



Answer the following questions

1) Use linear finite difference method to solve the following Boundary Value Problem

(BVP) given by $y'' + (x + 1)y' - 2y = 4x - 2$,
 $y(0) = 1$, $y(1) = 0$, $0 \leq x \leq 1$, $h = 0.2$,

2) Derive an approximate backward difference representation for $f'''(x_i)$,
 given evenly spaced grid points $f(x_i), f(x_{i-1}), f(x_{i-2}), f(x_{i-3})$ by means of :

i) Taylor Series Expansions , ii) A Forward Difference Recurrence Formula

3) If $f(x) = x e^x$.

(i) find an approximate value for $f'(x)$ at $x = 2$ with $h = 0.2$ using :

- a) Forward difference formula
- b) Backward difference formula

(ii) Apply Richardson's extrapolation process to evaluate $N_1(h)$, and $N_2(h)$.

4) Solve the following BVP

$y'' - \frac{3}{x}y' + \frac{3}{x^2}y = 2x^2e^x$, $1 \leq x \leq 2$, with $y(1) = 0$ and $y(2) = 4e^2$,
 using the linear shooting method with $h = 0.1$.

5) Find the curve of best fit of the form $y = ab^x$ to the following data using method of least squares.

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|---|-----|------|------|------|-------|
| x | 2 | 3 | 4 | 5 | 6 |
| y | 8.3 | 15.4 | 33.1 | 65.2 | 127.4 |

6) Given $y_1' = y_1y_2 + x$, $y_2' = xy_2 + y_1$, $y_1(0) = 1$, $y_2(0) = -1$, find $y_1(0.1)$ and $y_2(0.1)$
 by using Runge – Kutta Method of fourth order .

7) The vertical distance in meters covered by a rocket from $t = 8$ to $t = 30$ s is given by

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use Romberg's rule and three-point Gauss quadrature rule to find the distance covered. Use the 1, 2, 4, and 8-segment trapezoidal rule ,

b) Find the true error for part (a) ,