CHARACTERISTICS OF SEEPAGE UNDERNEATH A HEADING-UP STRUCTURE AND A SUBSIDIARY ONE

خصائص التسرب تحت منشأ حجز و آخر مساعد

By

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خلاصة:

أحيانا يكون من الضرورى وجود منشا مساعد خلف منشا حجز رئيسى يكون الغرض من إنشائه تخفيف الضاغط الهيدروليكى على المنشأ الأساسى. فى هذا البحث تم دراسة الخصائص الهيدروليكية للتسرب اسفل المنشأين فى حالة حجز المياه أمامهما مع جفاف القناة فى الخلف. تمت الدراسة باستخدام نظرية العناصر المحيطة (العنصر الخطى). تم دراسة تأثير كلا من: طول الجزء المنفذ بين المنشأ الرئيسى والمساعد وكذا سمك الطبقة المسامية أسفل المنشآت. تناولت الدراسة تحديد قوة التعويم أسفل المنشات والتصرف و الميل الهيدروليكى للمسافة بين المنشأين وحلف المنشأ المساعد وتمثيل خطوط تساوى الضغط. تم تمثيل النتائج على صورة منحنيات لا بعدية.

ABSTRACT

Sometimes, it is necessary to build a subsidiary weir downstream the heading-up structures to relief the acting head. In the present study, water prevented between the main and subsidiary structures was considered. The boundary element technique using linear elements was used to analyze the uplift pressure underneath a simple flat floor of the heading-up structure and the subsidiary weir. Seepage flow and the exit gradients were also considered. Seven cases of the distance between the heading-up structure and the subsidiary one had been considered with five thickness of the permeable layer under the structure. Obtained results were illustrated and diseussed.

INTRODUCTION

A subsidiary weir is built downstream heading-up structures to relief the acting head during the various cases of operation. In this study, the water was prevented upstream the main structure during high water levels period while the water upstream the subsidiary one was on the crest level and its downstream was dry. Many analytical and experimental studies had been carried out to evaluate the uplift pressure distribution and exit gradients for different boundary conditions and floor configurations, Harr [9]. Chawla [4] and Gray and Chawla [8] presented analytical solutions using conformal mapping technique for a floor founded on a permeable soil of infinite and finite depths, respectively, provided with finite pervious inlet and outlet surfaces and a cut-off at any general position along the

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floor. The analysis however did not cover the case of very narrow permeable portions, which considered in this study with the presence of another structure such as a subsidiary weir, Fig. 1.

El-Masry [5,6] studied the characteristics of seepage underneath a heading-up structure with insufficient pervious length downstream the structure due to the presence of lining extended to the end of the canal. El-Masry and Abd-Allah [7] studied the same present problem for different conditions. They investigated the case of just after construction, which mean maximum water level upstream the main heading-up structure and no water up and downstream the subsidiary one.

The main objective of the study is, to study the development of uplift pressure, exit gradient and seepage flow, which affect the stability of heading-up structure and subsidiary one. Influence of changing the distance of the pervious length between the structures (S) and the permeable soil thickness beneath the floors (T) to be studied. The target is to develop the curves that express the relationships between the variables involved in this study.

In the present study, the Boundary Element Method (BEM) as a numerical technique was used to analyze the practical problem of seepage under simple flat floors of heading-up structure and the subsidiary one. The ratio between the up-stream head on the subsidiary structure (H_2) and the head on the main one (H_1) was considered to be constant and expressed as 0.4 (H_2=0.4H_1). The middle surface length between floors (S) varied from half to double the head (H_1) with an interval of 0.25. Different permeable layer thickness under the floor (T) was considered as: 0.80, 1.00, 1.20, 1.50 and 2.00 of the head (H_1). To define the considered domain (D), the inlet surface length (L) was assumed equal to the length of the first floor (L_1); end surface length (Z) was assumed 1.5 L_1. Length of subsidiary floor (L_2) was assumed to be half the length L_1. Soil beneath the structures was assumed to be homogeneous and isotropic with permeability coefficient (K). Herein, variables were allocated along the middle surface length (S) besides the thickness of soil layer (T), while other parameters were kept constant. Inflow upstream the main structure (Q_{i1}), inflow upstream the subsidiary structure (Q_{i2}), middle surface outflow (Q_{o1}) and end surface outflow (Q_{o2}) were computed and illustrated, Q is the flow per unit length.

