SHADING EFFECT OF A SOUTH BUILDING ON THE SOLAR COLLECTOR APERTURE

تأثير الظلال الناتجة عن مبنى جنوبي

على السطح الماص لمجمع شمسي

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الخلاصة – يتناول هذا البحث النظرى تأثير الظلال الناتجة عن وجود مبنى جنوبي على السطح الماص لجهاز طاقة شمسية في النصف الشمالي للكرة الارضية . وقد تم اعداد نموذج رياضي لحساب قيمة مساحة الظل لحظيا كدالة في بعد المبنى وارتفاعه وعرضه وكذلك زاوية حيوده عن الجنوب وأبعاد السطح الماص . وقد تم عرض نتائج البحث بيانيا على صورة معامل الظل (النسبة بين الاشعاع الكلي الفعلي على السطح والذي بدون ظلال) لحظيا والقيمة المتوسطة له يوميا وسنويا كدالة في الابعاد الهندسية في صورة لا بعدية لكي يسهل تطبيقها في جميع الظروف المشابهة .

ABSTRACT - In the present work, the shading effect of a south building on the aperture of a solar device in the north latitudes is theoretically investigated. A mathematical model to calculate the instantaneous value of shaded area as a function of building horizontal distance, height, width and the angle of deviation from south direction as well as aperture dimensions has been obtained. The shading factor, which is the ratio between the actual total incident solar radiation and that without shading on the aperture, has been calculated for different geometrical parameters. The annual average shading factor, which may be considered as a reasonable measure of the shading effect is also given in terms of the system dimensionless parameters.

INTRODUCTION

In large cities, buildings are usually crowded, making shadows on each other and may produce a shading effect on the aperture of an existing solar device. This fact is one of the most important reasons why the majority of city residents do not prefer the solar choice. On the other hand, the reported work in this field is very little compared to the problem size, and is not sufficient in practice. For example, an analytical method for calculating daily and monthly average insolation on overhanging shaded windows of arbitrary azimuth is presented by Jones [1]. An extension of this work for a finite width overhang has been reported [2]. The problem of window shaders has also been handled by Barozzi and Grossa [3], using the numerical technique.

On the other hand, the shading effects of large scale solar systems are studied by Jones and Burkhart [4]. An analytical method for calculating the daily total radiation on rows of fixed collectors facing equator has been reported. The effect of row length is neglected, since it is pronounced only close to sunrise and sunset as reported. Similar work on solar cell array has been performed by Feldman et al [5]. They proposed a non-regular distribution of cells to reduce the shadow effects. Mathematical models for shading calculations, suitable for computer - aided design of complex systems are also available [6].

However, the shading problem of buildings on solar devices seems to be very significant, especially in cities and towns. This work is therefore aimed to investigate the effect of a south building shading on a solar device, since the style of buildings is mostly east - west, in north latitudes. A simple model and practical graphical results are also presented.

THEORETICAL APPROACH

An aperture of a solar device of width W, and height L is fixed in a place where the latitude is ϕ, with an angle of inclination β, to the horizontal and facing south. In front of the aperture, there exists a nearby building at a horizontal distance from the aperture bottom edge of z times the aperture height (i.e. = z L). The building height is equal to v L, while its width is d W, and is assumed to have an east-west direction. The deviation of building from the south direction is measured by the angle c, between the line connecting the mid points of the bottom aperture edge and the building width, and the south directioin in a horizontal plane. The system geometry is shown in Fig. 1. In the elevation view, the significant parameters which explain the geometrical relations between the beam solar radiation, aperture and the building are the extreme values of the profile angles λ_o and λ_f, where,

$$\tan \lambda = \frac{\tan \alpha}{\cos(\phi - \gamma)} , \qquad (1)$$

where α, is the solar altitude angle and γ, is the aperture azimuth angle which is equal to zero (aperture facing south).
In the plan view, the system geometry is characterized by the angle c, and extreme values of the solar azimuth angle a_o and a_f.

