

بسم الله الرحمن الرحيم

Forced Convective Laminar Flow In Elliptic Pipes Of Different Aspect Ratios السريان الركامي ذو الحمل الجبري في الأنايب بيضاوية المقطع

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خلاصة في هذا البحث تم دراسة السريان الركامي ذي الحمل الجبري في منطقة الدخول المبرودة لأنايب بيضاوية المقطع في حالة تيارات التيفض المبراري. في هذا العمل تم تحليل هذا السريان نظريا ومعليا. في الدراسة النظرية تم عمل نموذج رياضي واحصاف للسريان وذلك بإعادة تعريف كل من المتغيرات المستقلة والتابعة في معادلات الحركة والطاقة. نتيجة لهذا التعريف المحدد تم حادان النسبة البعدية للمقطع كعامل مؤثر على السريان وعلى ذلك يكون الحدال الناتج من هذا النموذج صليح للتطبيق بلسمع الأنايب بصرف النظر عن النسبة البعدية للمقطع. تم - معليا - تصميم وتعبيد تجربة معمولة: نتحقق حالة تيارات التيفض المبراري وذلك عن طريق تعيين حدان الأتيرة كبريما. تم عقد مقارنة بين النتائج النظرية والمعلمة وكذلك تم مقارنة النتائج الحلفية بنتائج الدراسات السابقة. تم اختيار أنايب ذات نسبة بعبدية ٢٥.٤ إلى ١٢.٤ كملر ذات / من مبرع.

Abstract In present work, the laminar forced convective flow in elliptic pipes is studied. The flow in the thermal entrance region is considered assuming that it is hydrodynamically, fully developed. In this work the flow in pipes is, theoretically and experimentally, analyzed. Constant heat flux at the walls of the pipe is considered. In theoretical study, a mathematical model describing the flow is suggested. According to this model the dependent and independent variables of momentum and energy equations are redefined, such that the aspect ratio as a parameter of the problem is eliminated. Consequently, the solution of this model is no longer dependent on the aspect ratio and hence is valid for all elliptic pipes. In experimental work, a test rig is designed and constructed. The walls of the tested pipes are electrically heated to satisfy the constant heat flux boundary condition. A comparison between the experimental and theoretical results is carried out and also a comparison between these results and those of previous studies is made. Pipes of aspect ratios of 0.404, 0.505, 0.636 and 1.00 are examined. The considered range of Reynolds number is 1600- 2200. The heat flux range is taken as 12.4 - 25.4 kW/m²

1. Introduction

The study of heat transfer process inside tubes is of great importance from industrial application point of view. The heat transfer from horizontal tubes is studied, theoretically and experimentally, by many researchers. According to the available previous studies, the deep understanding of hydrodynamic and thermal developing forced convective flow inside tubes requires more investigations.

Convective heat transfer in non-circular tube for fully developed flow was studied by many investigators [1-5]. Henry Barrow et al. [1] studied the flow and heat transfer in ducts of elliptic cross-section. Water ($Pr = 6.5$) was tested in tubes of aspect ratio of 0.316 and 0.415. The range of Reynolds was 1000 to 3000. J. Malak et al. [2] studied the effect of channel geometry on friction pressure losses and heat transfer process. M. A. Ebadian et al. [3] studied the convective heat transfer in tubes of elliptic cross-section of constant wall-temperature boundary condition. In theoretical analysis, the successive approximations

method was employed using elliptic coordinates. Jalil Ouazzan [4] studied the fully developed mixed convective flow between horizontal plates. He used the available data for laminar forced convective flow in pipes to explore the effect of length and shape of the entrance section. M. Kaviany [5] studied laminar combined convection in a horizontal annulus subjected to constant heat flux inner wall and adiabatic outer wall. finite difference approximations were applied to analyze the problem numerically. Heat transfer in tubes with deformed inner surface was examined by many investigators [6-8]. R. M. Manglik et al. [6] and S. W. Hong et al. [7] studied laminar flow heat transfer in semi-circular tube with uniform wall temperature. The result define the lower bound of heat transfer augmentation in circular tubes with twisted-strip inserts. C. Prakash et al. [8] studied laminar flow and heat transfer in entrance region of an internally finned circular duct.

