fourier transform (FFT).

Although it is immune to harmonics, it is not to decaying component. This decaying component causes undesirable oscillations which may lead to improper operation of the protection system especially for distance protection. So, practical digital relaying scheme requires additional techniques to reduce effects of decaying dc component [6]- [9].

A mathematical technique for estimating the decaying dc component in current input signals of digital relays is presented in [2]. A

method is introduced in [5] for ground fault resistance compensation in distant relays. It did not make the compensation on basis of assumed value of fault resistance. It first estimated the apparent impedance then made the compensation using the deviation vector and deviation angle. A more developed algorithm for fault resistance compensation is given in [6]-[8]. It uses the current signals from both sending end and receiving end. It is based on phase coordinates so it works well for unbalanced systems as well as balanced ones. Phase coordinates are applied to consider the effect of the three phases reflected by the mutual impedance between the three phases. The expression for the error in impedance due to fault resistance is derived. Then, treatment is continued to get an iterative expression to simultaneously estimate fault resistance and compensate for the measured relay-to-fault impedance.

This paper proposes a high-performance digital distance relay with a dedicated decaying dc component estimation algorithm and a new method for compensation of ground fault resistance. The proposed relay algorithm can estimate and eliminate the dc decaying component from fault current signals within one cycle from the fault instant. Then, it uses FFT to extract accurate phasor quantities of fault currents. The dc decaying magnitude and time constant are estimated accurately by integrating fault currents during one cycle. The dc decaying component is eliminated from current signals by subtracting the dc value at each sampling instant. Also, an innovative algorithm for compensation of fault resistance is introduced. The later technique is based on supposing a virtual distance relay at the other end of the line. Using the preliminary estimation of the apparent relay-to-fault impedance at both sides, the unknown fault resistance and its related error are determined. Hence, this error can be compensated for in computing the actual relay-to-fault impedance. A dynamic model of the proposed relay and its associated test power

system is constructed in Matlab/Simulink. Many test cases are performed under different fault locations and several values of ground fault resistance.

## II. Decaying dc component elimination

### A. Proposed dc component estimation algorithm

One can express the fault current as a sum of sinusoids and a decaying component as a decaying exponential function:

$$i(t) = I_0 e^{-t/\tau} + \sum_{n=1}^P (I_n \sin n\omega_1 t + \theta_n) \quad (1)$$

where

$I_0$  is the magnitude of the decaying dc offset,  $\tau$  is the time constant of the decaying dc offset,  $n$  is the harmonic order,  $I_n$  is the magnitude of the  $n$ th harmonic component,  $\theta_n$  is the phase angle of the  $n$ th harmonic component, and  $P$  is the maximum harmonic order.

Integrating (1) over one period  $T$ , the integral of the second term vanishes and only the first term which contains the exponential component remains as:

$$\begin{aligned} \int_0^T i(t) dt &= \int_0^T [I_0 e^{-t/\tau} + \sum_{n=1}^P (I_n \sin n\omega_1 t + \theta_n)] dt = \int_0^T I_0 e^{-t/\tau} dt \\ &= [-I_0 \tau e^{-t/\tau}]_0^T = I_0 \tau [1 - e^{-T/\tau}] = Z(0) \end{aligned} \quad (2)$$

The function  $Z(0)$  represents the integral of the fault current over one cycle starting from zero to  $T$ . If one started from a small time step after zero  $\delta t$ , then the function value is  $Z(\delta t)$  which represents the integral of fault current after a small time step and it is expressed as:

$$Z(\delta t) = \int_{\delta t}^{\delta t+T} I_0 e^{-t/\tau} dt = [-I_0 \tau e^{-t/\tau}]_{\delta t}^{\delta t+T} = Z(0) e^{-\delta t/\tau} \quad (3)$$

If the integrals of the fault current in (3) were known, the time constant ( $\tau$ ) and the magnitude ( $I_0$ ) of the decaying dc component can be estimated as:

$$\tau = \delta t / \ln [Z(0) / Z(\delta t)] \quad (4)$$

$$I_0 = Z(0) / \tau [1 - e^{-T/\tau}] \quad (5)$$

The required values to estimate ( $\tau$ ) and ( $I_0$ ) are  $Z(0)$  and  $Z(\delta t)$ .