The shaded area for a certain aperture-building geometrical

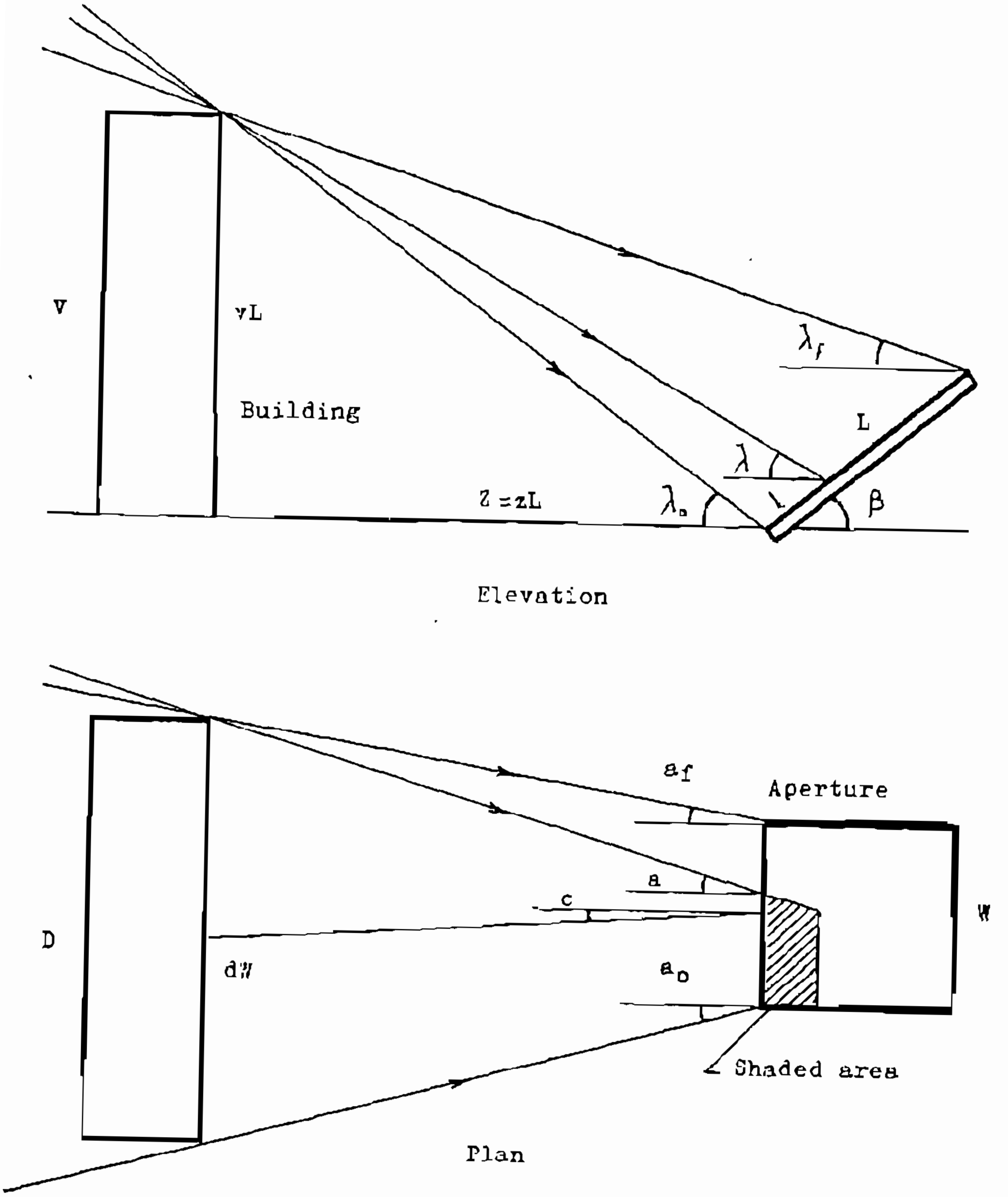

Fig. 1. Building - aperture system geometry.

configuration is dependent only on the angles λ, and a, as shown in the figure. The aperture is fully illuminated if $\lambda \geq \lambda_o$, while it is completely shaded if $\lambda \leq \lambda_f$ (and $a \leq a_f$). The aperture is partly shaded if $\lambda_o \geq \lambda \geq \lambda_f$, and $a_o \geq a \geq a_f$, where

$$\tan \lambda_o = v / z \qquad \text{and} \qquad \tan \lambda_f = \frac{v - \sin \beta}{z + \cos \beta} \tag{2}$$

It can be shown from the front view geometry that the profile angle λ, is related to z, v and β, at any time by

$$\tan \lambda = \frac{v - (1/L) \sin \beta}{z + (1/L) \cos \beta} , \tag{3}$$

where l, is the shade length in the aperture height direction. Equations 1, 2 and 3 could be used to obtain the value of (l/L), in terms of the angle λ, as

$$(l/L) = \frac{\tan \lambda - \tan \lambda_o}{\tan \lambda_f - \tan \lambda_o} \times \frac{\cos \beta \cos \lambda_f - \sin \beta}{\cos \beta \cos \lambda - \sin \beta} \tag{4}$$

This equation is valid only when $\lambda_o \geq \lambda \geq \lambda_f$. The value of (l/L) is equal to zero when $\lambda \geq \lambda_o$, and is unity if $\lambda \leq \lambda_f$.
From the plan system geometry, the following relations can be easily derived,

$$\tan a_o = \frac{d - 1}{2 e z} + \tan c \quad ; \tag{5}$$

$$\tan a_f = \frac{d - 1}{2 e z} - \tan c \quad ; \tag{6}$$

where $e = L/W$
The shade length at the aperture lower edge in the width direction w, is given by,

$$(w/W) = 1 + e z (\tan a_o - \tan a) \qquad \text{if} \quad a \geq a_o \quad ; \tag{7}$$

$$\text{and} \quad (w/W) = e d - e z (\tan a_o + \tan a) \quad , \quad \text{if} \quad a \leq a_f . \tag{8}$$

Equation 7 is valid only a.m., while the other is valid only p.m. The value of (w/W) = 1 if $a_o > a > a_f$ and zero otherwise.
The ratio A_r, of the shaded area to that of the aperture can be approximated by,

$$A_r = (l/L) (w/W) \tag{9}$$

The "ASHRAE" Handbook of Fundamentals [7] recommends the following equation for the prediction of the normal beam solar radiation H_{bn},

$$H_{bn} = H_o \exp(- B/\sin \alpha) \quad , \qquad (10)$$

where B, is the atmospheric extinction coefficient.
The values of H_o and B are representative of conditions on average cloudless days for north latitudes (from 0 to 64 degrees) The "ASHRAE" Handbook of fundamentals also gives a simplified general relation for the diffuse solar radiation H_{da}, from a clear sky that falls on any terrestrial surface as,

$$H_{da} = 0.5 \, f \, H_{bn} (1 + \cos \beta) \quad , \qquad (11)$$

where f, is the diffuse radiation factor which is given numerically for each month in the Handbook.
The average insolation on the aperture surface is given by,

$$H_a = H_{bn} (1 - A_r) \cos \theta + 0.5 \, f \, H_{bn} (1 + \cos \beta) \quad , \qquad (12)$$

where θ, is the incident angle of beam radiation on the aperture. The shading effect of the beam radiation is taken into account in the above equation, where the value of A_r = 0 to 1. However, a shading factor ζ, may be defined by the ratio between the actual insolation on the aperture surface and that without shading,

$$\zeta = \frac{(1 - A_r) \cos \theta + 0.5 \, f (1 + \cos \beta)}{\cos \theta + 0.5 \, f (1 + \cos \beta)} \qquad (13)$$

According to this equation, the shading factor is only a function of time for any given aperture-building configuration. A computer program is systematically constructed to calculate ζ, with a time step of 0.25 hour, during every day over a year for different values of system geometrical parameters as shown in Fig. 2.

