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Planar Convection in The Presence of External Fields

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1- ABSTRACT

The topic of this work is study of convection in conducting viscous fluids bounded by an infinite horizontal layer under the influence of both Coriolis and Lorentz forces [18,12]. This work is devoted to the solution and comparison of two problems mainly dealing with stress-free and rigid boundary. The basic equations are continuity, the momentum, the energy and magnetic induction along with the auxiliary relations based on Boussinesq approximation [9]. The basic phenomenon of stability for the steady state solution is studied. The complex eigen values determine the boundaries of region for instability.

Keywords:- Convection, Magnetic field, Coriolis forces, Lorentz forces, Galerkin methods, Eigen value problems, stability problems, nonlinear problems and Benard cell.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

2.1 Basic Equations

Consider a horizontal layer of fluid contained between two boundaries separated by the distance d. The lower and upper surfaces with constant temperature as maintained at T_1 and T_2 respectively. The layer is rotating about a vertical axis with an angular velocity Ω. We assume that the centrifugal force is negligible in comparison with gravity. This layer is permeated by a vertical magnetic field with a constant flux density $\underline{B}$. The analysis is based on the Boussinesq approximation of the equation of motion and the heat conduction equation. Using d, d^2/χ, $(T_2 - T_1)/R$ and $\underline{B}$ as scales of length, time, temperature and magnetic flux density, where:

$$\underline{B} - \bar{k} = \chi \, \underline{d} / \lambda, \quad R = \gamma (T_1 - T_2) \, g d^3 / \chi \nu, \; p = \nu / \chi, \; Q = b^2 d^2 / \rho \mu \nu \lambda$$

And $\underline{k}$ is the vertical unit vector opposite to the direction of gravity g, λ is the magnetic difusivity, μ the magnetic permeability, R is the Rayleigh number, P is the Prandtl number, Q the Chandrasekhar number, γ is the coefficient of the thermal expansion, ρ is the mean density and ν is the kinematics viscosity of the fluid, χ is the thermal difusivity of the fluid.

The dimensionless equation of motion according to Boussinesq approximation [5] is given by:

$$\frac{\partial V}{\partial t} + \underline{V}\,\nabla\,\underline{V} = \nabla^2 \underline{v} + \bar{k}\,\theta - \nabla\Pi - 2\Omega k \times \underline{V} + Q(\underline{B}.\nabla)\underline{b} \qquad (1)$$

The equation of continuity and of the equation of the source free magnetic field are given by

$$\nabla.\underline{v} = 0 \qquad \nabla.\underline{b} = 0 \qquad (2)$$

The energy equation is

$$P\left(\frac{\partial \theta}{\partial t} + \underline{v}.\nabla\theta\right) = \nabla^2\theta + R\,k.\underline{v} \qquad (3)$$

The equation of magnetic induction is

$$\frac{\chi}{\lambda}\left(\frac{\partial}{\partial t}\,\underline{b} + \underline{v}.\nabla\underline{b} - \underline{b}.\nabla\underline{v}\right) = k.\underline{v} + \nabla^2\underline{b} \qquad (4)$$

The solenoidal vector fields $\underline{v}$ and $\underline{b}$ are represented in the form [1,20]

$$\left.\begin{aligned} \underline{v} &= \delta\phi + \varepsilon\varphi \\ \underline{b} &= \delta\,h + \varepsilon\,\tilde{g} \end{aligned}\right\} \qquad (5)$$

where; $\delta\phi = \nabla\times(\nabla\times\, k\phi), \varepsilon\,\varphi = \nabla\times\, k\,\varphi$
$\delta h = \nabla\times(\nabla\times\, k\,h)$ and $\varepsilon\tilde{g} = \nabla\times\, k\tilde{g}$

The boundary conditions for free surfaces can be expressed [4] in the form

$$\phi = \partial_z^2\phi = \partial_z\varphi = \theta = \partial_z\tilde{g} = 0 \quad \text{at} \quad z = \pm\frac{1}{2} \qquad (6)$$

And the boundary conditions for rigid surfaces can be expressed in the form

$$\phi = \partial_z\phi = \varphi = \theta = \partial_z\tilde{g} = 0 \qquad (7)$$