**B. Implementation of the proposed algorithm**

In a practical digital relaying scheme, all of the calculations are performed in a discrete time base using sampled data and should be completed in each sampling interval. In (3), decaying dc component can be mathematically calculated. However, since the calculation of the natural logarithm should be performed for every sample, it can be a computational burden from the practical perspective. In order to reduce the computational load, the time constant can be obtained by using a Taylor series expansion in (3):

$$e^{-\delta t/\tau} = 1 + (-\delta t/\tau) + (-\delta t/\tau)^2/2 + \dots = Z(\delta t)/Z(0) \tag{6}$$

In (6), the time step  $\delta t$  is very small so that higher order terms of the expansion can be ignored without potentially affecting the accuracy. One can use only the first two terms to calculate the time constant  $\tau$  simplifying the calculations to be:

$$\tau = \delta t / [1 - Z(\delta t) / Z(0)] \tag{7}$$

Using (3), the value of dc component can be written as:

$$I_0 e^{-(t+\delta t)/\tau} = I_0 e^{-t/\tau} e^{-\delta t/\tau} = I_0 e^{-t/\tau} [Z(\delta t) / Z(0)] \tag{8}$$

One can extract the dc-free current signal by subtracting the calculated dc value in (8) from the corresponding current sample. After one cycle needed to correctly calculate (8), the error in the dc-free current signal reduces to zero and the decaying dc component is eliminated.

**III. Apparent relay-to-fault impedance calculation**

**A. Conventional method**

Traditionally, effect of fault resistance is not considered on calculating impedance seen by the distance relay. Traditional formulae adopted for calculating the apparent impedance from relay to fault point for each fault type are

given in [3], [4]. Table 1 summarizes these formulae.

Table 1 Fault impedance calculation formulae for difference fault types

Fault type	Formula
A-G (phase A to Ground)	$E_a / (I_a + K_0 I_0)$
B-G (phase B to Ground)	$E_b / (I_b + K_0 I_0)$
C-G (phase C to Ground)	$E_c / (I_c + K_0 I_0)$
A-B (phase A to phase B) or A-B-G	$(E_a - E_b) / (I_a - I_b)$
B-C (phase B to phase C) or B-C-G	$(E_b - E_c) / (I_b - I_c)$
C-A (phase C to phase A) or C-A-G	$(E_c - E_a) / (I_c - I_a)$

where

$$K_0 = (Z_0 - Z_1) / Z_1 ;$$

$Z_0, Z_1$  are the zero and positive sequence line impedance, respectively.

$I_0$  is the zero sequence-current.

On estimating the impedance seen by the relay using formulae in Table1, a considerable error exists due to ignoring the effect of fault resistance. This can cause overreach problem and relay maloperation. This section proposes an algorithm to compensate for the effect of ground fault resistance.

**B. Proposed fault resistance compensation technique**

The idea behind the algorithm is to make use of all available measurements at sending end as well as receiving end, i.e. the voltages and currents of the three phases at both ends. Taking into account the error due to ground fault resistance, the fault resistance and the actual impedance from the sending end (relay point) to the fault location can be estimated. The required inputs are the voltages of the sending end  $V_{S_{a,b,c}}$ , the receiving-end voltages  $V_{R_{a,b,c}}$ , the sending-end currents  $I_{S_{a,b,c}}$ , and the receiving-end currents  $I_{R_{a,b,c}}$ .

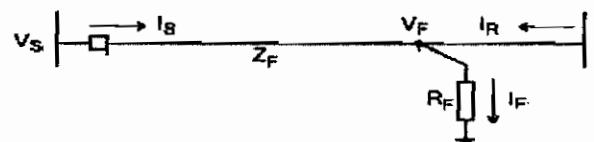


Fig.1 Faulted transmission line

Referring to Fig.1, the error in the impedance seen from sending end due to fault resistance  $Z_{SErr}$  is given in [6] by (9):

$$Z_{SErr} = R_F [ I_R / I_S + 1 ] \quad (9)$$

Similarly, the error in the impedance seen from receiving end due to fault resistance  $Z_{RErr}$  is given by (10):

$$Z_{RErr} = R_F [ I_S / I_R + 1 ] \quad (10)$$

Summing the actual value of apparent impedance seen from the sending end and the value of error given by (9), one gets the traditional value of sending-end apparent impedance  $Z_{MS}$  estimated by Table 1 based on  $V_{Sa,b,c}$ ,  $I_{Sa,b,c}$ :