RESULTS AND DISCUSSION

Because the problem involves a large number of parameters, it is necessary to keep some of them constant and investigate the influence of the others. On the other hand, the instantaneous values of shading factor during any day show only its behavior in the same day, but the daily average value is expected to be more useful in calculating the energy gain. However, in order to investigate the influence of system different parameters, the annual average value of the shading factor is essentially

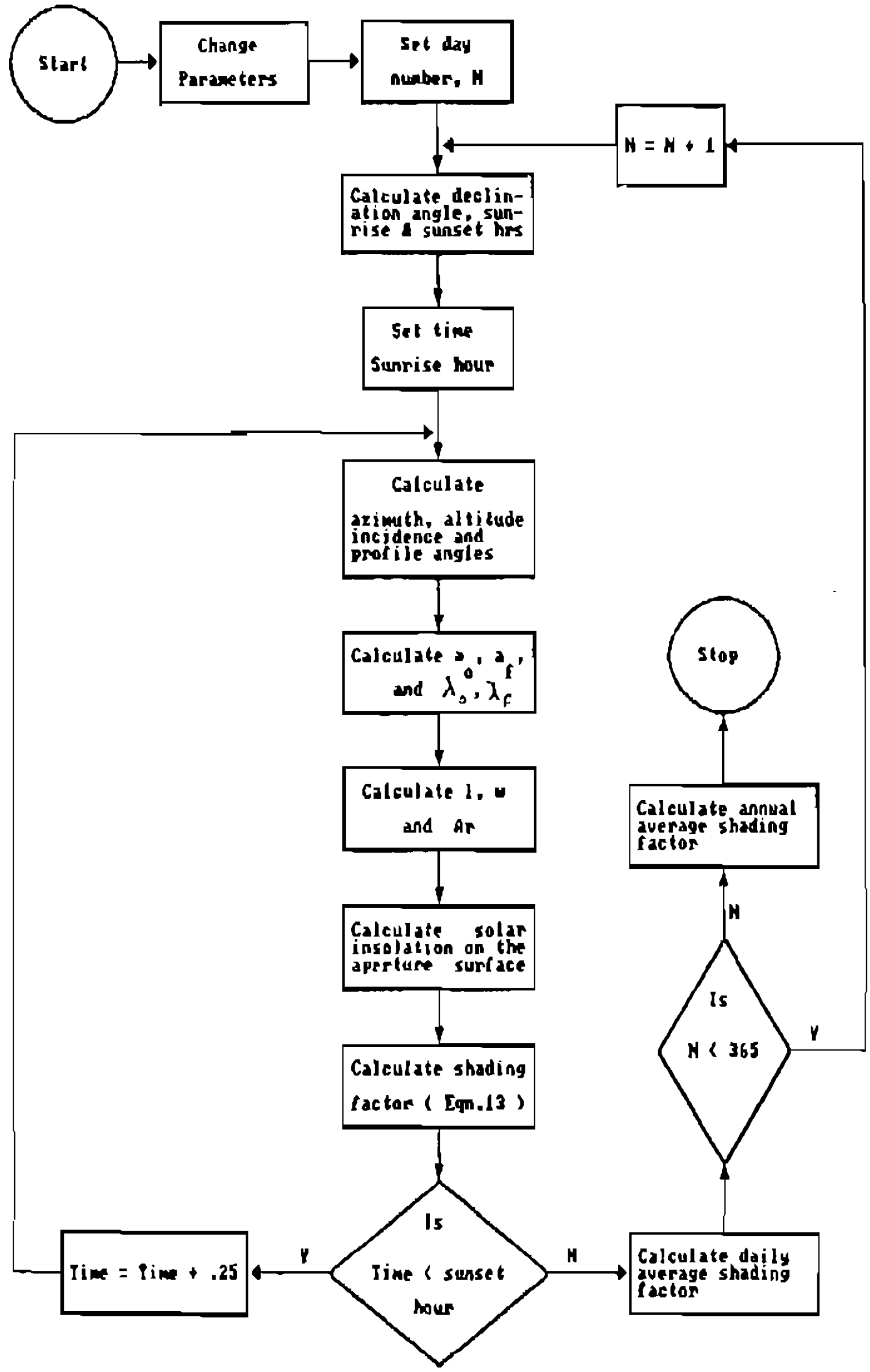

Fig. 2. The flow diagram of the computer program

required. Results of this analysis, which are given in graphical form, show the hourly values of shading factor in some days, daily and the annual average values under different parameters.

Figure 3. shows the hourly values of the shading factor resulting from a very long horizontal object at a distance L, from the aperture lower edge ($z = 1$). This object may be a building or an east-west wall in front of the aperture. The shading factor is plotted for different building heights during a summer day ($N = 172$). For all values of v/z, the shading factor decreases with time, from unity, reaching a minimum value at noon and then increases again to unity. It is clear also that shading factor decreases with increasing height. It is to be noted that the flat minimum in the case of $v/z = 20$, is only due to diffuse radiation where the aperture is completely shaded during this period. Figures 4 and 5. show similar results for a winter and a spring days respectively. In winter, the shading factor has a maximum value at noon for small building heights as seen in Fig. 4. However, in equinoxes, it is almost independent of time as shown in Fig. 5, where $N = 80$.