Operating with δ and ε on (1) (see [3]) and rewriting as the following system for the unknown scalar fields ϕ,φ,θ, h and $\tilde{g}$; and using these representations in the basic equations we get

$$\partial_t\left(\nabla^2\nabla_2\phi\right)+\delta.\left(\underline{V}.\nabla\underline{V}\right)$$
$$=\nabla^4\Delta_2\phi-\Delta_2\theta-2\Omega\ \partial_z\Delta_2\varphi+Q\partial_z\left(\nabla^2\Delta_2 h\right) \quad (8)$$

$$\partial_t\nabla^2\Delta_2\varphi+\nabla^2\varepsilon.\left(\underline{v}.\nabla\underline{v}\right)$$
$$=\nabla^4\Delta_2\varphi+2\Omega\partial_z\nabla^2\Delta_2\phi+Q\partial_z\nabla^2\Delta_2\ \tilde{g} \quad (9)$$

$$P\left(\frac{\partial}{\partial t}\theta+\underline{v}.\nabla\theta\right)=\nabla^2\theta-R\Delta_2\phi \quad (10)$$

$$\left.\begin{aligned}\partial_z\nabla^2\Delta_2\Phi+\nabla^4\Delta_2\ \underline{h}=0\\ \partial_z\Delta_2\varphi+\nabla^2\Delta_2\quad 9=0\end{aligned}\right\} \quad (12)$$

where; $\Delta_2=\partial^2_{xx}+\partial^2_{yy}$.

Hence the equations of motion take the following form using (11) we get:

$$\partial_t\left(\nabla^2\Delta_2\phi\right)+\delta.\left[\left(\delta\phi+\varepsilon\varphi\right).\nabla\left(\delta\phi+\varepsilon\varphi\right)\right]$$
$$=\nabla^4\Delta_2\phi-\Delta_2\theta-2\Omega\partial_z\Delta_2\varphi-Q\partial_z^2\Delta_2\phi \quad (12)$$

$$\partial_t\nabla^2\Delta_2\varphi+\nabla^2\varepsilon.\left[\left(\delta\phi+\varepsilon\varphi\right).\nabla\left(\delta\phi+\varepsilon\varphi\right)\right]$$
$$=\nabla^4\Delta_2\varphi+2\Omega\partial_z\nabla^2\Delta_2\phi-Q\partial_z^2\Delta_2\varphi \quad (13)$$

$$P\left(\frac{\partial\theta}{\partial t}+\left(\delta\phi+\varepsilon\varphi\right).\nabla\theta\right)=\nabla^2\theta-R\Delta_2\phi \quad (14)$$

2.2 The Steady State Problem

Assuming the functions ϕ,φ and θ depend only on z and x. I assuming also that $\lambda >> \kappa$ such that all terms multiplied by $\frac{\chi}{\lambda}$ can be neglected. We find that the general solution which satisfies the stress-free boundary conditions by applying steady solution due to Galerkin's method [10] has the form

$$\phi = \sum_{n,m} a_{nm}\phi_{nm} = \sum_{n,m} a_{nm} \cos m\,\alpha x\, \sin n\pi(z+\frac{1}{2}) \quad (15a)$$

$$\varphi = \sum_{n,m} b_{nm}\varphi_{nm} = \sum_{n,m} b_{nm} \cos m\,\alpha x \cos (n-1)\pi(z+\frac{1}{2}) \quad (15b)$$

$$\theta = \sum_{n,m} c_{nm}\theta_{nm} = \sum_{n,m} c_{nm} \cos m\,\alpha x\, \sin n\pi(z+\frac{1}{2}) \quad (15c)$$

Also the rigid boundary conditions [5,11];

$$\phi = \sum_{n,m} a_{nm}\phi_{nm} = \sum_{n,m} a_{nm} e^{im\alpha x} g_n(z); \quad (16a)$$

$$\varphi = \sum_{n,m} b_{nm}\varphi_{nm} = \sum_{n,m} b_{nm} e^{im\alpha x} f_n(z); \quad (16b)$$

$$\theta = \sum_{n,m} c_{nm}\theta_{nm} = \sum_{n,m} c_{nm} e^{im\alpha x} g_n(z); \quad (16c)$$