Convective heat transfer in entrance region were studied by many investigator [9-13]. D. Butterworth et al. [9] studied forced convective laminar flow heat transfer in the entrance region theoretically and experimentally. In this work, heating was started at different stages of flow developing. R. F. Babus 'Hag [10] studied ,experimentally, steady state local heat transfer coefficient for laminar flow of air inside an electrically heated pipe. Q. M. Lei et al. [11] studied, numerically, forced convection of thermally developing laminar flow in circular sector ducts. Hlan Quarmby et al. [12] the effect of finite length on forced convection heat transfer from cylinders. L. S. Yeo [13] studied the laminar flow convective heat transfer in a channel and pipe. An analytical solution of flow in the entry region of a heated vertical channel is presented.

In the present work, the heat transfer by forced convection for the case of the flow inside elliptic tubes is studied experimentally and theoretically. The case of thermally developing flow is considered. The boundary condition of uniform heat flux at the pipe wall is examined.

2. Experimental Work

A layout of the test loop, used in this study, is shown in figure(1). Water leaves the collecting tank and flows through the test section, where it is heated electrically by heating coil. Then water enters the shell and tube heat exchanger to decrease its temperature to the original value (before entering the test tube). The water is circulated through out the test loop by the circulating pump.

Temperature at inlet- and outlet-cross sections of the test section is measured by glass thermometer while the values of wall-temperature along the tested pipe are measured by copper-constantan thermocouple. Water flow rate is measured using calibrated orifice plate. The pressure drop across the test section is measured by U-tube manometer. Four tubes of aspect ratio of 0.404, 0.505, 0.636 and 1.00 are tested. The dimensions of the unreformed tube are 0.9 m long, 19.8 mm outer diameter and 0.4 mm thickness. Experiments are carried out for water flow rate range of 4-10 kg/hr. The applied power to the electrical heating coil is calculated according to the relation;

$$w' = V^2 / R_k$$

where V , w' and R_k are the applied voltage, produced power and resistance of the heating coil, respectively. According to the foregoing equation, one can calculate the net heat transferred to water through the pipe wall as;

$$Q_{net} = \dot{m} - Q_{loss} \quad ,$$

where Q_{net} and Q_{loss} are the net heat transfer to water and heat loss from insulation layer, respectively. Consequently, the heat flux can be obtained as;

$$q = Q_{net} / A_s \quad ,$$

where A_s is the total surface area of the tube. To calculate the bulk temperature of water along the tube, one can divide the tube into a set of elements and carries out a heat balance of each one. The bulk temperature at certain position can be determined using the following relation;

$$T_{i+1} = \frac{Q_{net} z / l}{m \cdot c_p} + T_i \quad ;$$

where T_{i+1} , T_i , m , c_p , z and l are the outlet temperature from the element, inlet temperature to the element, mass flow rate through the tube, specific heat of water, length of element and the tube length, respectively. The mean temperature of the water T_m of each element is estimated by the relation;

$$T_m = (T_{i+1} + T_i) / 2 \quad .$$

Local heat transfer coefficient, local Nusselt number and Peclet number are evaluated according to the following relations;

$$h_l = q / (T_w - T_m) \quad ; \quad Nu_z = h_l z / k \quad ; \quad Pe_z = (\rho c_p u z) / k \quad ,$$

where k , ρ , and u are the thermal conductivity of water, density of water and mean velocity of water.