An example of of the effect of building width on the hourly values of the shading factor is given in Fig. 6. This figure gives results for the day $N = 172$, where the specific values of z, v and c are 1, 13 and 0 respectively. As expected, the period during which shading factor is less than unity, increases with the building width. Also, all curves representing different values of d, are symmetrical around the noon line. However, figures from 3 to 6 are given under specially decided parameters to show reasonable variations of shading factor to demonstrate its daily behavior. Otherwise, the shading factor is either unity or at its minimum corresponding to the diffuse radiation.

On the other hand, the daily average value of the shading factor for any system can be very useful in estimating the output energy distribution over the year to compare with the energy demand. In large cities, the apparent picture of building shading may be not encouraging,but the actual energy distribution may fit the demand. Calculations can also be used for an already existing solar device. Figure 7. is an example of daily average shading factor for a system with $z = 10$, $d = 20$ and $c = 0$, at different values of v/z, over the year. As seen from the figure, the shading factor is at its minimum value during winter months if $v/z = 1$, then it increases rapidly to unity in other days. For values of $v/z > 1$, shading factor increases gradually from its minimum and then rapidly. However,the shading factor is unity all over the year if $v/z \leq \tan^{-1}(1.16-\phi)$, for all values of c and d.

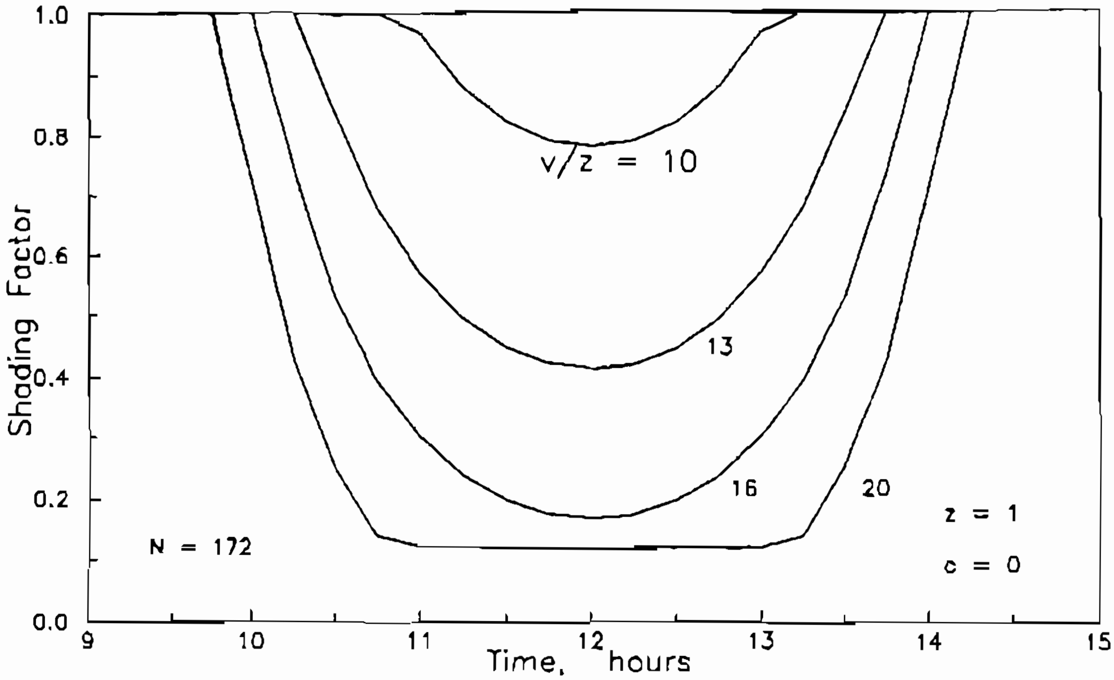

Fig. 3. Hourly variation of shading factor for a summer day.

