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NONEQUILIBRIUM THERMODYNAMIC TREATMENT OF A WARM PLASMA IN STRONG MAGNETIC AND ELECTRIC FIELFS

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ABSTRACT

In the framework of the irreversible thermodynamics we study a rarefied and collisional warm electron plasma under the effect of external strong magnetic and electric fields which generate small wave amplitudes. We adopt the linear theory and normal mode solution in the MHD model to calculate the perturbation in pressure, mass density, components of velocity, electric and magnetic fields. By applying the second law of thermodynamics it is concluded that the change in the internal energy of the plasma particles predicts whether they gain from or loose energy to the generated waves .The obtained results agree with the physical ground bounded by the positive nature of the entropy production. The predictions have been carried out within the range of the frequency of the generated waves and the distance from the Debye sphere.

KEYWORDS: MHD flows, Plasma, Non-equilibrium Thermodynamics

INTRODUCTION

Considerable interest has been aroused in the study of a small amplitude waves in plasma by applying the normal mode solutions, for example,Stix [1] investigated the natural modes of oscillation of a cylindrical plasma of finite density at zero pressure in a longitudinal magnetic field, he considered frequencies well below the electron plasma and electron cyclotron frequencies, at different cases he found the dispersion relations and the plasma current.

Willett [2] studied magnetoplasma-filled waveguides, he applied the linear theory where the perturbation quantities was given by the factor $\exp[i(kz-\omega t)]$. The components of the electric and magnetic fields are estimated and the dispersion relation of the system is obtained.

Recently, Cranmer [3] studied the state of the plasma in a magnetic field by applying the linearly perturbed Vlasov equation, in which the distribution function is expressible in the Boltzmann equation and then investigated the dispersion relation. He tackled the case of the initial problem where $\omega = \omega_r + i\,\alpha$.

Khalil et al. [4] investigated warm, magnetized plasma in waveguides and applied the linear theory with perturbations in the form $f(x,y,z,t) = f(x,y)\exp[i(kz-\omega t)]$. They calculated the average power flux from the pointing vector.

Loverich and Shumlak [5] solved numerically the equations describing the two-fluid plasma system in one dimension. This consists of electron and ion continuity, momentum and energy equations. They solved the full Maxwell equations, by including displacement current and electron and ion currents. The fluids were assumed to be collisional and non-relativistic.

In this paper the MHD model together with the non-equilibrium thermodynamic viewpoints are adopted to study the case of warm plasma under the effect of external strong magnetic and electric fields.

In the beginning we mention a nomenclature of the variables and parameters used in our study:

- **c = speed of light,**
- **C_p = specific heat at constant pressure,**
- **e = electron charge,**
- **E = electric field,**
- **H = magnetic field,**
- **K_B = Boltzmann's constant,**
- **m = electron mass,**
- **n = electrons number density,**
- **P = electrons pressure,**
- **S = entropy,**
- **T = electrons temperature,**
- **U = internal energy,**
- **u = fluid velocity (the mean velocity of electrons in a plasma),**
- **V = phase velocity of a small amplitude waves,**
- **V_S = speed of sound,**
- **V_{th} = thermal velocity,**
- **ρ = electrons mass density,**
- **σ = entropy production,**
- **υ_{ee} = collision frequency between electrons,**
- **ω = frequency of a small amplitude waves,**
- **ω_{ce} = electron cyclotron frequency,**
- **ω_P = plasma frequency,**
- **λ_d = Debye length .**

The physical situation and mathematical formulation

We shall investigate theoretically the characters of the small amplitude waves generated in a warm rarefied collisional plasma influenced by external strong electric and magnetic fields[6], that is to unsure the confinement and equally to avoid boundary effects. At t=0 we assume that the plasma is in thermal equilibrium. The ions are too massive to move at the frequencies involved and form a fixed, uniform neutralizing background of positive charges. Therefore the collisions between the electrons are only considered [4].