MATHEMATICAL IDEALIZATION OF THE PROBLEM

A typical two-dimensional problem of flow through the fully saturated porous media is shown in Fig. 2. Using principles of continuity of incompressible flow and Darcy's law, the governing equation of seepage in a two-dimensional flow domain (**D**) can be described by Laplace equation (1).

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad , \text{ in } D \qquad (1)$$

in which:

$h = (p/\gamma + y)$ is the potential head, and

p/γ : is the pressure head, while y: is the elevation head.

For a confined domain of seepage flow, there are two types of boundaries:

1. A prescribed head on the upstream and downstream permeable surfaces (A-B, C-D and E-F),
2. A prescribed flux on boundaries (B-C, D-E, F-G, G-J and J-A).

The boundary conditions on A-B, C-D and E-F can be described by the following set of equations:

$h = H_1$	on A-B	(2-a)
$h = H_2$	on C-D	(2-b)
$h = 0$	on E-F (D.S. is dry)	(2-c)

The boundary conditions for the impervious boundaries can be described as:

$$\frac{\partial h}{\partial n} = 0 \qquad (3)$$

Where

$\partial h/\partial n$ is the hydraulic gradient in the direction perpendicular to the boundary surface.

BOUNDARY ELEMENT FORMULATION

In the boundary element method, the boundary of a flow region was divided into several linear segments connected by nodal points as shown in Fig. 2. The heads and the gradients on the boundary nodes were denoted as $\{h\}$ and $\{\partial h/\partial n\}$, respectively. The value of head or gradient at any point on a line segment between two nodes could be obtained by using interpolation functions $\langle N \rangle$ and $\{M\}$, Paris and Canas[12], as follows:

$$h = \langle N \rangle \{h\} \tag{4}$$

$$\frac{\partial h}{\partial n} = \langle M \rangle \left\{ \frac{\partial h}{\partial n} \right\} \tag{5}$$

where

$\langle\ \rangle$ and $\{\ \}$ represent a row vector and a column vector, respectively. In this presentation, $\langle N \rangle$ is chosen to be as $\langle M \rangle$, both being linear functions using Green's theorem, the volume integral of the Laplace equation can be reduced to a boundary integral, Brebbia [3]:

$$\alpha(x)\, h(x) \int_{\Gamma} \left\{ G(\zeta, x) \frac{\partial h}{\partial n}(\zeta) - F(\zeta, x)\, h(\zeta) \right\} d\Gamma(\zeta) \tag{6}$$

where

$h(x)$ is the potential at x, $G(\zeta, x)$ and $F(\zeta, x)$ represent the potential head and gradient at field point ζ due to a unit concentrated source at source point x (i.e., the fundamental solution), respectively. For two dimensional problems, Brebbia [3]:

$$G(\zeta, x) = \frac{1}{2\pi} \ln\left(\frac{1}{r}\right) \quad r = |\zeta - x| \tag{7}$$

$$F(\zeta, x) = \frac{1}{2\pi} \frac{\partial r}{\partial n} \tag{8}$$

Using the rigid-body analogy, Banerjee and Butterfield [1], the value of α can be evaluated by:

$$\alpha(x) = \begin{cases} 1 & x \text{ in } D \\ -\int F(\zeta, x) d\Gamma(\zeta) & x \text{ on } \Gamma \\ 0.5 & x \text{ on } \Gamma \text{ and } \Gamma \text{ is smooth} \end{cases} \tag{9}$$

Substituting the interpolation functions into eq. (6). The relationship between heads and gradients at the boundary nodal points of a given domain is given by:

$$[H]\{h\} = [G]\left\{ \frac{\partial h}{\partial n} \right\} \tag{10}$$

In the above, the matrices [H] and [G] are obtained from:

$$[H]=[\delta]\{\alpha(x^m)\}-\int_\Gamma\{F(x^m,\zeta)\}\langle N(\zeta)\rangle\,d\Gamma(\zeta) \tag{11}$$

$$[G]=\int_\Gamma\{G(x^m,\zeta)\}\langle N(\zeta)\rangle\,d\Gamma(\zeta) \tag{12}$$

in which:

δ : is the Kronecker delta, $\mathbf{x}^m$: is the point of node m,

ζ : is the field point on the boundary surface Γ and

$\langle N(\zeta)\rangle$: is the interpolation function.

In a boundary value problem, either gradient or potential head is known for a given node on the boundary. Therefore, eq.(10) gives a set of simultaneous equations that can be solved for the unknown variables. The boundary element method which has been outlined for a homogeneous flow domain is fairly standard and can be found in the literature, [1,2,10,11].