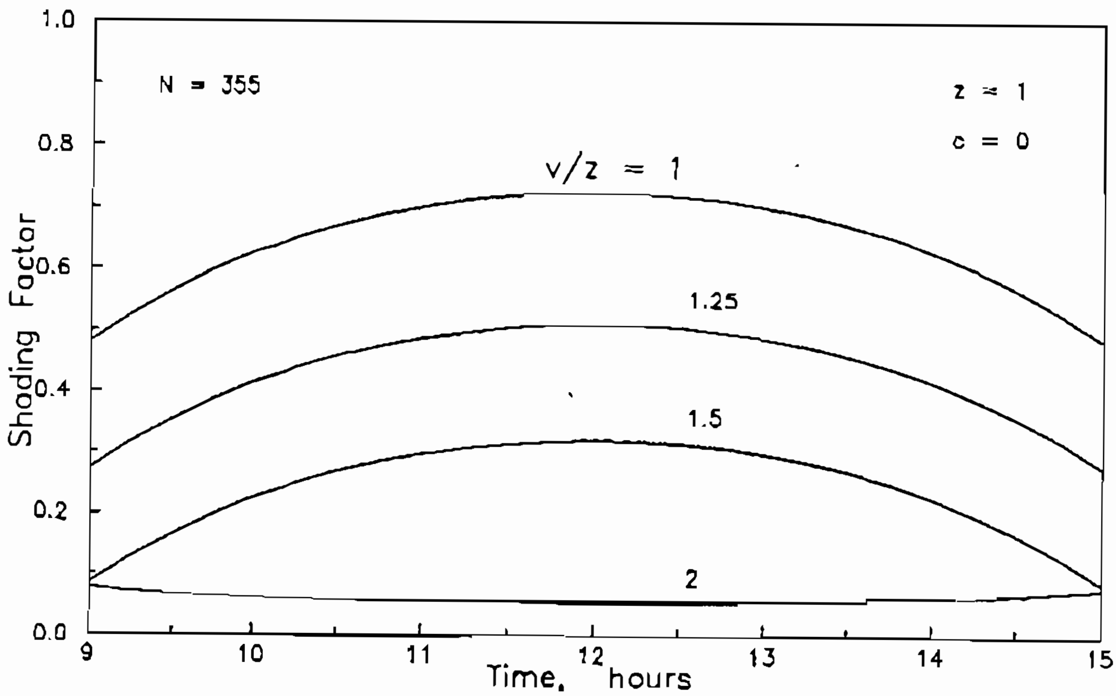

Fig. 4. Hourly variation of shading factor for a winter day.

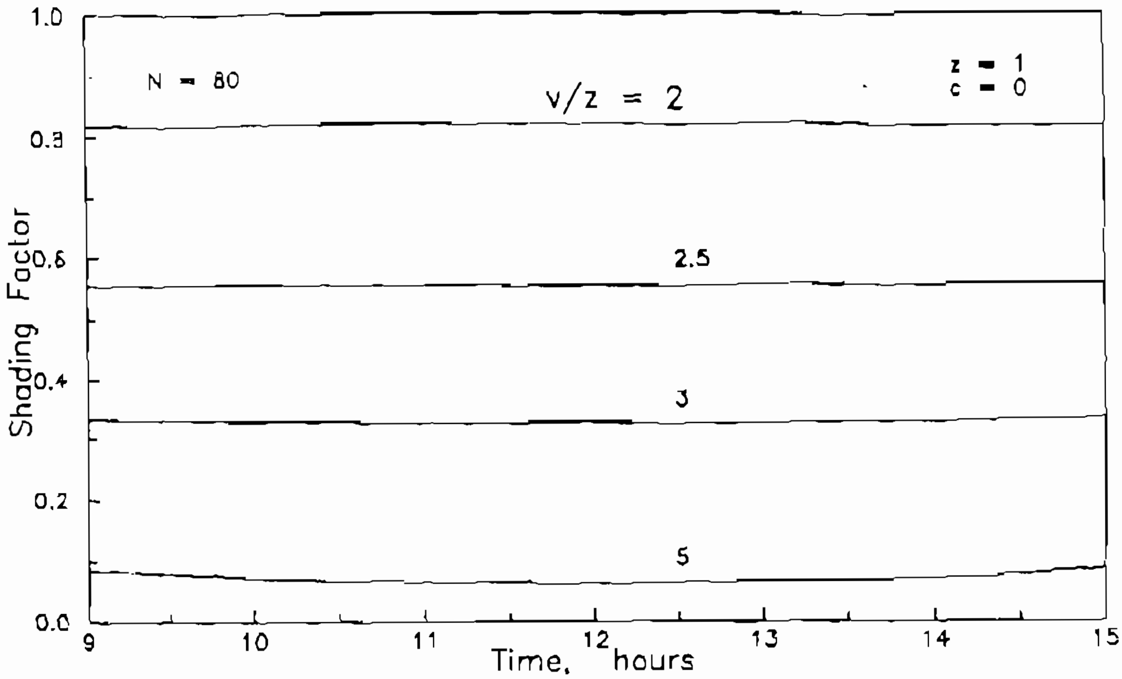

Fig. 5. Hourly variation of shading factor for a spring day.

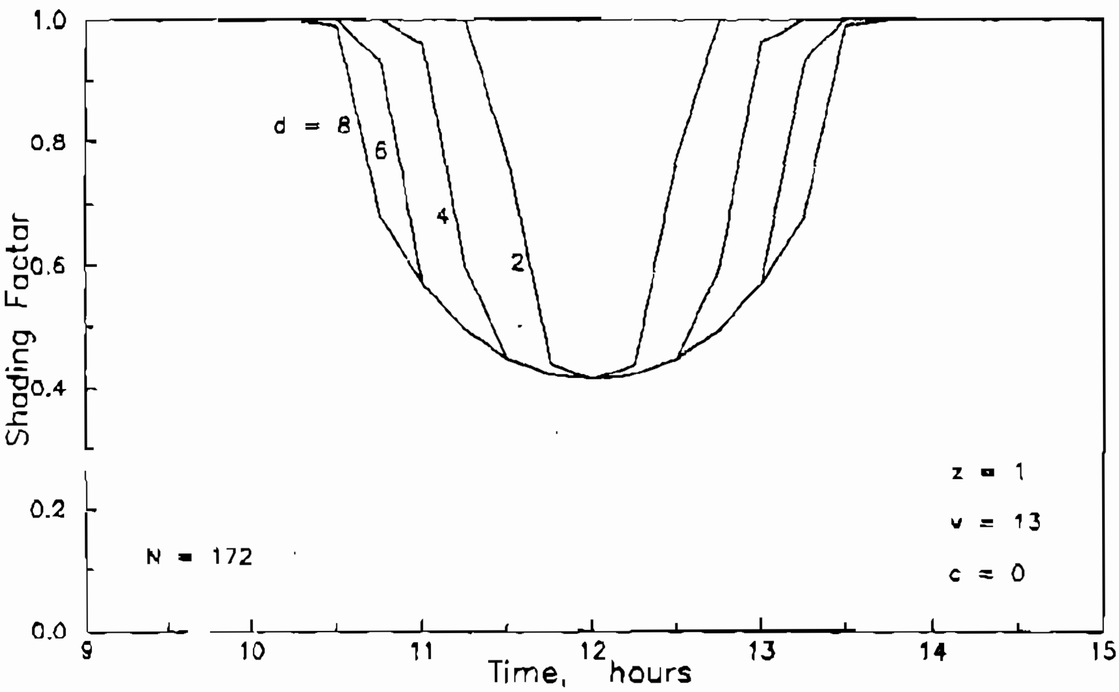

Fig. 6. Hourly variation of shading factor in a summer day at different values of building widths.