Now applying the MHD model in which the essential equations will be the continuity of mass, equation of motion, energy equation, and Maxwell's equations [7,8,9]. Replacing each variable by its equilibrium value plus a small amplitude of oscillations such that the terms containing higher amplitude powers can be neglected. We consider the normal mode solutions in which the perturbed quantities vary like $\exp[i(\bar{k}\cdot\bar{r}-\omega t)]$,where ω is a real number and solving for a complex wave number, where energy dissipation is expected, $k = k_r + i\alpha$. The real part is $k_r = \frac{\omega}{V}$, while the imaginary part α is the damping (attenuation) coefficient and we assume a very weak instability with $0 < \alpha \ll k_r$. This is appropriate for the case of source problem [10]

The one fluid equations are [4,7,8]

. <u>*The continuity equation*</u>

(1) $$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = 0 \quad .$$

. *The equation of motion*

(2) $$mN\frac{d\vec{u}}{dt} = eN\left[\vec{E} + \frac{1}{c}\vec{u}\times\vec{H}\right] - mN\upsilon_{ee}\vec{u} - \vec{\nabla}P \ .$$

.*The energy equation*

(3) $$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho u^2 + \frac{3}{2}P\right) + \vec{\nabla}\cdot\left[\vec{u}\left(\frac{5}{2}P + \frac{\rho u^2}{2}\right)\right] = -\rho\upsilon_{ee}u^2 + \rho T\frac{dS}{dt} + \frac{e\rho}{m}\left[\vec{u}\cdot\vec{E}\right] \ .$$

. *Maxwell's equations*

(4) $$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t} \ ,$$

(5) $$\frac{1}{c}\frac{\partial\vec{E}}{\partial t} = \vec{\nabla}\times\vec{H} - \frac{4\pi}{c}\vec{J} \ .$$

In this section a detailed investigation of the behavior of plasma waves is developed starting from the linearization of the system (1-5).

The linearized equations are obtained by setting [4,5],

(6) $$\begin{aligned} P &= P_0 + \tilde{P} \\ T &= T_0 + \tilde{T} \\ U &= U_0 + \tilde{U} \\ N &= n_0 + \tilde{n} \\ u &= v_0 + \tilde{v} \\ \rho &= \rho_0 + \tilde{\rho} \end{aligned}$$

The terms with 0 subscripts are the background constants and the terms with a tilde are small linear perturbations from the background value. These variables are substituted into the nonlinear equations. We assume that $\vec{v}_0 = v_0\vec{e}_z$, $\vec{H}_{ext} = H_0\vec{e}_z$, $\vec{E}_{ext} = E_{0x}\vec{e}_x + E_{0y}\vec{e}_y + E_{0z}\vec{e}_z$ **and the condition** $\frac{\partial}{\partial z} >>> \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$ **is fulfilled [4].**

The perturbation of the mass density and temperature are expressed in terms of the pressure and entropy through the equations of state [10], so that to replace

$$\tilde{\rho} \rightarrow \left(\frac{\partial \rho}{\partial P}\right)_S \tilde{P} + \left(\frac{\partial \rho}{\partial S}\right)_P \tilde{S}$$

(7)
$$\tilde{T} \rightarrow \left(\frac{\partial T}{\partial P}\right)_S \tilde{P} + \left(\frac{\partial T}{\partial S}\right)_P \tilde{S}$$

where the perturbed variables are expressed by the normal mode solutions in the form

(8)
$$\begin{aligned} \tilde{P}(\vec{r},t) &= \Gamma e^{i(\vec{k}.\vec{r}-\omega t)} \\ \tilde{S}(\vec{r},t) &= \varphi e^{i(\vec{k}.\vec{r}-\omega t)} \\ \tilde{v}(\vec{r},t) &= v e^{i(\vec{k}.\vec{r}-\omega t)} \\ \tilde{H}(\vec{r},t) &= H e^{i(\vec{k}.\vec{r}-\omega t)} \\ \tilde{E}(\vec{r},t) &= E e^{i(\vec{k}.\vec{r}-\omega t)} \end{aligned}$$

We take $\tilde{v}(\vec{r},t) = \vec{v}_\perp + v_z \vec{e}_z$, $\vec{k}.\vec{v}_\perp = 0$, **where** $\vec{v}_\perp = v_x \vec{e}_x + v_y \vec{e}_y$, **and** $\vec{k}$ **is directed along the z-direction [4].**

The non-equilibrium thermodynamic predictions of the problem

In order to study the irreversible thermodynamic properties of the system we begin with the evaluation of the law of increase of entropy and the law of change in internal energy.