3. Mathematical Model

In the following analysis, steady incompressible laminar flow is considered. The flow is assumed to be, hydrodynamically, fully developed flow but, in the same time, it is thermally developing. According to the boundary layer theory, this assumption appears to be valid for liquids of high values of Prandtl number ($Pr > 1$) since the hydrodynamic entry length is relatively very small. As shown in figure (2), cylindrical coordinate system (r, θ, z) is used to express the flow governing equations. In the present analysis, Newtonian and constant properties fluids are considered. The axial pressure gradient is assumed to be constant. Both axial heat conduction and viscous dissipation are assumed to be very small. The laminar forced convective flow is described through the following equations;

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = \frac{1}{\mu} \frac{\partial p}{\partial z} \quad , \quad (1)$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = \frac{\rho C_p u}{k} \frac{\partial T}{\partial z} \quad , \quad (2)$$

equation (1) is the momentum equation, where the inertia terms are not appeared because the flow is assumed to be axial fully developed flow. In the energy equation, equation (2), the convective terms including tangential and radial components are neglected. The energy dissipation term is assumed to be nil. Equations (1) and (2) must satisfy the following boundary conditions;

$$\frac{\partial u}{\partial r} = 0.0 \quad ; \quad \frac{\partial T}{\partial r} = 0.0 \quad \text{at} \quad r = 0.0 \quad , \quad (3)$$

$$u(r, \theta) = 0.0 \quad ; \quad \frac{\partial T}{\partial r} = - \frac{q}{k} \quad \text{at} \quad r = r_w \quad .$$

To express the governing equations (1-3) in suitable simpler form, one defines r in terms of aspect ratio of tube cross section and θ as:

$$r = a \sqrt{\cos^2 \theta + m^2 \sin^2 \theta} \quad , \quad (4)$$

where m is the aspect ratio defined as the ratio between the minor axis (B) and the major axis (A) of the tube cross section ($m = B/A$) and r represents any point on the ellipse of major axis a corresponding to r - θ domain. Accordingly, the value of r corresponding to the tube wall (r_w) is given by;

$$r_w = A \sqrt{\cos^2 \theta + m^2 \sin^2 \theta} \quad , \quad (5)$$

where A is the major axis of tube cross section. To put the flow describing equations in dimensionless form, one introduce the following definitions of the dependent and independent variables as;

$$R = \frac{r}{A \sqrt{\cos^2 \theta + m^2 \sin^2 \theta}} \quad , \quad Z = \frac{z}{A P_r} \quad ,$$

$$\varphi = \frac{T_w - T}{q_w l / k} \quad , \quad U = \frac{u}{U^*} \quad , \quad (6)$$

$$U^* = \frac{dP}{dz} A^2 / \mu \quad , \quad Pe_x = \frac{C_p U^* \rho A}{k} \quad .$$

where Pe_a is Peclet number based on the major axis (A) and Z is the dimensionless horizontal distance. With the aid of the previous definitions and the governing equations of the flow (equations 1-3), the dimensionless forms of momentum and energy equations and their boundary conditions can be derived as;

$$\frac{d^2 U}{dR^2} + \frac{1}{R} \frac{dU}{dR} = 1 \quad (7)$$

$$-\frac{\partial^2 \phi}{\partial R^2} + \frac{1}{R} \frac{\partial \phi}{\partial R} = U \frac{\partial \phi}{\partial Z} \quad (8)$$

equations (7-8) must satisfy the following dimensionless boundary conditions;

$$U = 0.0 \quad ; \quad \frac{\partial \phi}{\partial R} = -1.0 \quad \text{at the wall } (R = 1.0) \quad (9)$$

$$\frac{\partial U}{\partial R} = 0.0 \quad ; \quad \frac{\partial \phi}{\partial R} = 0.0 \quad \text{at the tube center } (R = 0.0)$$