$$Z_{MS} = Z_S + Z_{SErr} \quad (11)$$

That is,

$$Z_{MS} = Z_S + R_F [ I_R / I_S + 1 ] \quad (12)$$

Summing the actual value of apparent impedance seen from the receiving end and the value of error given by (10), one gets the traditional value of receiving end apparent impedance  $Z_{MR}$  estimated by Table 1 based on  $V_{Ra,b,c}$ ,  $I_{Ra,b,c}$ :

$$Z_{MR} = Z_R + Z_{RErr} \quad (13)$$

That is,

$$Z_{MR} = Z_R + R_F [ I_S / I_R + 1 ] \quad (14)$$

The sum of the actual impedance value from sending end to fault point ( $Z_S$ ) and the actual impedance value from receiving end to fault point ( $Z_R$ ) equals the total line impedance from the sending end to receiving end as given by (15):

$$Z_S + Z_R = Z_l \quad (15)$$

Adding (12) and (14) and substituting from (15) gives:

$$Z_{MS} + Z_{MR} = Z_l + R_F [ 2 + I_R / I_S + I_S / I_R ] \quad (16)$$

from (16), the value of fault resistance  $R_F$  is given by (17):

$$R_F = [ Z_{MS} + Z_{MR} - Z_l ] / [ 2 + I_R / I_S + I_S / I_R ] \quad (17)$$

One can use the value of  $R_F$  given in (17) to calculate the actual value of impedance from sending-end relay point to the fault location from (12) as:

$$Z_S = Z_{MS} - R_F [ I_R / I_S + 1 ] \quad (18)$$

Fig.2 shows the schematic diagram of fault resistance compensation procedure.

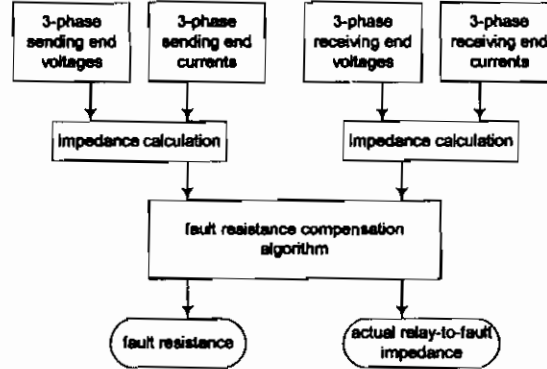


Fig.2 Block diagram of fault resistance compensation procedure

#### IV. Construction of proposed relay

The overall functional block diagram of the proposed distance relay is depicted in Fig. 3. A full dynamic model of the relay and the associated model power system given in Fig.4 is constructed in Matlab/Simulink. The Simulink model of the relay is illustrated in Fig.5. The test system data is given in Table 2.

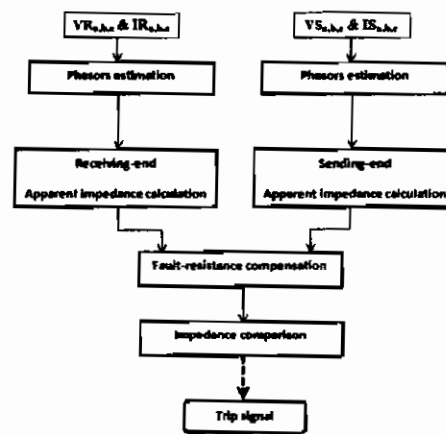


Fig.3 Overall functional block diagram of the proposed distance relay

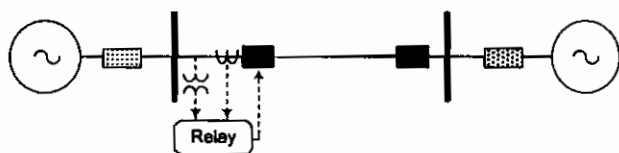


Fig.4 One-line diagram of test system

Table 2 Test system data

source	230KV, 50Hz
Source impedance	$0.89+j 5.2 \Omega$
Source connection	Yg
line length	100km
Line Inductance L1,L0	0.933mH/km, 4.12mH/km
Line capacitance C1,C0	$12.74e-9, 7.75e-9F/km$
Line resistance R1,R0	$0.013 \Omega/km, 0.38 \Omega/km$