The use of linear elements on the boundary leads to a problem at corner points, which have two values for the head h and the normal derivative $\partial h/\partial n$ depending on the side under consideration. At these points, it is essential to select which of the two variables h or $\partial h/\partial n$ will be prescribed. As $\partial h/\partial n$ can not be defined, one generally will choose to prescribe h. This however, does not produce a very accurate computed value for the derivatives at the corners. This problem does not occur in finite elements due to the way in which the natural boundary conditions are prescribed and the fact that the solution is also approximated in the domain, i.e. errors tend to be more distributed. To avoid the corner problem, it is considered that, there are two points very near to each other but belong to different sides Fig. 2. At one node, h condition is prescribed while $\partial h/\partial n$ is prescribed at the other one.

RESULTS AND DISCUSSION

In this study, thirty-five runs were computed using the boundary element method. For the considered study, the following parameters were considered:

1. Relative distance (S/H_1) varied from 0.50 to 2.0 (0.50, 0.75, 1.0, 1.25, 1.50, 1.75 and 2.00).
2. Relative thickness of the permeable layer (T/H_1) changed from 0.8 to 2.0 (0.8, 1.0, 1.2, 1.5 and 2.0).

From the obtained results, Figs. 3 & 4 were plotted to show the relationship between the relative discharge (Q/KH_1) and both of the relative thickness of.the permeable layer (T/H_1) and the relative distance (S/H_1). Figure 3 shows that, increasing the relative thickness of the permeable layer increases the upstream inflow (Q_{i1}) and decreases the downstream inflow (Q_{i2}). Increasing the separate distance, S, increases the downstream inflow and decreases the upstream inflow.

From Fig.4, it is observed that, increasing the permeable layer thickness increases the downstream out flow (Q_{o2}) while the flow seeps downstream the first floor decreases. Increasing the separate distance, S, increases the out flow downstream the first floor, (Q_{o1}), while the final out flow that seeps at the rear of the downstream structure, Q_{o2}, decreases. For the considered cases, the computed error between the summation of the inflow discharge, ($Q_i = Q_{i1} + Q_{i2}$) and the total out flow ($Q_o = Q_{o1} + Q_{o2}$) is considered good as the error is not more than 0.30%.

Figure 5 shows the distribution of the computed inflow-outflow seepage discharge for two cases of $S=H_1$, $T=0.8\ H_1$, and $S=2H_1$, $T=2H_1$. From these figures, it is observed that, the separate length, S, is divided into outflow and inflow portions at point O, that point lies at about 0.55 S apart from the downstream end of the first floor.

Figure 6 shows the variation between Q/KH_1 versus S/H_1 for $T=0.8\ H_1$. In this figure, inflow-outflow curves are represented with the summation of them. From these curves, it is clear that, the summation of the inflow discharge ΣQ_i is coincided with the outflow one, ΣQ_o.

The computed uplift pressures that acting upon the main and subsidiary structures for one case of the separate distance, $S = H_1$, for all the considered thickness of the permeable layer, T/H_1 which are illustrated as shown in Fig. 7. From this figure, it is clear that, changing the relative thickness of the permeable layer has no effect on the distribution of the uplift pressure.

The influence of the relative thickness of the permeable layer on the exit gradient at the separate distance, S, is illustrated for the case of $S/H_1=2.0$ as shown in Fig. 8. From this figure, it is clear that, changing the relative thickness T/H_1, has no considerable effect. On the other hand, Fig.9 shows the illustration of the hydraulic gradient at exit (downstream the first floor) for the case of $T/H_1=2.0$. From this figure, one can observe that, increasing the distance S, increases the values of exit gradient. From these Figs. 8 & 9, it is observed that, the distribution of exit gradient curve can be divided into two portions, the first one that lies downstream the first floor, the second lies at the distance upstream the second floor. The point that divided the two portions was found to be at about 0.55 S measured from the downstream end of the first floor. These two portions represent the outflow and inflow gradients, respectively.

Hydraulic gradients at the rear exit surface downstream the second floor are represented as shown in Fig. 10. Considering the case of $T/H_1 = 2.0$, changing the relative distance $S/H_1 = 0.5$ to 2.0, the obtained hydraulic gradients are illustrated. At the second floor end, the exit gradients tend to have an infinite value.

As a result of the computed cases, equipotential lines are plotted for three cases as shown in Fig. 11.