. The entropy production equation can be written as [12]

(9)
$$\frac{\partial S}{\partial t} + (\vec{u} \cdot \vec{\nabla})S = \sigma \geq 0$$

. The first and second laws of thermodynamics can be combined in the form [13],

(10)
$$dU = TdS + \frac{P}{\rho^2} d\rho$$

All equations of the one fluid system can be put in more convenient form if we use some simple thermodynamic relations [10]. Thus,

(11) $$\left(\frac{\partial T}{\partial S}\right)_P = \frac{T_0}{C_P} \quad ,$$

(12) $$\left(\frac{\partial T}{\partial P}\right)_S = \frac{2T_0}{\rho_0 V_S^2} \quad ,$$

(13) $$\left(\frac{\partial \rho}{\partial S}\right)_P = -\frac{\rho_0}{3C_P} \quad ,$$

(14) $$\left(\frac{\partial \rho}{\partial P}\right)_S = \frac{1}{V_S^2}$$

Several non-dimensional factors are developed that are common plasma variables; these variables characterize the system under study. The non-dimensional barred variables are defined as follows:

- $\bar{z} = \dfrac{\bar{z}}{\lambda_d}$,
- $\bar{\rho} = \dfrac{\rho}{\rho_0}$,
- $\bar{V} = \dfrac{V}{c}$,
- $\bar{P} = \dfrac{P}{\rho_0 V_S^2}$,
- $\bar{t} = t\upsilon_{ee}$,
- $\bar{S} = \dfrac{S}{C_P}$,
- $\bar{\omega} = \dfrac{\omega}{\upsilon_{ee}}$,
- $\bar{E} = E\left(\dfrac{e}{mc\upsilon_{ee}}\right)$,
- $\bar{H} = H\left(\dfrac{e}{mc\upsilon_{ee}}\right)$,
- $\bar{U} = \dfrac{U}{T_0 C_P}$,

Therefore the complete system of equations will be rewritten as:

. *The continuity equation*

(15) $$(\omega - kL_1 v_0)\Gamma - \frac{(\omega - kL_1 v_0)}{3}\varphi - kL_1 v_z = 0$$

where $$L_1 = \frac{c}{\lambda_d \upsilon_{ee}}$$

. *The equation of motion*

Along the x-axis:

(16)

$$(1 - i(\omega - kL_1 v_0))v_x - \omega_{ce} v_y - E_x + v_0 H_y - E_{0x}\Gamma + \frac{E_{0x}}{3}\varphi = E_{0x} e^{-i(kz-\omega t)}$$

Along the y-axis:

(17)

$$(1 - i(\omega - kL_1 v_0))v_y + \omega_{ce} v_x - E_y - v_0 H_x - E_{0y}\Gamma + \frac{E_{0y}}{3}\varphi = E_{0y} e^{-i(kz-\omega t)}$$

Along the z- axis:

(18) $$\begin{aligned}&(i(\omega - kL_1 v_0) - 1)v_z + E_z - (v_0 + 9ikL_1 a_1 - E_{0z})\Gamma \\ &- \left[2ikL_1 a_1 - \frac{v_0}{3} + \frac{E_{0z}}{3}\right]\varphi = (v_0 - E_{0z})e^{-i(kz-\omega t)}\end{aligned}$$

where $$a_1 = \left(\frac{V_{th}}{c}\right)^2,$$

. *The energy equation*

(19)

$$\left(-iv_0\left(\omega - \frac{3}{2}kL_1 v_0\right) + 2v_0 + \frac{15ikL_1 a_1}{2} - E_{0z}\right)v_z - E_{0x}v_x - E_{0y}v_y$$

$$-v_0 E_z + \Gamma\left[-i\frac{v_0^2}{2}(\omega - kL_1 v_0) - v_0 E_{0z} - \frac{9ia_1}{2}(3\omega - 5kL_1 v_0) + v_0^2\right]$$