The effect of building width on the shading factor for the case of $v/z = 2$, is shown in Fig. 8. It is clear that the number of days with unity shading factor is not affected by the building width. Only the change from minimum is seen to be more gradual, and the value of minimum shading factor increases with smaller building widths. The shading factor is of course unity when d=0.

However, the average value of shading factor over a year is more practical for design purposes. Figure 9. shows the annual shading factor as a function of z, at different values of v/z, for d=10 and c=0. Since z, v and d are dimensionless parameters, this graphical representation can be used to determine shading factor for any system geometry having the same values of d and c. As an example, the annual shading factor resulting from a building with height to horizontal distance of unity, and at a distance of 5 times the aperture height is about 0.8. In this case, a layman may think that this site is not suitable for any solar system.

Figure 10. shows the same results at different values of building widths (d = 5, 20 and 40), for a wide range of horizontal distances($z \leq 20$). The shading factor decreases with the increase in building width for all values of v/z. This effect increases with the horizontal distance. Also, it can be seen from the figure that the shading factor decreases with z, to a minimum value and then increases again and tends to have a constant value at larger z, for any v/z. The value of z, at which this minimum occurs increases with the building width. Finally, the effect of angle c, on the shading factor for the system geometry with d=z, (i.e. the building width is equal to the horizontal distance) is shown in Fig. 11. The angle c, is seen to have a marked effect on the shading factor. It increases rapidly with c, where its value approaches unity for $c = 60°$, as shown in the figure.

CONCLUSIONS

The shading effect of a south building block on the aperture of a solar device in the north latitudes (0-64°) is theoretically investigated. A mathematical model to calculate the shading factor, which is defined by the ratio of actual total solar radiation on the aperture surface to that without shading, as a function of system geometrical parameters is presented. The hourly, daily and annual average values of shading factor, which can be easily calculated by this model, may be of great help when a site is to be tested for the installation of a solar device in large cities. Results which are given in graphical form can be used in a quick and accurate estimation of shading factor for any

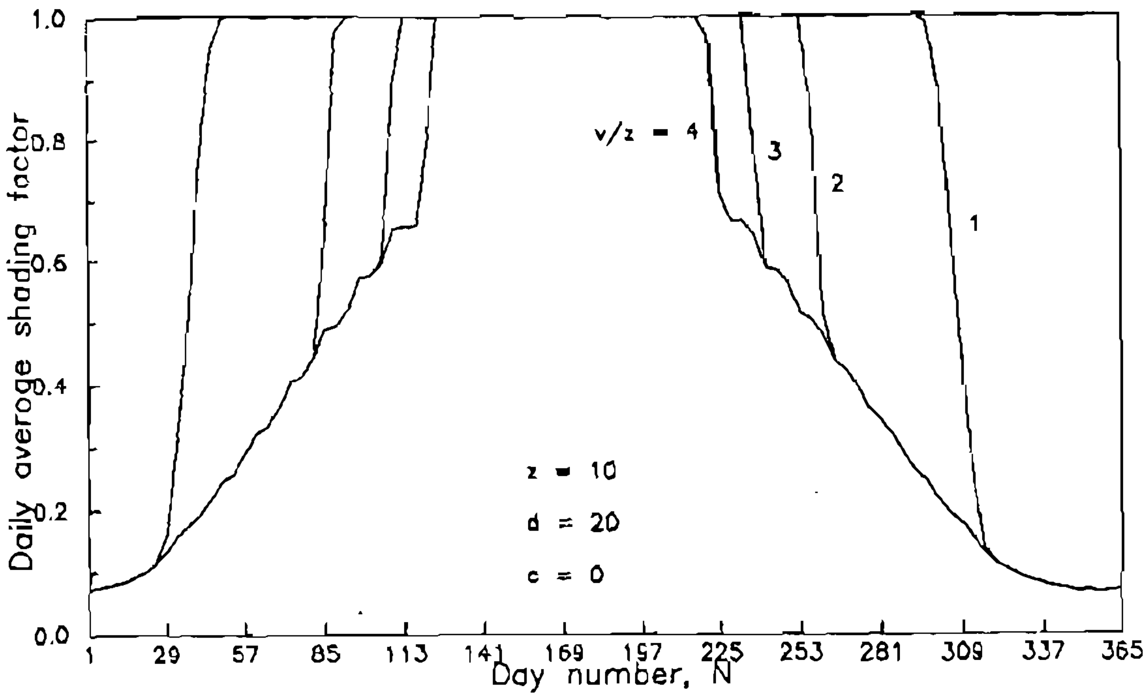

Fig. 7. Daily average shading factor for different building heights.