CONCLUSIONS

Characteristics of seepage underneath a heading up structure and a subsidiary one was studied numerically using the well-known Boundary Element Method. Spacing between the structures and the permeable layer thickness were considered. Thirty-five cases were considered taking into account the above mentioned parameters.

According to the present study, it can be concluded that:

- For the cases studied, the presence of the subsidiary structure divided the outflow discharge into two parts, downstream the first floor and downstream the rear one.
- The separate distance between the heading-up structure and the subsidiary one have been divided into two portions, one belongs to the outflow discharge downstream the first floor, and the second belongs to the inflow discharge from the prevented water upstream the subsidiary structure.
- The outflow portion of the separate distance is about 0.55 its total length.
- Increasing the permeable layer thickness leads to the following:
  - increasing the inflow upstream the first floor,
  - increasing the outflow downstream the rear floor,
  - decreasing the inflow upstream the second floor, and
  - decreasing the outflow downstream the first floor.
- The separate distance between the heading-up structure and the subsidiary one had a reverse influence comparing with the thickness of the permeable layer for inflow-outflow discharge.
- The permeable layer thickness had no a considerable effect on the uplift pressure under the floor of the structures.
- The hydraulic gradient for the separate distance between the structures represents the inflow – outflow portions.
- Increasing the separate distance increases the exit gradient.
- Equipotential lines have been illustrated as a result of the present solutions.
- More future studies considering shorter and longer separate distance between the structures and the effects on seepage streams should be accomplished.

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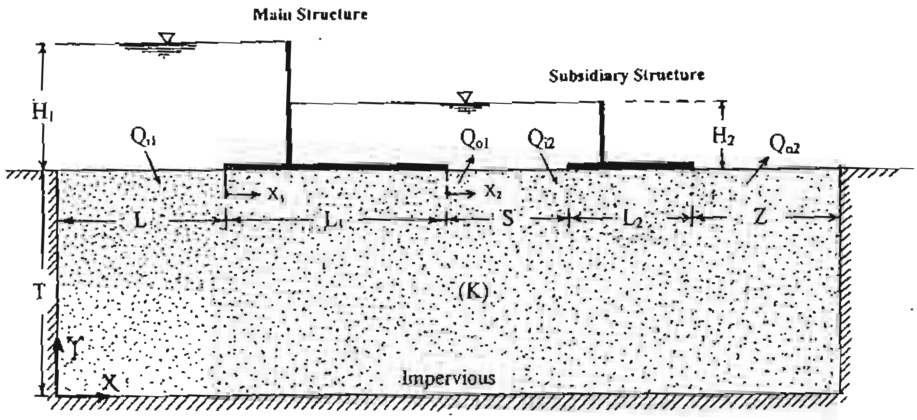


Fig. 1 Layout of the considered problem.

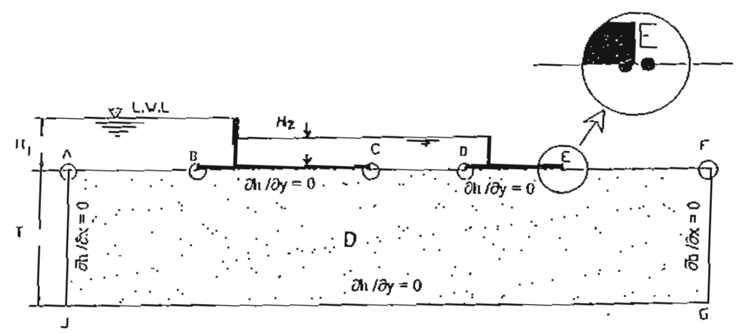


Fig. 2 Idealization of the problem.