$$+\varphi\left[i\frac{v_0^2}{6}(\omega - kL_1 v_0) + \frac{v_0}{3}E_{0z} - ia_1(3\omega - 5kv_0) - \frac{v_0^2}{3} + iL_2(\omega - kL_1 v_0)\right]$$

$$= (v_0 E_{0z} - v_0^2)e^{-i(kz-\omega t)}$$

where $L_2 = \frac{T_0 C_P}{c^2}$,

. *Maxwell's equations*

(20) $\omega H_x + kL_1 E_y = 0$,

(21) $\omega H_y - kL_1 E_x = 0$,

(22) $i\omega E_x - ikL_1 H_y - L_1 \omega_P^2 v_x = 0$,

(23) $i\omega E_y + ikL_1 H_x - L_1 \omega_P^2 v_y = 0$,

(24) $\frac{i\omega}{\omega_P^2} E_z - v_z - v_0 \Gamma + \frac{v_0}{3}\varphi = v_0 e^{-i(kz-\omega t)}$.

. *The change in internal energy*

(25) $d\tilde{U} = i\left[\left(1 - \frac{a_1}{L_2}\right)\varphi + 3\frac{a_1}{L_2}\Gamma\right](kdz - \omega dt)e^{i(kz-\omega t)}$.

. ***The entropy production***

(26) $\sigma = i(kL_1 v_0 - \omega)\varphi e^{i(kz-\omega t)}$.

In the process of non-dimensionalization, three factors have been appeared, namely a_1, L_1, L_2. The linear system of equations (15-24), can be written in the matrix form AB = C, where,

(27) $B = (v_x \quad v_y \quad v_z \quad \Gamma \quad \varphi \quad E_x \quad E_y \quad E_z \quad H_x \quad H_y)^T$

(28) $A = [a_{ij}]$ **, where** $i, j = 1 \to 10$.

The nonzero matrix elements are :

$a_{11} = a_{22} = -a_{43} = (1 - i(\omega - kv_0 L_1))$, $a_{21} = -a_{12} = \omega_{ce}$, $a_{14} = a_{51} = -E_{0x}$,

$a_{15} = \frac{E_{0x}}{3}$, $a_{25} = \frac{E_{0y}}{3}$, $a_{85} = \frac{v_0}{3}$, $a_{88} = \frac{i\omega}{\omega_P^2}$, $a_{101} = a_{92} = -L_1\omega_P^2$,

$a_{33} = a_{66} = -a_{77} = ia_{1010} = -ia_{99} = -L_1 k$, $a_{16} = a_{83} = a_{27} = -a_{48} = -1$,

$a_{84} = a_{29} = a_{58} = -a_{110} = -v_0$, $a_{79} = a_{610} = ia_{97} = ia_{106} = \omega$,

$a_{24} = a_{52} = -E_{0y}$,

$a_{53} = iv_0\left(\frac{3}{2}kL_1 v_0 - \omega\right) + 2v_0 + \frac{15ikL_1 a_1}{2} - E_{0z}$,

$$a_{54}=i\frac{v_0^2}{2}\left(kL_1v_0-\omega\right)+v_0^2+\frac{9ia_1}{2}\left(5kL_1v_0-3\omega\right)-v_0E_{0z}\ ,$$

$$a_{55}=-i\frac{v_0^2}{6}\left(kL_1v_0-\omega\right)-\frac{v_0^2}{3}+ia_1\left(5kL_1v_0-3\omega\right)+\frac{v_0E_{0z}}{3}-iL_2\left(kL_1v_0-\omega\right)\ .$$

and

(29) $$C=\left(E_{0x}e^{\alpha z}\quad E_{0y}e^{\alpha z}\quad 0\quad C_{14}\quad C_{15}\quad 0\quad 0\quad v_0e^{\alpha z}\quad 0\quad 0\right)^T\ ,$$

where $C_{14}=\left(v_0-E_{0z}\right)e^{\alpha z}$, $C_{15}=\left(v_0E_{0z}-v_0^2\right)e^{\alpha z}$.