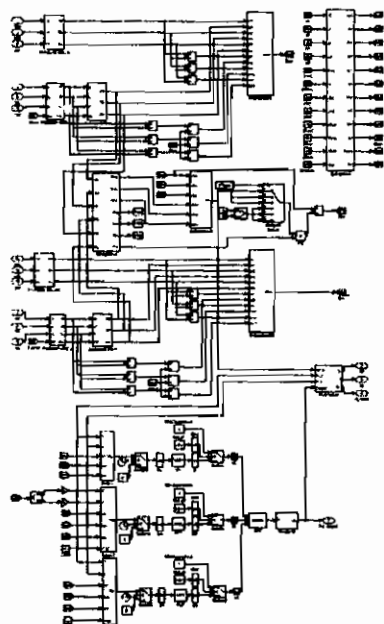


Fig.5 Simulink model of the proposed relay

## V. Relay Performance

### A. Elimination of dc component

A single phase to ground b-g fault is initiated at 0.04 s at the transmission line midpoint. Fig.6 shows the phase b current for this fault before and after the application of the dc eliminating algorithm. The fault current has an obvious decaying dc component. The application of the algorithm could adequately eliminate the dc component in about a half

cycle. The dc-free current waveform is ready to accurately extract the fundamental component using FFT.

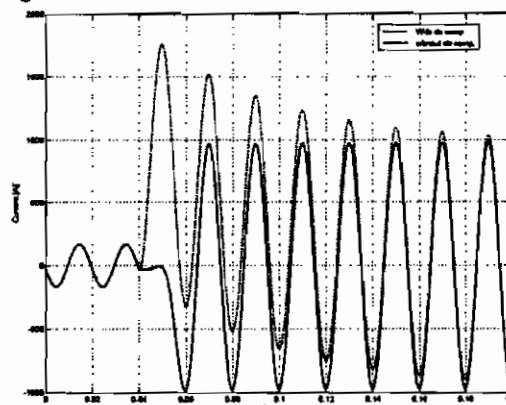


Fig.6 Fault current with and without dc component

### B. Effect of dc elimination on fundamental component extraction

As the dc component is quickly eliminated in less than one cycle, this helps the FFT to extract the fundamental component much faster. Fig.7 demonstrates the application of FFT for extracting the fundamental amplitude of the current waveforms of Fig.6. It is noted that when dc component is eliminated the FFT can extract the fundamental component accurately in one cycle time. This is much faster and more steady compared to when the dc component exists. For the later case, FFT takes more than 10 cycles to extract the fundamental amplitude correctly.

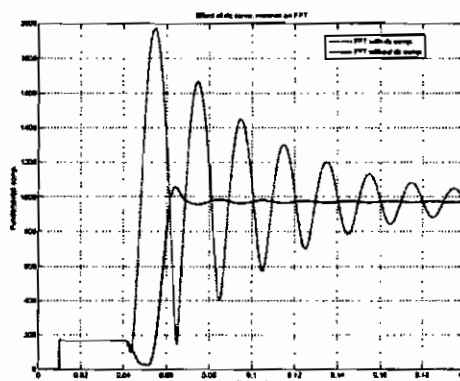


Fig.7 Effect of dc component elimination on fundamental extraction

C. Relay response to faults

Fig.8 shows the relay performance for single phase to ground fault at 40 km with fault resistance ( $R_f$ ) of 100  $\Omega$ . The figure reveals the development of trajectory of the estimated relay-to-fault impedance relative to the setting circles of the relay. The trajectory starts away from the circles and moves towards them gradually. This is due to the progress of fundamental extraction with time. Eventually, the trajectory settles at a point inside the circle of zone 1 (zone 1 covers 85% of line length). A close-up around the final operating point of the relay is also illustrated in Fig.8. The relay correctly locates the fault despite the fairly high fault resistance. This confirms the potential of its enforced algorithm for fast dc removal and fault resistance compensation.

The fault is on border between the first and second zones which is a possible situation for mal-operation. Also, the fault resistance is tangibly high. Nevertheless, the relay could correctly locate the fault in the first protection zone.