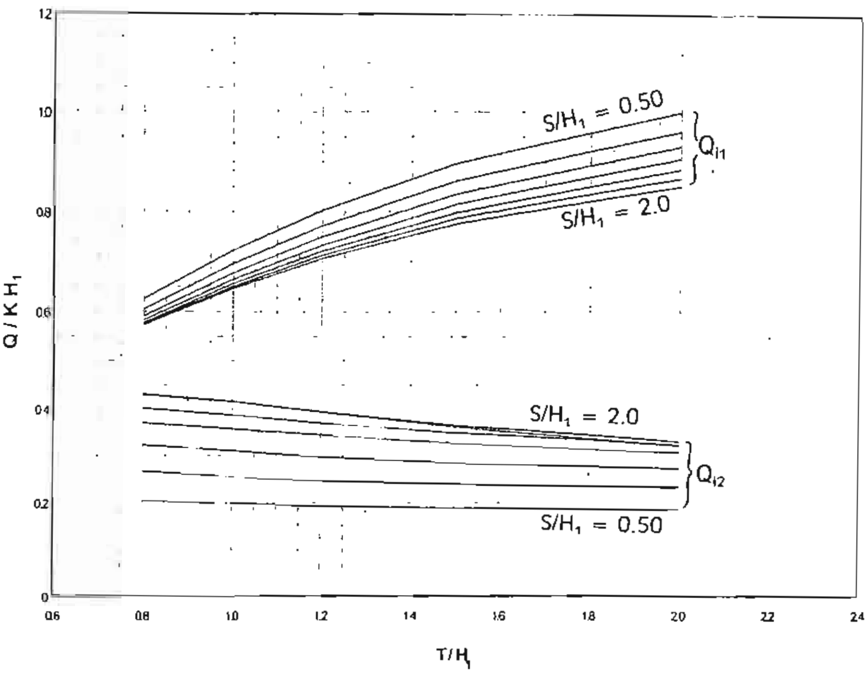


Fig. 3 Inflow upstream the first and the second floors for all S/H_1 ratios.

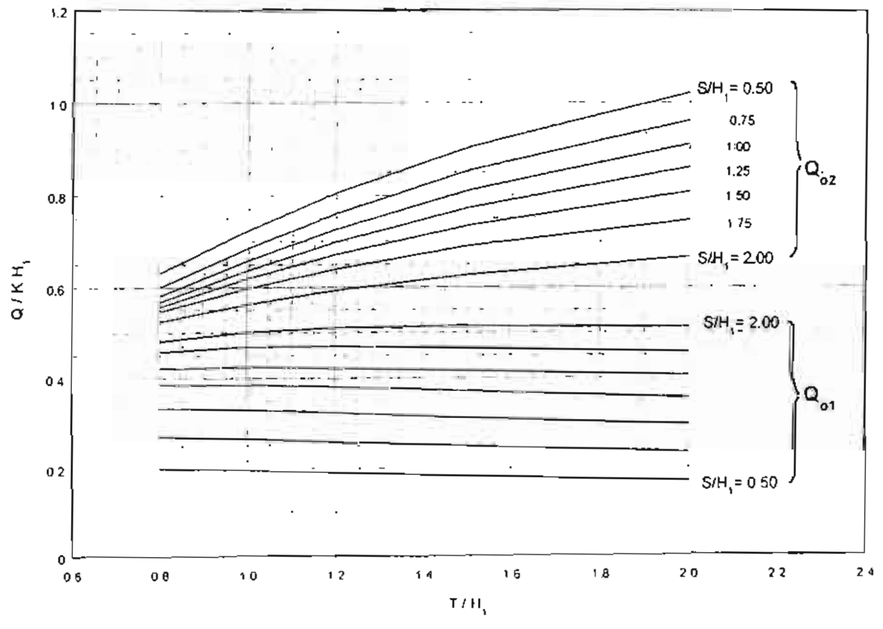


Fig.4 Outflow downstream the first and the second floors for all S/H_1 ratios.