DISCUSSION AND CONCLUSIONS

The irreversible thermodynamics and the normal mode solutions of the linearized MHD equations are applied to study the behavior of a warm, collisional, rarefied plasma in external strong electric and magnetic fields.

It is worth noting to mention some general remarkes at the start,namely:

1-The estimated variables are in complex form, where we take those parts which satisfy the physical ground bounded by the positive nature of the entropy production.

2-The computations are performed according to the works [9,11,12] for warm hydrogen plasma subjected to the following conditions and parameters :

$n = 10^{14}$ cm^{-3}, T = 10 eV, $\omega_{Pe} = 5.639x10^{11}$ rad/sec, $\upsilon_{ee} = 144$ sec^{-1} ,
$\lambda_d = 2.35x10^{-4}$ cm , $H_0 = 3.2x10^4$ Gauss , e = $4.8032x10^{-10}$ statcoulomb,
$V_{th} = 1.326x10^8$ cm/sec, c = $2.998x10^{10}$ cm/sec, $V_S = 5.654x10^6$ cm/sec,
$m_e = 9.1x10^{-28}$ g, $C_p = 3.793x10^{11}$ erg/g-deg(K) [10], $\ln\Lambda = 19.54$,
$K_B = 1.38x10^{-16}$ erg/deg(K), and $\chi = 1.032x10^7$ erg/sec-cm-deg(K) [12].

These solutions are estimated under the assumptions that the non-dimensional value of both the external magnetic and electric fields is taken w.r.t. the factor $\frac{mc\upsilon_{ee}}{e} = 8.19x10^{-6}$ so that the external magnetic field will amount to $H_0 = 2.6x10^{11}$ to unsure the confinement of the plasma, whereas $E_{0x} = E_{0y} = E_{0z} = 10^9$ .The characteristic frequencies are $\omega_P = \omega_{ce} = 3.9x10^9$ such that $\omega \leq \omega_P, \omega_{ce} \leq \omega$ while $\alpha = 2.35x10^{-4}$ is evaluated in the ranges $\omega = [10^8, 10^{11}]$ and $V = [10^{-5}, 10^{-3}]$, which are bounded by the positive nature of the entropy production.

3- The graphs show that the components of the velocity will follow the tracks of the corresponding components of the electric field.

Since the thermodynamically non-equilibrium state of the system is in the focus of our attention therefore we shall start with;

The entropy production σ :

It is seen in figure (1) that all over the surface the entropy production σ is positive within the intervals of the frequency and phase velocity; which is in a good agreement with the H-Theorem [14]. We note that σ increases smoothly and nonlinearly within the interval $V = [10^{-5}, 10^{-3}]$ at $\omega = 10^{11}$ while it takes a constant value at $\omega = 10^8$ within the same V interval .It is observed that σ is a constant within the interval $\omega = [10^8, 10^{11}]$ at $V = 10^{-5}$ but increases smoothly and nonlinearly within the same interval of the frequency at $V = 10^{-3}$.

The entropy S :

The warm plasma is far from the equilibrium state in the beginning of the frequency interval and approaches to the equilibrium state as the frequency increases. It is seen from figure (2) that the maximum amount of entropy equals to zero.

The pressure P and mass density ρ :

The amplitudes of the pressure and mass density are substationally less than unity, this agree with the assumption of small perturbations; see figures (3,4). They decrease smoothly and nonlinearly within the interval $\omega = [10^{8}, 10^{10.1}]$ at $V = [10^{-5}, 10^{-4.5}]$ and clearly decrease slowly within the interval $V = [10^{-5}, 10^{-4.5}]$ at $\omega = 10^{10.1}$.

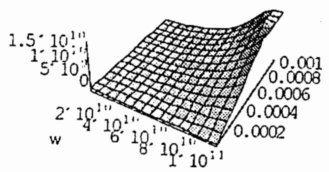


Figure (1): The entropy production σ vs. the frequency and phase velocity.