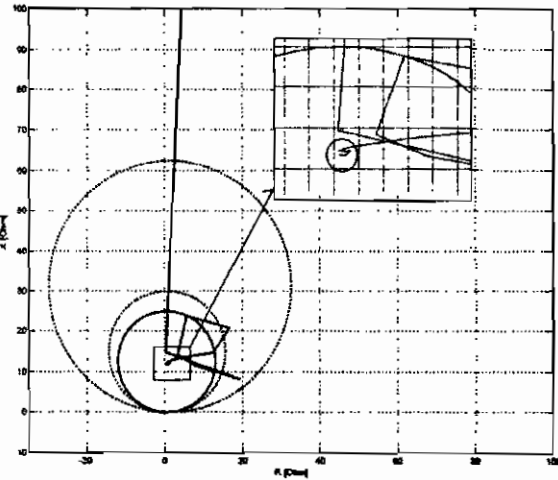


Fig.8 Measured impedance trajectory for an A-G fault at 40 km with  $R_f = 100 \Omega$

Fig.9 is similar to Fig.8 but for single phase to ground fault at 60 km with fault resistance of 500  $\Omega$ . Despite the high value of fault resistance, the relay could correctly locate the fault in the first protection zone.

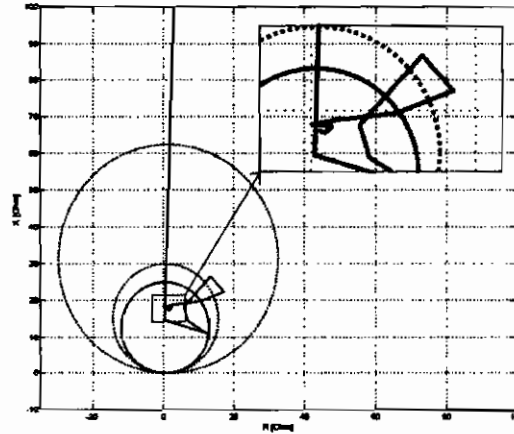


Fig.9 Measured impedance trajectory for an A-G fault at 60 km with  $R_f = 500 \Omega$

For second zone A-G fault at 88 km with 100  $\Omega$  fault resistance, the relay could operate properly as illustrated in Fig.11. The relay responds correctly for A-G fault at 88 km with 500  $\Omega$  fault resistance as shown in Fig.12.

Fig.10 is similar to Fig.8 but for single phase to ground fault at 80 km with high fault resistance of 900  $\Omega$ .

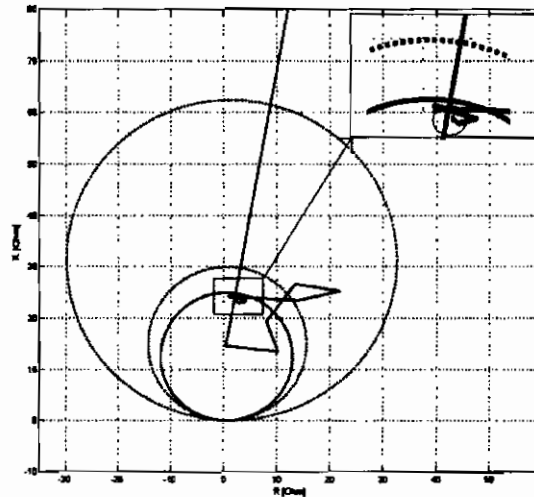


Fig.10 Measured impedance trajectory for an A-G fault at 80 km with  $R_f = 900 \Omega$

Table 3 Overview of the simulations results

Type	Distance [%]	Relay-To-Fault Impedance [ $\Omega$ ]			Fault Resistance [ $\Omega$ ]		
		Actual	Estimated	Error [%]	Actual	Estimated	Error [%]
Internal	50	0.6365 +j14.6665	0.6374+j14.6711	0.03	40	39.87	0.32
Eternal	88	1.1202 +j25.8131	1.1938 +j25.8340	0.09	40	39.98	0.05
Internal	60	0.7638 +j17.5998	0.8310 +j17.6408	0.25	100	99.97	0.03
Eternal	90	1.1457 +j26.3997	1.3536 +j26.4264	0.13	100	100.16	0.16
Internal	40	0.5092 +j11.7332	0.3637 +j11.7206	0.15	300	301.65	0.55
Eternal	88	1.1202 +j25.8131	1.7289 +j25.8345	0.21	300	301.86	0.62
Internal	50	0.6365 +j14.6665	0.6498 +j14.657	0.06	500	504.12	0.82
Eternal	90	1.1457 +j26.3997	2.2211 +j26.3857	0.21	500	504.96	0.98
Internal	70	0.8911 +j20.5331	1.7077 +j20.4944	0.06	750	758.52	1.12
Eternal	90	1.1457 +j26.3997	2.7585 +j26.2969	0.06	750	752.22	0.29
Internal	60	0.7638 +j17.5998	1.2640 +j17.5552	0.09	900	910.45	1.16
Eternal	88	1.1202 +j25.8131	2.9537 +j25.6417	0.10	900	911.25	1.25
Internal	50	0.6365 +j14.6665	0.6572 +j14.6249	0.28	1200	1212.40	1.03
Eternal	90	1.1457 +j26.3997	3.6917 +j26.012	0.57	1200	1213.36	1.113