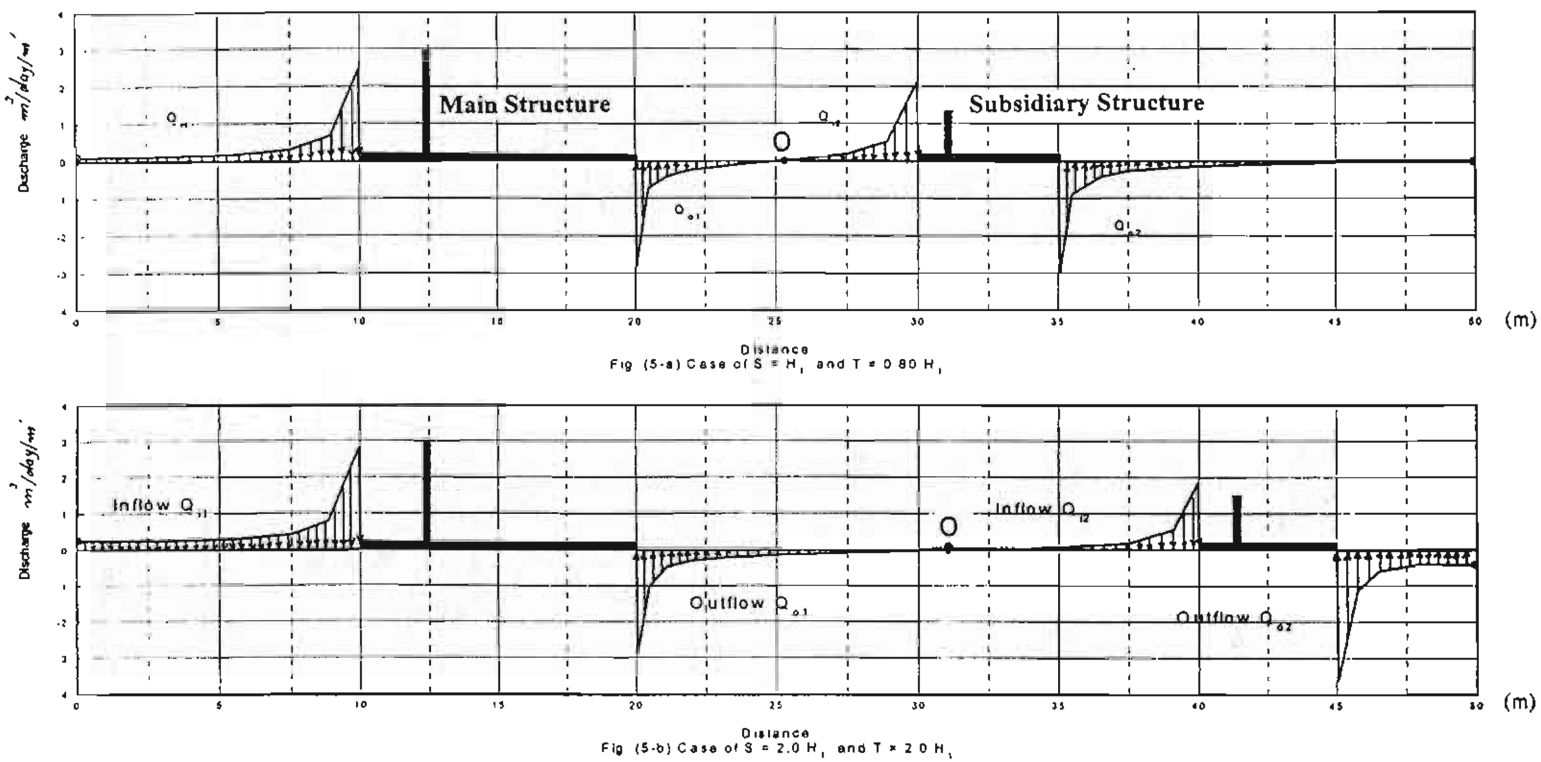


Fig. (5) The Distribution of The Inflow - Outflow Seepage Discharge

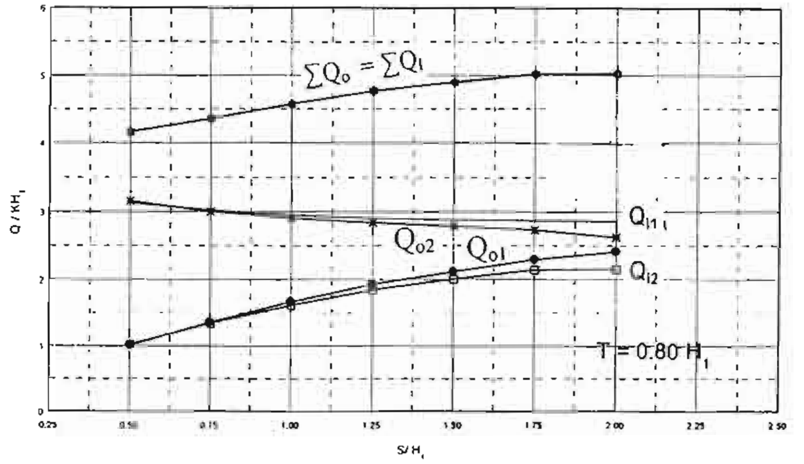


Fig. 6 Inflow- Outflow versus S/H_1 for $T/H_1 = 0.80$.