Equations (7-9) are the dimensionless form of the governing equations. Due to the transformation of dependent and independent variables, this derived dimensionless form of governing equations describes an axisymmetric flow, see the modified system of coordinate (R , θ and Z) in figure (3). Solving equations (7-9), velocity distribution across the tube and the temperature distribution across the tube at different positions along the tube can be predicted. Accordingly, the physical quantities of the flow such as the heat transfer coefficient, Nusselt number and coefficient of friction can be determined. Nusselt number and coefficient of friction are defined through the following relations;

$$Nu = \frac{hA}{k} \quad ; \quad C_f = \frac{\tau_w}{\rho u^2 / 2} \quad (10)$$

where Nu and C_f are local Nusselt number and local coefficient of friction, respectively. h and τ_w are the local heat transfer coefficient and shear stress at the tube wall, respectively. They are defined according to the following relations;

$$h = \frac{q}{(T_w - T_m)} \quad ; \quad \tau_w = \mu \left. \frac{du}{dr} \right|_w \quad (11)$$

with the aid of equation (6) and equations (10-11), one can define Nusselt number and coefficient of friction in terms of dimensionless variables as;

$$C_f Re = \frac{d U}{d R} \Big|_{R=1.0} ; \quad Nu = \frac{1}{\varphi} \frac{d \varphi}{d R} \Big|_{R=1.0} \quad (12)$$

Average Nusselt number can be determined as;

$$\overline{Nu} = \frac{1}{Z} \int_0^Z Nu \, dZ \quad (13)$$

Momentum equation (eqn. 7) and energy equation (eqn. 8) are solved, numerically, using finite divided difference technique. According to this technique the governing differential equations (7-8) are transferred to two sets of linear algebraic equations as;

$$A_i U_{i+1} + B_i U_i + C_i U_{i-1} = D_i \quad (14)$$

$$A I_i \varphi_{i+1} + B I_i \varphi_i + C I_i \varphi_{i-1} = D I_i \quad (15)$$

where the coefficients of equations (14-15) are defined through the following expressions;

$$\begin{aligned} A_i &= 2 R_i + \Delta R & B_i &= -2 R_i \\ C_i &= 2 R_i - \Delta R & D_i &= 2 (\Delta R)^2 R_i \\ A I_i &= 2 R_i \Delta Z + \Delta R \Delta Z & B I_i &= -[4 R_i \Delta Z + 2 \bar{U}_i (\Delta R)^2 R_i] \\ C I_i &= 2 R_i \Delta Z - \Delta R \Delta Z & D I_i &= -2 U_i (\Delta R)^2 R_i \varphi_i \end{aligned} \quad (16)$$

Since the momentum equation is an ordinary second order differential equation, it is enough to solve it along the co-ordinate R (from $R=0.0$ to $R=1.0$), as it is illustrated in figure (4). Because the energy equation (eqn. 8) is a partial differential equation of second order and of parabolic type, it is convenient to solve it, for certain value of Z , along the vector R (from $R=1.0$ to $R=0.0$) and then the solution is carried out in a repetitive manner along the tube axis (at different values of Z), see figure (4). A computer program is designed to solve the previously describe theoretical model to analyze such present flow.

4. Results And Discussions

Although four pipes of different aspect ratios at different flow rates and heat fluxes are, experimentally, examined; some results of the test of the pipe of aspect ratio 0.636 are presented in this paper. Temperature distribution along the pipe length is shown in figures (5-6). The temperature of water increases as either mass flow rate or heat flux decrease. Nusselt number versus Peclet number is presented in figures (7-8). It is clear that, Nusselt number has a highest value at the inlet of the tube and then it decreases rapidly in the neighboring part of the pipe. Then it decreases slowly until it reaches an asymptotic value. This asymptotic value seems to be the same for all values of mass flow rates.