Where the functions $g_n(z)$ and $f_n(z)$ are represented as follows

$$g_n(z) = \sin n\pi(z+\frac{1}{2}) \qquad (17)$$

$$f_n(z) = \begin{cases} \dfrac{\sinh \mu nz}{\sinh \mu n/2} - \dfrac{\sin \mu nz}{\sin \mu n/2} = s_n(z) & \text{odd } n \\ \dfrac{\cosh \lambda nz}{\cosh \lambda n/2} - \dfrac{\cos \lambda nz}{\cos \lambda n/2} = c_n(z) & \text{even } n \end{cases} \qquad (18)$$

Where $\lambda_n and \mu_n (n = 1,2,3,...)$ are the positive roots of the equations

$$\tanh(\lambda/2) + \tan(\lambda/2) = 0 \qquad (19)$$

$$\coth(\mu/2) - \cot(\mu/2) = 0 \qquad (20)$$

When defined in this manner, the functions $c_n(z)$ and $s_n(z)$ have the properties

$$\int_{-1/2}^{1/2} c_m(z)\, c_n(z)dz = \int_{-1/2}^{1/2} s_m(z)\, s_n(z)dz = \delta_{nm} \qquad (21)$$

Where, δ_{nm} is the Kronecker delta

The substitution of ϕ, φ and θ from equation (15) and (16) into the system of equations of steady motion (12-14) in the case stress-free and rigid boundary respectively, the partial differential equations reduce to a system of nonlinear

Algebraic equations. Employing the orthogonality of the trigonometric functions ϕ_{nm}, φ_{nm} and θ_{nm} over the interval $-1/2 \le Z \le 1/2$, the following system of equations is Obtained which allow the determination of the coefficients a_{nm}, b_{nm} and c_{nm} [7,9,12,21] in the form

$$\sum_{n,m} a_{nm}\, a_{kl}\, I^{(1)}_{nmrskl} + a_{nm} I^{(2)}_{nmrs} + c_{nm} I^{(3)}_{nmrs} + \Omega b_{nm}\, I^{(4)}_{nmrs} = 0 \qquad (22a)$$

$$\sum_{n,m} a_{nm}\, b_{kl}\, I^{(5)}_{nmrskl} + b_{nm} I^{(6)}_{nmrs} - \Omega\, a_{nm}\, I^{(7)}_{nmrs} = 0 \qquad (22b)$$

$$\sum_{n,m} P\, a_{nm}\, c_{kl}\, I^{(8)}_{nmrskl} - c_{nm} I^{(9)}_{nmrs} + Ra_{nm}\, I^{(10)}_{nmrs} = 0 \qquad (22c)$$

In order to compute the unknown coefficients a_{nm}, b_{nm} and c_{nm} it is necessary to truncate the representation (18) at a sufficiently high level. Hence, choosing a truncation parameter N such that all coefficients with n+m>N are negligible. Assuming that the truncation parameter N is an even integer we have $\frac{1}{2}N(N+1)$ equations to be solved. Starting with a guessed solution, we use a $\frac{1}{2}N(N+1)$ dimensional Newton-Raphson iteration procedure to obtain the solution [15,19]. The satisfactory approximation is reached if it differs by a sufficiently small amount from the solution obtained with N+2 instead of N terms. The summation convention have been applied to (15) or (16). The calculation of the matrices $I^{(n)}$ from the terms

Given in (22) are straightforward. Using angular brackets to indicate an average over the fluid layer, we can write

$$I^{(1)}_{nmrskl} = \left(\phi_{rs}\ \delta.\left[(\delta\phi+\varepsilon\varphi).\nabla(\delta\phi+\varepsilon\varphi)\right]\right).$$