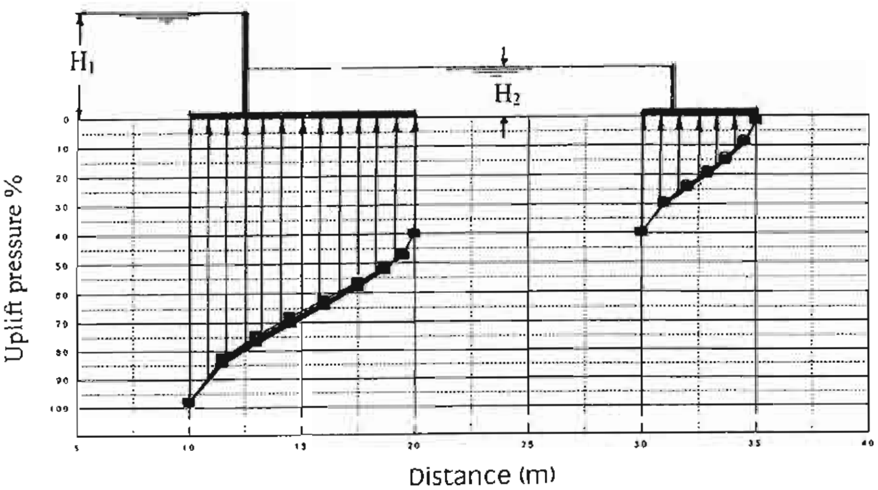


Fig. 7 Total uplift pressures on the first and second floors
For the considered cases of T/H_1, $(S=H_1)$

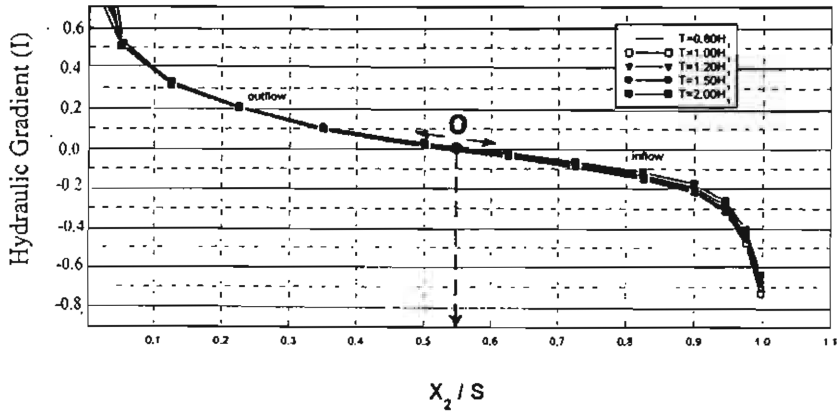


Fig. 8 Hydraulic gradient downstream the first floor ($S/H_1 = 2.0$)

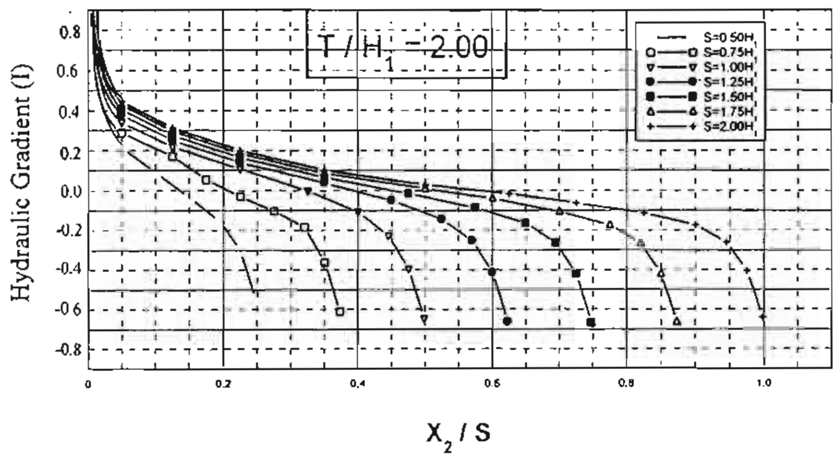


Fig. 9 Hydraulic gradient downstream the first floor ($T/H_1 = 2.0$)