In the following figures the results obtained by solving the mathematical model are presented. Figure(9) shows velocity profile. As a result of the definition of the dimensionless variables of the problem, single velocity profile satisfies all hydrodynamic fully developed flow, whatever the aspect ratio is. On the other hand, figure (10) shows the dimensionless temperature-distribution along the pipe length at different dimensionless positions Z . As it is mentioned before, these temperature profiles are valid for all pipes of different aspect ratios. At every position along the pipe, the temperature has the maximum value at the wall and then it decreases rapidly near the wall and beneath this region it decreases, relatively, slowly till it reaches its minimum value at the center of the pipe. As it is expected, the temperature increases in the down stream direction. Figure (11) shows local Nusselt number along the pipe. At the entrance of pipe, Nusselt number goes to infinity then it decreases rapidly nearby the entrance cross section. Then it decreases slowly going to an asymptotic value of 4.385. A comparison between the experimental and theoretical present work is shown in figure(12). As it is clear, the deviation between the experimental and theoretical results is smaller as the value of aspect ratio increases. Also a comparison between present and previous work is presented in the same figure. The validity of theoretical model is clear in this figure. A correlation for average Nusselt number as a function of aspect ratio, Prandtl number and Reynolds number is made. This correlation and the corresponding experimental result are shown in figure (13). This derived correlation can be written as;

$$\overline{Nu} = 1.25 Re^{0.16} Pr^{0.0011} m^{-0.5661}$$

5. Conclusion

In the present work a theoretical model is proposed to analyze the forced convective laminar flow in pipes of different aspect ratios. This model is valid for all pipes, whatever the aspect ratio is. A series of experiments are carried out, such that they cover the flow in pipes of wider range of aspect ratio compared with the previous available works. A correlation for average Nusselt number as a function of the flow parameters is derived.

Nomenclature

A	Major axis of the elliptic cross section of the pipe , m
A_s	Surface area of pipe , m^2
B	Minor axis of the elliptic cross section of the pipe , m
C_p	Specific heat of fluid , $kJ / kg ^\circ C$
D	Diameter of the tube , m
D_h	Hydraulic diameter of the tube , m
h	Convective heat transfer coefficient , $W / m^2 ^\circ C$
k_f	Thermal conductivity of water , $W / m ^\circ C$
l	Length of the pipe , m
\dot{m}	Mass flow rate , kg / s
m	Aspect ratio
p	Pressure , N / m^2
Q	Heat rate , W
q	Wall heat flux , W / m^2
R	Radial Coordinate , m
R_{heater}	Electric resistance of the heater , Ω

R_w	outer tube radius, m
T	Temperature, °C
T_w	Wall temperature, °C
U	Dimensionless axial velocity
U^*	Characteristic axial velocity, m/s
u	Axial velocity, m/s
Z	Dimensionless axial coordinate
z	Axial coordinate, m

Greek Symbols

θ	Angular position, rad.
ρ	Density, kg/m ³
μ	Dynamic viscosity, N s / m ²
ϕ	Dimensionless temperature

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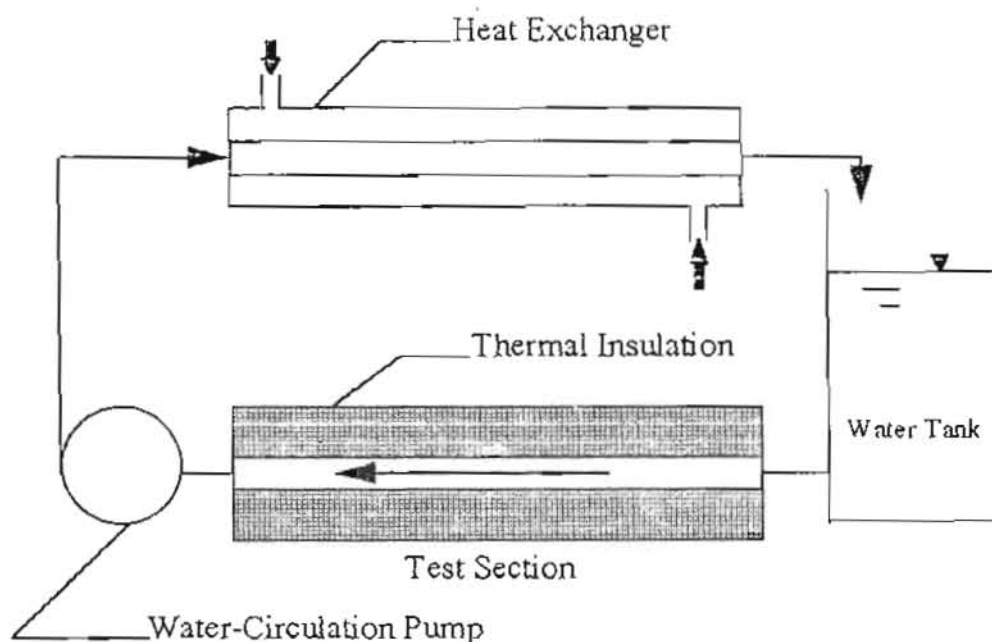