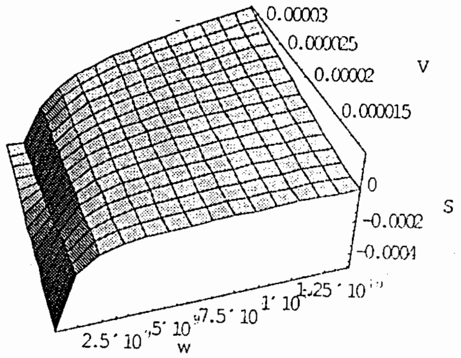


Figure (2): The entropy S vs. the frequency and phase velocity.

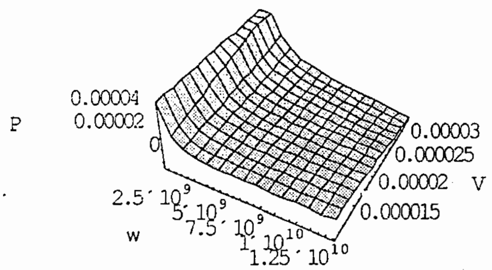


Figure (3): The pressure P vs. the frequency and phase velocity.

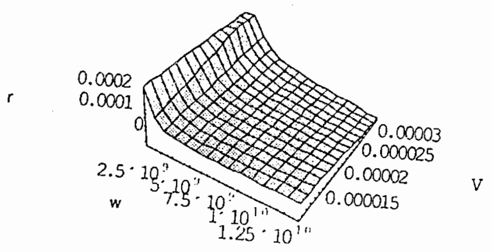


Figure (4): The mass density ρ vs. the frequency and phase velocity.

<u>The x-component of the electric field</u> :

The amplitude of E_x increases slowly nonlinearly within the interval $V = [10^{-5}, 10^{-3}]$ at $\omega = [2x10^{10}, 10^{11}]$. It increases suddenly within the interval $V = [10^{-5}, 2x10^{-4}]$ at $\omega = [10^{8}, 2x10^{10}]$. E_x takes a constant large value within the interval $V = [2x10^{-4}, 10^{-3}]$ at the same frequency interval. It is

directed along the positive x-direction with amplitude less than the external strong electric field E_0 ; see figure (5) .

The x-component of the velocity :

The amplitude of v_x takes positive values less than 1, it increases in a quasilinear manner within the intervals V = $[10^{-5}, 10^{-3}]$ at $\omega = [2x10^{10}, 10^{11}]$, and nonlinearly decreases at $\omega = [10^8, 2x10^{10}]$ for the same V interval; see figure (6) .

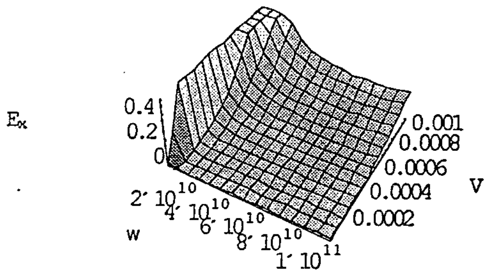


Figure (5): The E_x-component of the electric field vs. the frequency and phase velocity with respect to $E_0 = 10^9$.

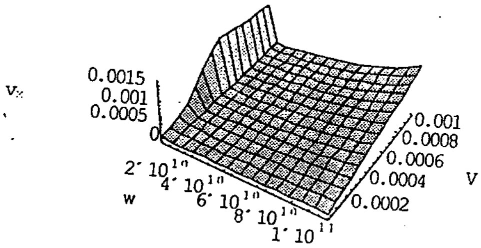


Figure (6):The v_x -component of the velocity vs. the frequency and phase velocity.

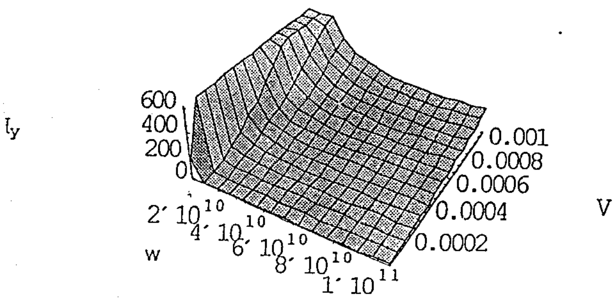


Figure (7): The H_y-component of the magnetic field vs. the frequency and phase velocity with respect to $H_0 = 2.6x10^{11}$.