2.3 The Stability Problem

We can superimpose infinitesimal disturbances in the case of stress-free of the form [6,14, 22]

$$\tilde{\phi} = \sum_{n,m} \tilde{a}_{nm}\ \tilde{\phi}_{nm} = \sum_{n,m} \tilde{a}_{nm}\ g_n^{(z)}\ \mathrm{Exp}\left[i(m\alpha+d)x+i\ by\ +\sigma t\right] \quad (23)$$

$$\tilde{\varphi} = \sum_{n,m} \tilde{b}_{nm}\tilde{\varphi}_{nm} = \sum_{n,m} \tilde{b}_{nm}\ g_n^{*(z)}\ \mathrm{Exp}\left[i(m\alpha+d)x+i\ by\ +\sigma t\right] \quad (24)$$

$$\tilde{\theta} = \sum_{n,m} \tilde{c}_{nm}\ \tilde{\theta}_{nm} = \sum_{n,m} \tilde{c}_{nm}\ g_n^{(z)}\ \mathrm{Exp}\left[i(m\alpha+d)x+i\ by\ +\sigma t\right] \quad (25)$$

Where; $g_n(z) = \sin(n\pi\ (z+\frac{1}{2}))$

$$g_n^{*}(z) = \cos((n-1)\pi(z+\frac{1}{2}))$$

Taking the rigid boundary in the form

$$\tilde{\phi} = \sum_{n,m} \tilde{a}_{nm}\ \tilde{\phi}_{nm} = \sum_{n,m} \tilde{a}_{nm}e^{im\alpha x}g_n(z)\ e^{[i(bx+dy)+\sigma t]} \quad (26)$$

$$\tilde{\varphi} = \sum_{n,m} \tilde{b}_{nm}\ \tilde{\varphi}_{nm} = \sum_{n,m} \tilde{b}_{nm}e^{im\alpha x}\ f_n(z)\ e^{[i(bx+dy)+\sigma t]} \quad (27)$$

$$\tilde{\theta} = \sum_{n,m} \tilde{c}_{nm}\, \tilde{\theta}_{nm} = \sum_{n,m} \tilde{c}_{nm} e^{im\alpha x}\, g_n(z)\, e^{[i(bx-dy)-\sigma t]} \tag{28}$$

Since $\tilde{\phi}, \tilde{\varphi}$ and $\tilde{\theta}$ satisfy the same boundary conditions as $\phi, \varphi,$ and θ (6) and (7) respectively in each case. The equations for the field $\{\tilde{\phi}, \tilde{\varphi}, \tilde{\theta}\}$ of infinitesimal disturbances are obtained by replacing ϕ, φ and θ in (8-10) by $(\phi + \tilde{\phi}, \varphi + \tilde{\varphi}$ and $\tilde{\theta} + \theta)$ respectively. And subtracting the equations for the steady solution $\{\phi, \varphi, \theta\}$ i.e. (12-14) we get

$$\begin{aligned} &\partial_t(\nabla^2 \Delta_2 \tilde{\phi}) + \delta.[(\delta\phi + \varepsilon\,\varphi).\nabla(\delta\tilde{\phi} + \varepsilon\,\tilde{\varphi}) \\ &\qquad + (\delta\tilde{\phi} + \varepsilon\,\tilde{\varphi}).\nabla(\delta\phi + \varepsilon\,\varphi)] \\ &= \nabla^4 \Delta_2 \tilde{\phi} - \Delta_2 \tilde{\theta} - 2\,\Omega\, \partial_z \Delta_2 \tilde{\varphi} - Q\, \partial_z^2 \Delta_2 \tilde{\phi} \end{aligned} \tag{29}$$

$$\begin{aligned} &\partial_t \nabla^2 \Delta_2 \tilde{\varphi} + \nabla^2 \varepsilon.[(\delta\phi + \varepsilon\,\varphi).\nabla(\delta\tilde{\phi} + \varepsilon\,\tilde{\varphi}) \\ &+ (\delta\,\tilde{\phi} + \varepsilon\,\tilde{\varphi}).\nabla(\delta\,\phi + \varepsilon\,\varphi)] \\ &= \nabla^4 \Delta_2 \tilde{\varphi} + 2\,\Omega \partial_z \nabla^2 \Delta_2 \tilde{\phi} - Q\, \partial_z^2 \Delta_2 \tilde{\varphi} \end{aligned} \tag{30}$$

$$\begin{aligned} &P\left(\frac{\partial\tilde{\theta}}{\partial t} + (\delta\phi + \varepsilon\varphi).\nabla\tilde{\theta} + (\delta\tilde{\phi} + \varepsilon\tilde{\varphi}).\nabla\theta\right) \\ &\qquad = \nabla^2 \tilde{\theta} - R\Delta_2 \tilde{\phi} \end{aligned} \tag{31}$$