Figure (1) Schematic Drawing of Experimental Test Rig

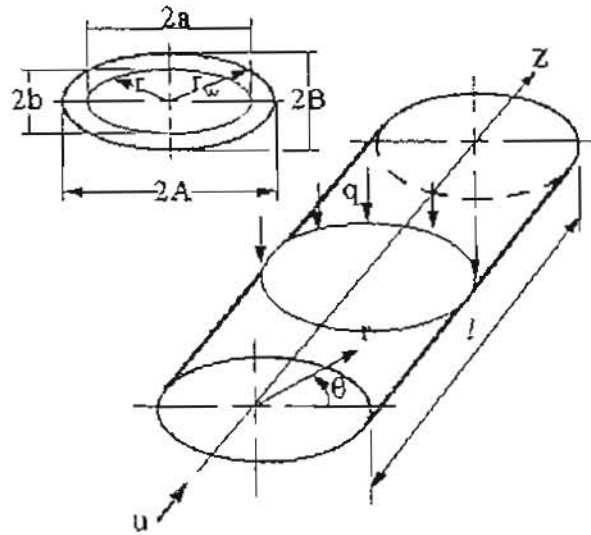


Figure (2) Actual Tube Configuration Using Cylindrical Coordinate System

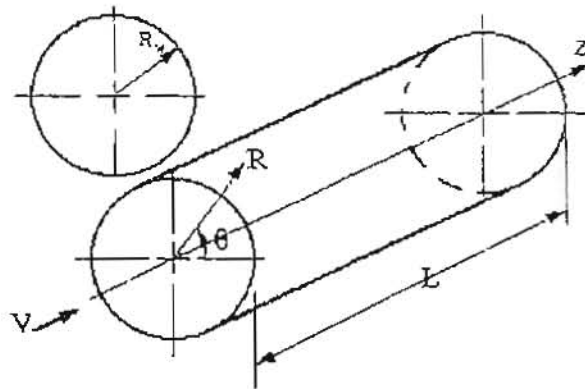


Figure (3) Transformed Tube Configuration Using Transformed Cylindrical Coordinate

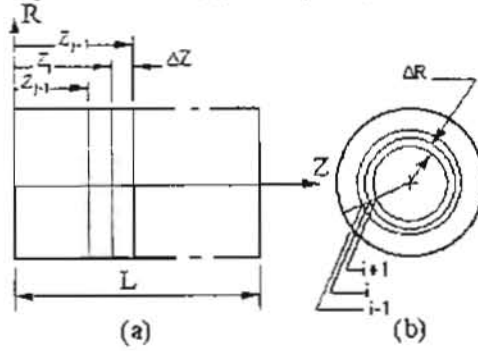


Figure (4) Tube Discretization Used in Numerical Solution
 a) Numerical Marching Scheme in Axial Direction
 b) Cross-Section Discretization