The y-component of the magnetic field :

The amplitude of H_y decreases nonlinearly for all values of V within the interval $\omega = [10^8, 10^{11}]$, and takes a nearly constant value at the beginning of the frequency scale for all the values of V; see figure (7). Its amplitude is of the order 10^2 less than the external strong magnetic field H_0 .

The y-component of the electric field :

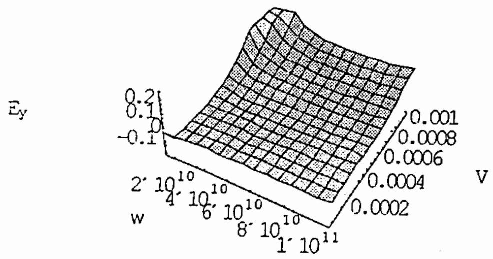
At the beginning of the frequency interval, the amplitude of E_y takes zero value and decreases to (-ve) values within the interval and $\omega = [10^8, 2x10^{10}]$ at $V = 10^{-5}$. It increases nonlinearly to (+ve) values within the interval $V = [10^{-5}, 10^{-3}]$ at $\omega = 10^8$, and then increases quasilinearly within the interval $V = [10^{-5}, 10^{-3}]$ at $\omega = 10^{11}$, the amplitude of E_y is less than the external strong electric field E_0 ; see figure (8) .

The y-component of the velocity v_y :

At the beginning of the frequency interval $\omega = [10^8, 2x10^{10}]$ for all of the V interval, the amplitude of v_y takes (-ve) values, and increases suddenly within the same interval of ω. But it increases slowly nonlinearly as the phase velocity V increases within the interval $\omega = [2x10^{10}, 10^{11}]$; see figure (9).

The x-component of the magnetic field :

At the beginning of the frequency interval, the amplitude of H_x takes zero value and decreases suddenly within the interval $\omega = [10^8, 2x10^{10}]$for all the V scale. It increases slowly within the interval $V = [10^{-5}, 10^{-3}]$ at $\omega = [10^8, 2x10^{11}]$; see figure (10) . It takes the negative x-direction where its amplitude of the order 10^2 less than the external strong magnetic field H_0 .



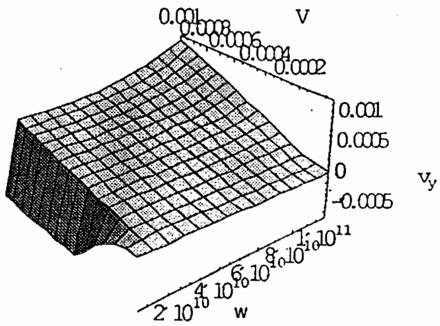


Figure (8): The E_y-component of the electric field vs. the frequency and phase velocity with respect to $E_0 = 10^9$.

Figure (9): The v_y -component of the velocity vs. the frequency and phase velocity.

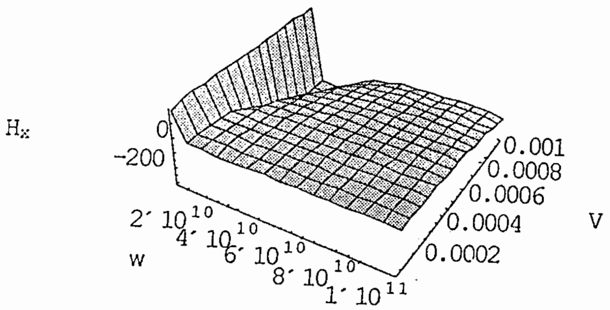


Figure (10): The H_x -component of the magnetic field vs. the frequency and phase velocity .

The z-component of the electric field:

The amplitude of E_z begins with zero at $\omega=10^8$,then increases suddenly to a maximum in the narrow interval of $\omega=[10^8 ,10^{10}]$ for all the range of V .Then decreases nonlinearly gradually within the intervals $\omega =[10^{10},10^{11}]$, $V = [10^{-5}, 10^{-3}]$. Its amplitude is of the order 10^4 less than the external strong electric field E_0 ; see figure (11).