2.4 Eigen Value Problems:

The variables $\tilde{\phi}, \tilde{\varphi}$ and $\tilde{\theta}$ are satisfied by same boundary conditions (6) and (7) in each case. From (29-31) we have, substitution of the expansions (23-25) and

(26-28) into (29-31) leads to the equations for the unknowns a_{nm}, b_{nm} and c_{nm} [2,6]. When the same procedure as in the steady problem is used i.e. (as in the Case of the steady problem, a system of algebraic equations is obtained by multiplying equations (29-31) by (ϕ_{rs}, φ_{rs} and θ_{rs}) Exp[-i(dx+by)-σ t] respectively and averaging it over the fluid layer in each case ($-1/2 \le z \le 1/2$) the resulting by using [9,21] are linear homogeneous equations representing an eigen values problem with eigen values σ

$$\sigma\ \tilde{a}_{nm}\ I^{(11)}_{nmrs} = -\tilde{a}_{nm} I^{(12)}_{nmrs} - \tilde{b}_{nm} I^{(13)}_{nmrs} + \tilde{c}_{nm} I^{(14)}_{nmrs} - \Omega \tilde{b}_{nm} I^{(15)}_{nmrs} - Q \tilde{a}_{nm} I^{(16)}_{nmrs} \tag{32}$$

$$\sigma\ \tilde{b}_{nm} I^{(17)}_{nmrs} = -\tilde{a}_{nm} I^{(18)}_{nmrs} - \tilde{b}_{nm} I^{(19)}_{nmrs} + \Omega\ \tilde{a}_{nm} I^{(20)}_{nmrs} - Q\ \tilde{b}_{nm} I^{(21)}_{nmrs} \tag{33}$$

$$\sigma\ P\ \tilde{c}_{nm} I^{(22)}_{nmrs} = -p(\tilde{c}_{nm} I^{(23)}_{nmrs} + \tilde{a}_{nm} I^{(24)}_{nmrs} + \tilde{b}_{nm} I^{(25)}_{nmrs}) + \tilde{c}_{nm} I^{(26)}_{nmrs} - R \tilde{a}_{nm} I^{(27)}_{nmrs} \tag{34}$$

Where; $I^{(12)}_{nmrs} = \sum < \phi_{rs} \nabla^2 \Delta_2 \tilde{\phi} >$

$$I^{(21)}_{nmrs} = \sum_{lk} a_{lk} < \phi_{rs}\ \delta.(\delta\ \phi_{lk}.\nabla\ \delta\tilde{\phi}_{nm} + \delta\ \tilde{\phi}_{lk}.\nabla\ \delta\phi_{lk}) > + b_{lk} < \phi_{rs} \delta.(\varepsilon\varphi_{lk}.\nabla\ \delta\tilde{\phi}_{nm} + \delta\tilde{\phi}_{nm}.\nabla\ \delta\varphi_{lk}) > + < \phi_{rs} \nabla^4 \Delta_2\ \tilde{\phi}_{nm} >$$

(10-12) is a system of linear homogeneous equations representing an eigenvalues problem with the eigenvalues σ. The same truncation parameter N is used for the disturbance representation (32-34) as for the steady solution. Usually the eigenvalues σ depends smoothly on the parameters of the problem and accurate results can be obtained by interpolation from a fairly computed eigenvalues σ. In order to apply the usual algebraic eigenvalues methods, the indices n and m,r and s had to be combined to form a single running subscript. In addition the coefficients $\tilde{a}_{nm}, \tilde{b}_{nm}$ and $\tilde{c}_{nm}$ were combined sequentially to form a single variable. The analysis of the eigenvalues problem (32-34) is further simplified by noticing that it separates into four subsystems.