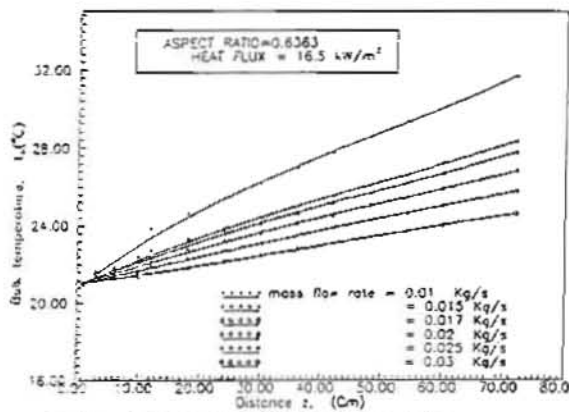


Fig. (5) Bulk Temperature along the Tube Length for Uniform Heat Flux. (Experimental Results).

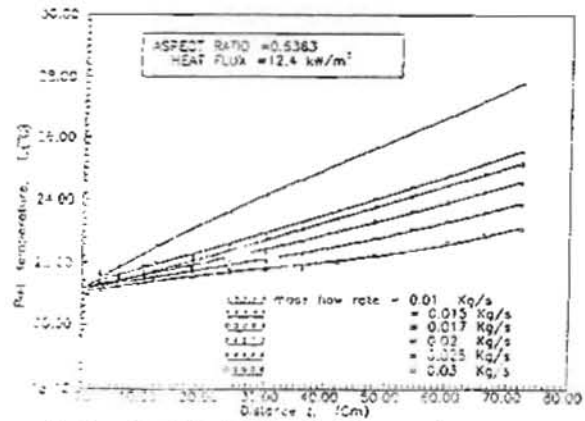


Fig. (6) Bulk Temperature along the Tube Length for Uniform Heat Flux. (Experimental Results).

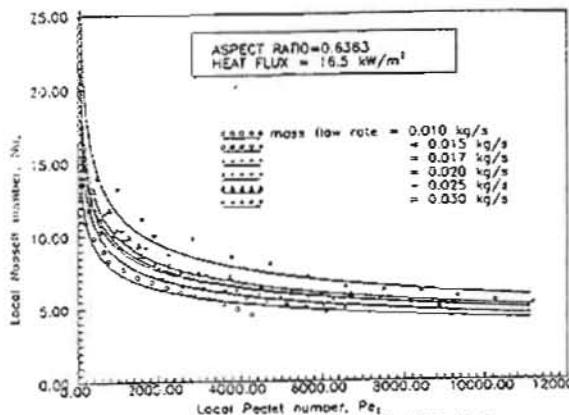


Fig. (7) Local Nusselt Number for Different Peclet Numbers for Uniform Heat Flux. (Experimental Results).

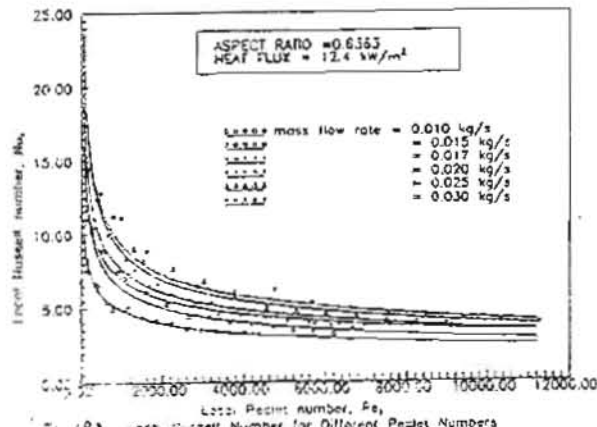
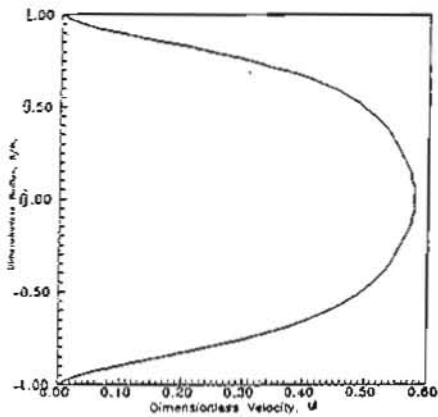
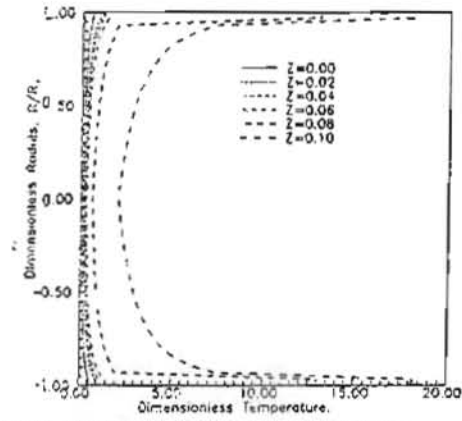