The z-component of the velocity :

At the beginning of the frequency $\omega=10^8$ the amplitude of v_z takes a zero value and increases suddenly to a maximum at $V=10^{-3}$. Within the interval $\omega =[10^{10},10^{11}]$, $V = [10^{-5}, 10^{-3}]$ it decreases nonlinearly gradually; see figure (12).

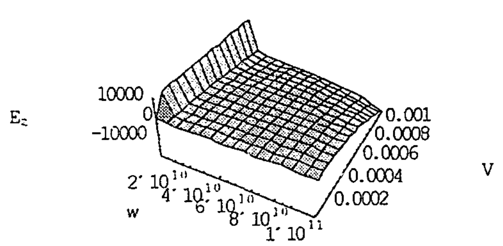


Figure (11): The E_z-component of electric field vs. the frequency and phase velocity.

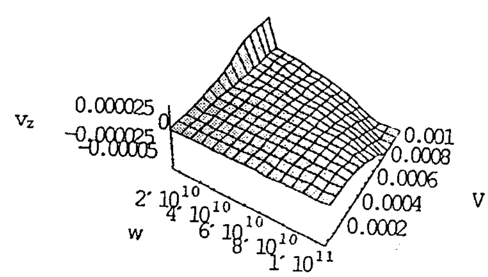


Figure (12): The v_z-component of the velocity vs. the frequency and phase velocity.

The internal energy change dU :

The change in internal energy of the electrons diminishes substantially in the course of time giving it to the propagated waves. It takes a

constant value within the distance interval z1 = [50, 100], this loss in the internal energy is of the order 10^{10} at $\omega = 10^8$ and $V = 10^{-5}$; see figure (13-a).

We notice that the loss in the internal energy change increases as ω increases at $V = 10^{-5}$; see figures (13-b,c). The value of dU is unchanged within the interval of the phase velocity; see figure (13-a,d).

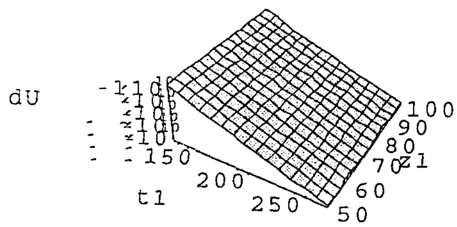


Figure (13-a) : dU at $\omega = 10^8$, $V = 10^{-5}$.

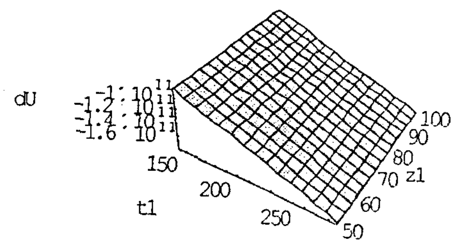
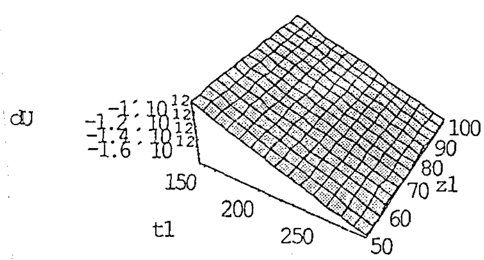
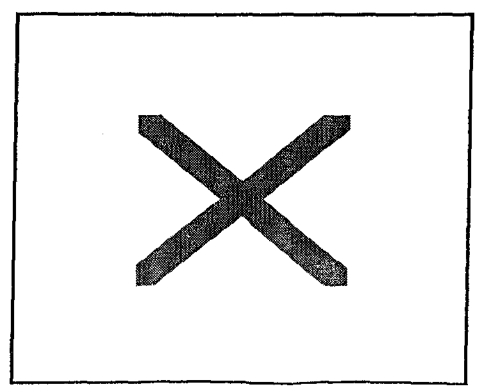


Figure (13-b) :dU at $\omega = 10^9$, $V = 10^{-5}$.

Figures (13, a-d) show the change in internal energy dU vs. the time and distance intervals.



(13-c) :dU at $\omega = 10^{10}$, $V = 10^{-5}$



(13-d):dU at $\omega = 10^8, V=10^{-4.5}$

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