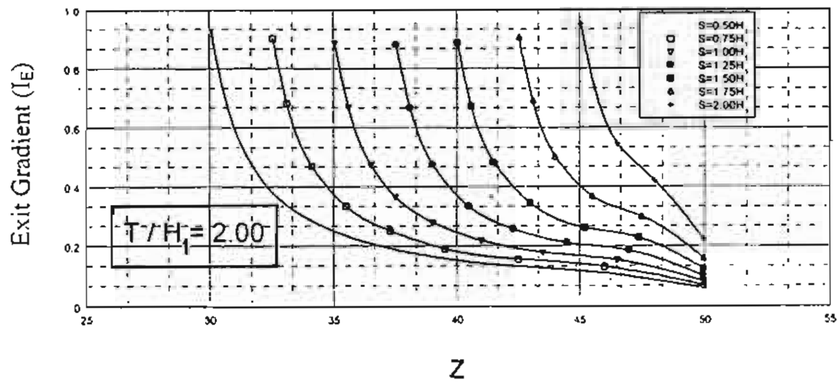
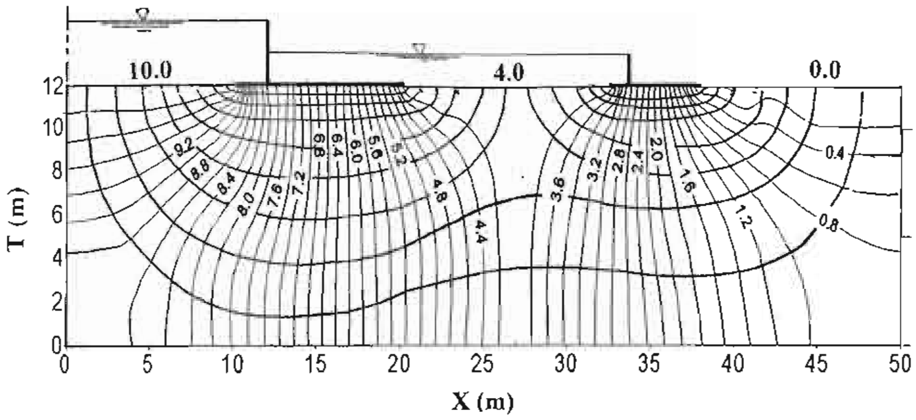
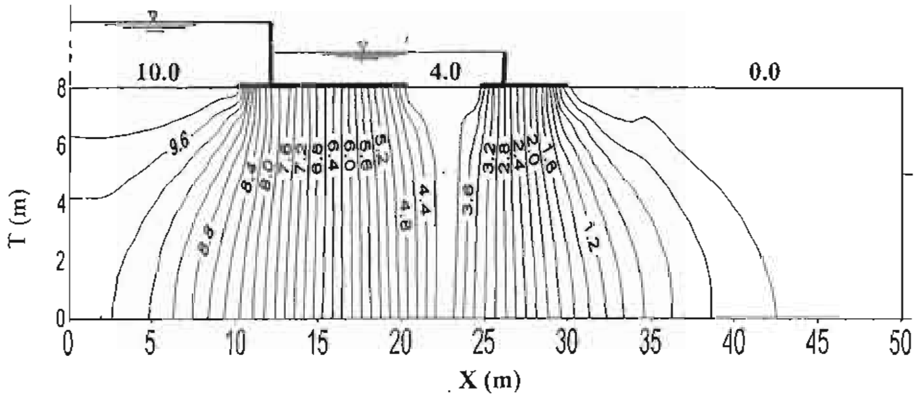


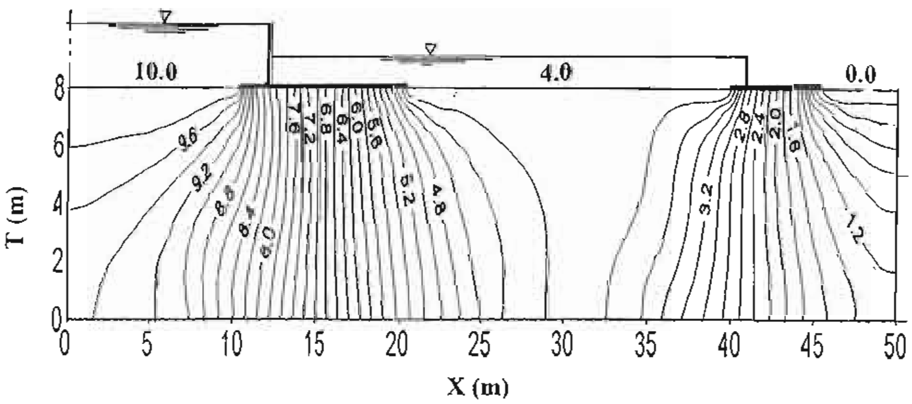
Fig. 10 Hydraulic gradient downstream the second floor ($T/H_1 = 2.0$)



Flow-net for the case of T=1.2 H_1 and S = 1.25 H_1



Equipotential lines for case of T = 0.8 H_1 and S= 0.5 H_1



Equipotential lines for case of T = 0.8 H_1 and S= 2.0 H_1

Fig. 11 Flow net and equipotential lines for some cases