(1) The equations with even n + m. coefficients

(2) The equations with odd n + m. coefficients

(3) In each case the symmetry of the steady solution with respect to the x direction allows a further separation into solutions which are either (i) symmetric or (ii) anti-symmetric in x. It should be noted that in all cases the coefficients $\tilde{b}_{nm}$ have the opposite symmetry properties to the coefficients $\tilde{a}_{nm}$ and $\tilde{c}_{nm}$. In the following, all references to symmetry properties will be made with respect to the latter coefficients. The primary objective of the analysis of problem (32-34) is to determine σ as a function of α, P and the Rayleigh number R at which the real part of the eigenvalues σ with largest real part vanishes. The corresponding eigenvector describes the

Disturbance leading to instability of the steady solution when the Rayleigh number passes through the marginal value. Even though system (32-34) has been separated into four subsystems the amount of computation for the determination of stability boundaries is still substantial because of the large number of parameters involved. For this reason only a few representative cases will be investigated. Fortunately the dependence of the eigenvalues σ on various parameters is smooth and interpolation formulae can be used successfully to reduce the cost of computation [6]. The infinitesimal transformation of the steady vortex solution at $\phi = \partial\phi/\partial x$ the eigenvalues σ should equal zero. The details of the dependence of the eigenvalues σ on wave value b are complicated by the fact that imaginary part vanishes for sufficiently small values of b. [14].

2.5 The Steady Solution

Most of the numerical computations of two-dimensional convection published in the literature have been carried out without consideration of the stability of the solution. Thus in many cases solution has been obtained for values of the parameters for which the two-dimensional solution is physically not realizable. This is particularly true in the case of rotating layer, in each case the stability domain of convection rolls is much smaller than in the non-rotating case. The heat transfer is usually represented in terms by the Nusselt number which describes the ratio between the heat transfer with convection and what it would be without convection at a given Rayleigh number.

In describing the numerical results for steady convection rolls we shall concentrate on the convective heat transfer, which not only is the parameter of primary physical interest, but also appears to characterize best the other aspects of convection as viscous dissipation occurs at the same as the convection heat transfer. A number of numerical studies of convection rolls have yielded results for the Nusselt number [17].

$$Nu = 1 + \langle \underline{v}.k\theta \rangle / R \qquad (35)$$

3. RESULTS:

Fig.(1) shows a relation between the Nusselt number Nu and reduced Rayleigh (R/Rc) for water at $\alpha = \alpha c$ with different boundary conditions. The onset of instability of convection occurs at the critical Rayleigh Rc with Nu=1. This figure is for an electric fluid with critical wave number αc=2.2 and no external fields. In the Case of stress-free boundary, we note that the convection increases faster than for the rigid boundary case. This figure illustrates the effect of presence of rigid boundary on the heat transfer. It is clear that the rigid wall inhibits the convection heat transport. Note that S stands for stress-free and R stands for rigid boundary.

Fig. (2) shows the real part of the eigenvalues σ as a function of the Rayleigh number. The objective of this figure is to determine the critical Rayleigh number RII by intersecting the line of zero σ with these curves. It is obvious that with the existence of the magnetic field, $Q = 100$ that RII is proportional to rotation for all

Cases. It could be concluded that increasing the rotation increases RII. Moreover it can also be observed that the two curves for rigid boundary case have different slopes while those with stress-free surface have the same slop.

Fig. (3) shows the dependence of the wave number (α) on the Rayleigh number R in the lowest mode for both cases of boundary conditions. The minimum At a finite value αc reflects the property that narrow convection cells corresponding to high values of α are ineffective because of the strong heat exchange between up and down going fluid. For very large-scale motions the vertical comparison to horizontal component and the release of potential energy is less efficient. The letter effect compensated by a decrease of thermal dissipation in the case of thermally insulating boundaries. The critical wave number αc indicates that the horizontal extent of the circulation motion comparable with the depth of the layer. In the stress-free case is less restricting boundary the horizontal extend is somewhat larger.

4. CONCLUSIONS

The study of the convection in an electric conducting viscous fluid under the influence of rotation and magnetic field was investigated.

Two kinds of boundary conditions were used (1) stress-free boundary, (2) rigid boundary. The basic equations for continuity, momentum, energy, and magnetic induction were written along with the auxiliary relations. Boussinesq approximation [5] was used. The basic equations were solvec by Galerkin spectral [10] method and Newton's method [8]. For each case of the boundary condition the

Steady state solution was obtained first, then the stability was investigated. The critical boundary stability region was determined.

Steady Solution:

1- The Coriolis force appears to have a constraining effect on steady convection by delaying the onset of convection.