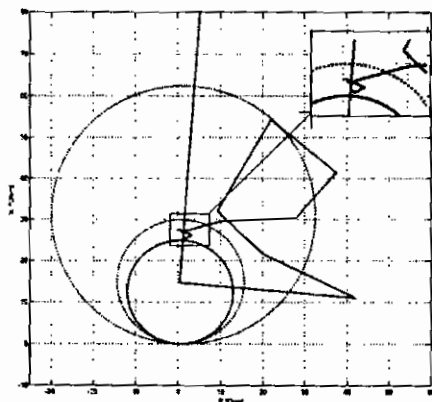


Fig.11 Measured impedance trajectory for an A-G fault at 88 km with  $R_f = 100 \Omega$

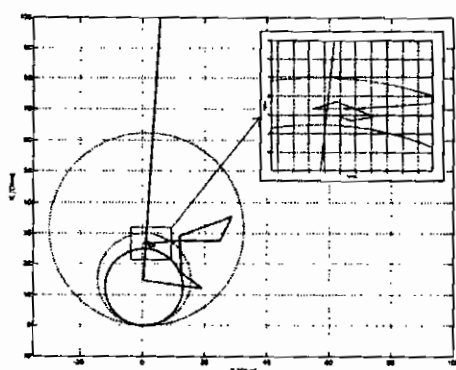


Fig.12 Measured impedance trajectory for an A-G fault at 88 km with  $R_f = 500 \Omega$

#### D. Comparison to other relaying algorithms

Table 3 shows an overview of the simulations results. Column one shows the fault type with respect to the first protection zone. Column 2 includes the percentage fault distance from the relay location. Columns 3, 4 and 5 show the actual relay-to-fault impedance, the estimated relay-to-fault impedance, and percentage error in estimation, respectively. Columns 6, 7 and 8 show the actual fault resistance, the estimated fault resistance, and percentage error in estimation, respectively. It is remarked that the proposed relay algorithm is accurate enough in deciding fault location for a wide range of fault resistance from 0 to 1200  $\Omega$  with a minor error of no more than 0.6%. Moreover, the fault resistance is estimated accurately with tolerable small error less than 1.25%. The estimation accuracy of relay-to-fault impedance and fault resistance is close to the results given in [8]. Though the method described in [8] is iterative and time consuming, it works properly for fault resistance below 750  $\Omega$ . The fault resistance can be correctly estimated up to only 20  $\Omega$  using the method presented in [5]. On the other hand, the proposed algorithm performs efficiently for fault resistance above 1200  $\Omega$ .

## VI. Conclusion

A high-performance digital distance relay with a dedicated decaying dc component estimation algorithm and a new method for compensation of ground fault resistance is proposed. The relay algorithm can estimate and eliminate the dc decaying component from fault current signals within one cycle from the fault instant. Then, it uses FFT to extract accurate phasor quantities of fault currents. The dc decaying magnitude and time constant are estimated accurately by integrating fault currents during one cycle. Also, an innovative algorithm for compensation of fault resistance is introduced. The later technique is based on supposing a virtual distance relay at the other end of the line. Using the preliminary estimation of the apparent relay-to-fault impedance at both sides, the unknown fault resistance and its related error are determined. Hence, this error can be compensated for in computing the actual relay-to-fault impedance. A dynamic model of the proposed relay and its associated test power system is constructed in Matlab/Simulink. The various test results presented assure that the proposed distance relay works properly for a wide range of fault resistance up to  $1200\Omega$ .

## VII. References

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