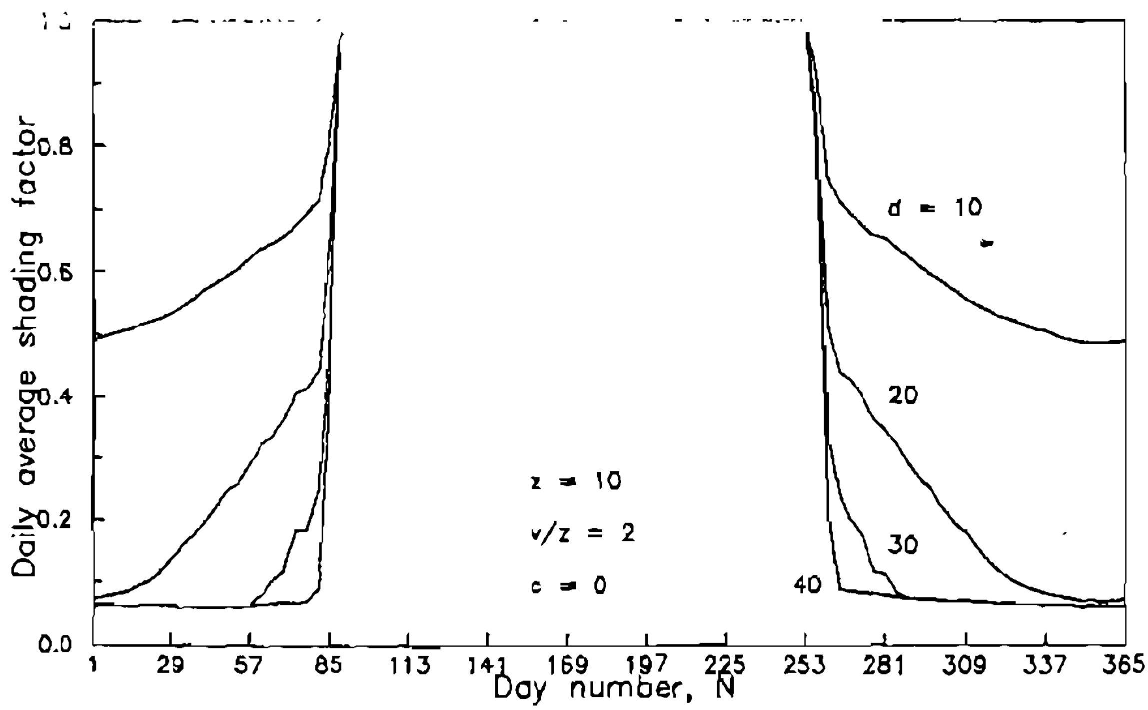

Fig. 8. Daily average shading factor for different building widths.

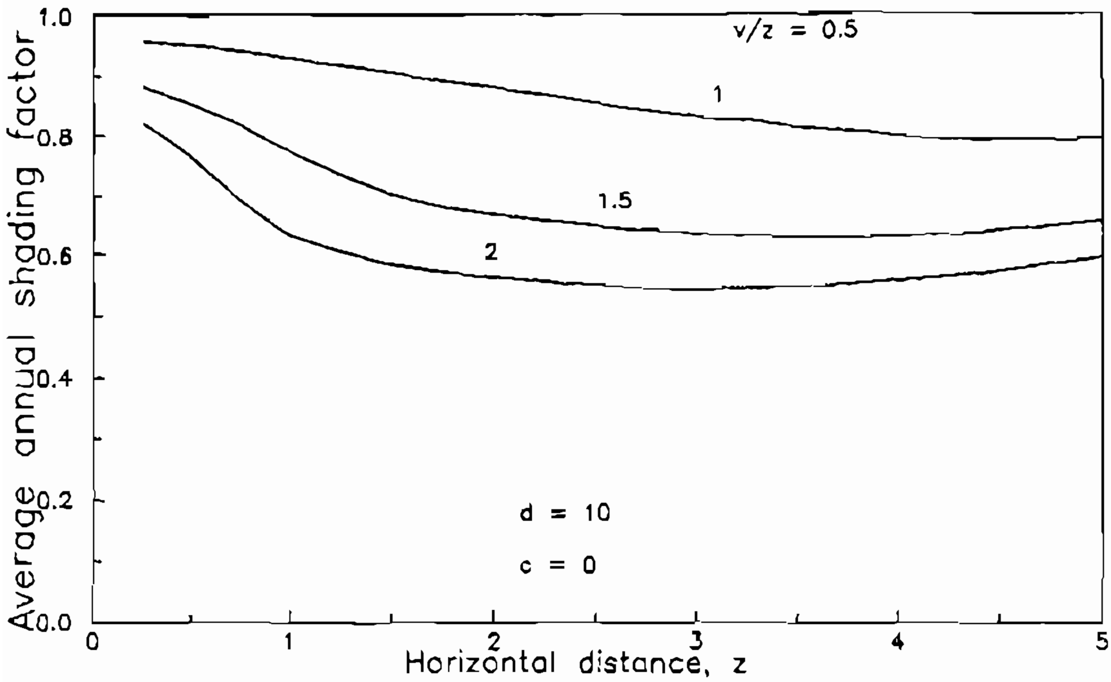

Fig. 9. Annual shading factor for a near southeren object.