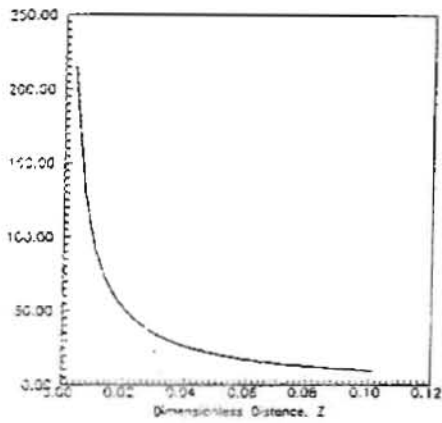
Fig. (8) Local Nusselt Number for Different Peclet Numbers for Uniform Heat Flux. (Experimental Results).



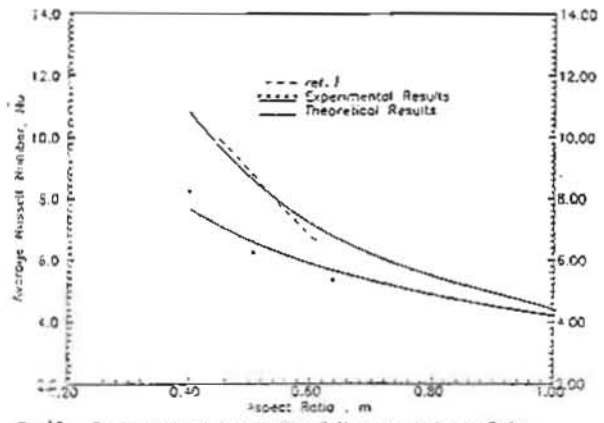
9) Dimensionless Velocity Profile at Any Cross Section. (Theoretical Results)



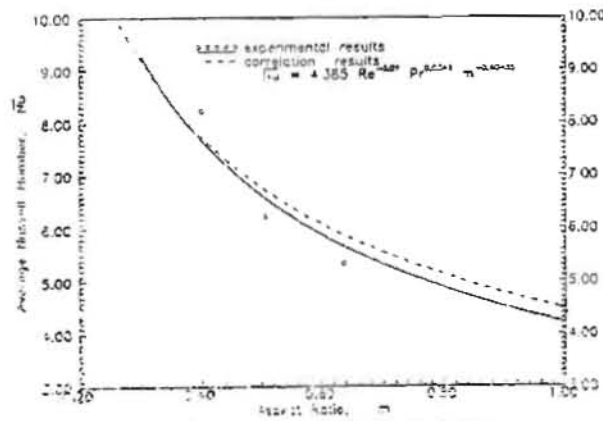
10) Dimensionless Temperature Profile at Different Positions Along the Tube. (Theoretical Results)



11) Local Nusselt Number Along the Tube (Theoretical Results)



12) Relation between Average Nusselt Number and Aspect Ratio.



13) Comparison between Experimental Results and Obtained Correlation for Average Nusselt Number