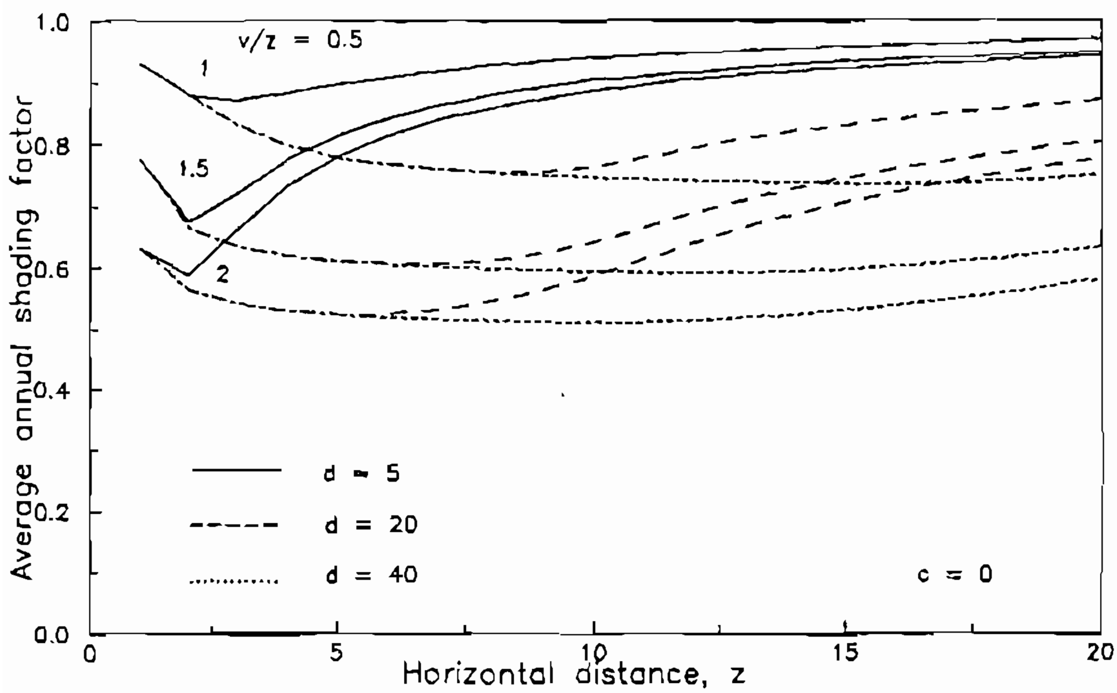

Fig. 10. Average annual shading factor vs. horizontal distance for different values of building heights and widhs.

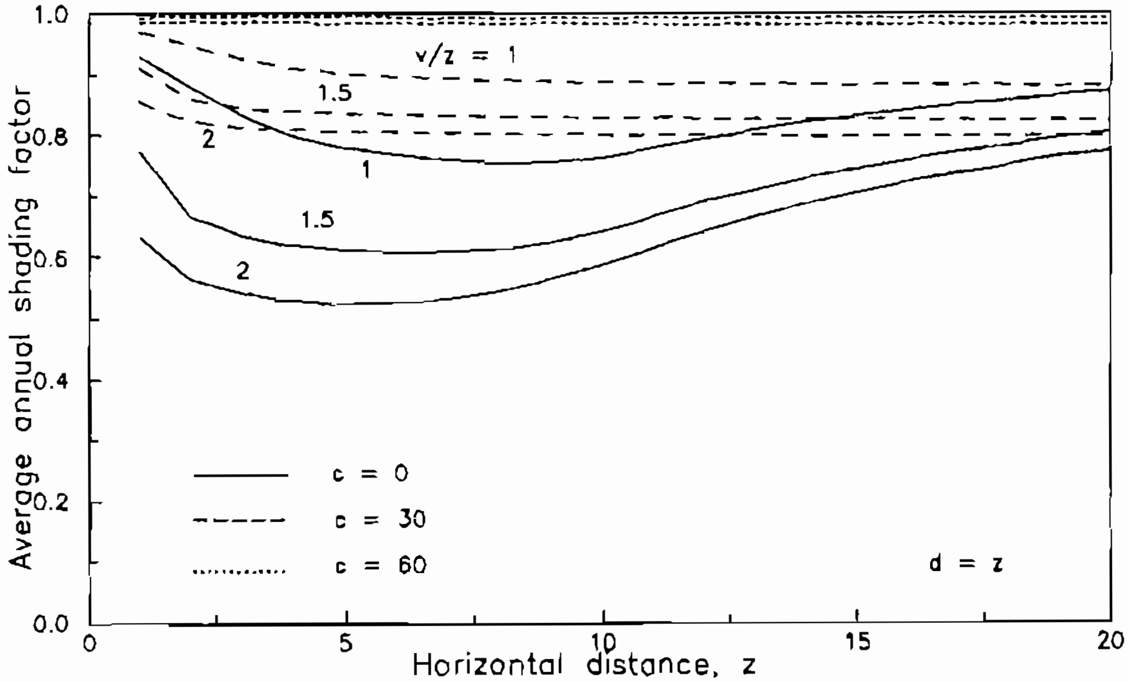

Fig. 11. Average annual shading factor vs. horizontal distance for different values of angle c, and building heights.

system geometry, since all parameters are dimensionless.

NOMENCLATURE

A_r	Ratio of the shaded area to that of the aperture
a	Solar azimuth angle, rad. (with normal sign convention).
c	Angular deviation of the building from south direction, rad.
d	Ratio of building width to that of aperture in E-W direction
e	Aperture aspect ratio
f	Diffuse radiation factor
H_{bn}	Beam solar radiation intensity at normal incidence, W/m^2
H_{da}	Diffuse solar radiation intensity on aperture surface, W/m^2
L	Aperture height, m
l	Length of shade in the aperture height, m
N	Number of days from January first
V	Building height, m
v	Ratio of building height to that of aperture
W	Aperture width, m
Z	Horizontal distance between building and aperture, m
z	Ratio between the horizontal distance and aperture height
α	Solar altitude angle, rad.
β	Aperture tilt angle, rad.
γ	Aperture surface zimuth angle, rad.
θ	Incident angle of beam radiation on aperture surface, rad
λ	Profile angle, rad.

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