2- The Lorentz force appears to have a constraining effect on steady convection by delaying the onset of convection.

3- Both forces (Lorentz force and Coriolis force) accelerate the onset of instability for steady convection.

4- In the case of rigid boundary we need a greater angular velocity Ω than in case of Stress-free boundary to get the same effect on steady convection.

Stability Problem:

The effect of the magnetic field on the stability is contradicted with influence of the Rotation on the stability while the increasing of the rotation leads to instability; it is contradicted with the influence of the magnetic field. The convection is insatiable

When the Rayleigh number increases beyond critical value. The steady solution of the small amplitude is considered insatiable with increase of the rotation rate.

The wave number remains constant in spite of the increase of the Rayleigh number. The dependence of the stability properties of convection rolls on the magnetic field strength are in marked contrast to the dependence of the stability on the rotation rate in a layer rotating about a vertical axis. While the regions of stable rolls is strongly reduced with increasing rotation rate and finally vanishes at the Critical value of the angular velocity, the opposite influence of the magnetic field is found. The mathematical origin of this contrasting behavior has already been discussed in (sec.2.2). The absence of vertical vorticity of steady convection rolls in the presence of a magnetic field is the basic physical reason for the difference in the Otherwise rather similar influences of magnetic field and rotation. The most surprising result of the present analysis is the phenomenon that convection rolls setting in at the critical Rayleigh number becomes unstable as soon as the Rayleigh number increases beyond the critical value. The steady solution with small amplitude is unstable when the rotation rate is sufficiently large. The value of the parameter α for the most stable solution remains essentially constant with increasing Rayleigh number. Like the stationary solution with maximum heat transfer, however less pronounced, the most stable solution has a slightly increasing wave number α. About the convictions with rotation; the rotation decreases the rate of the convection, Rigid boundary needs strong rotation to effect the convection more than Stress-free problem. In both cases Rigid and free surfaces for Q=100 appear a constraining effect on stability convection delay of the onset of instability with increasing angular velocity Ω.

REFERENCES

1- Bolton, E.W. and Busse, F.H. "Stability of convection rolls in a layer with Stress-free boundaries" J. Fluid Mech. vol. 150, pp. 487-498, (1985).

2- Busse, F.H. "Non-linear properties of thermal convection" Rep. Prog. Phys., vol. 41, (1978).

3- Busse, F.H. & Clever, R.M. "Stability of convection rolls in the presence of vertical magnetic field" Phys. Fluids vol. 25, pp. 931-935, (1982).

4- Busse, F.H. & Bolton, E.W. "Instabilities of convection rolls with Stress-free boundaries near threshold, J. Fluid Mech. vol. 146, pp. 115-125, (1984).

5- Chandrasekhar, S. "Hydrodynamic and Hydromagnetic Stability" Oxford; Clarendon press, (1961).

6- Clever, R.M. & Busse, F.H. "Transition to time-dependent convection" J. Fluid Mech. vol. 65, pp. 625-645, (1974).

7- Clever, R.M. & Busse, F.H. "Nonlinear properties of convection rolls in a horizontal layer rotating about a vertical axis" J. Fluid Mech. vol. 94, part 4, pp. 609-627, (1979).

8- Demidovich, B.P. & Maron, I.A. "Computational Mathematics" Moscow (1987).

9- Grosche, G. and Ziegler, V. "Taschen buch der Mathematik", (1981).

10-Gottlieb, D and Orszag, S.A. "Numerical Analysis of Spectral Methods" (Theory and Applications), (1989).

11-Harris, D.L. and Reid, W.H. "On Orthogonal functions which satisfy four boundary condition. I. Tables for use in Fourier-Type-Expansions", (1958).

12-IMSL. MATH/LIBRARY Subroutines for Mathematical Applications U.S.A., (1987).

13-Moore, D.R. and Weiss, N.O. "Two-dimensional Rayleigh-Benard convection" J. Fluid Mech. vol. 58, part 2, pp. 289-312, (1973).

14-Nagata, M and Busse, F.H. "Three-dimensional tertiary motion in a plane shear layer", J. Fluid Mech. vol. 135, pp. 1-26, (1983).

15-Press, W.H., Flannery, B.P., Teukolsky, S.A., and Vetterling, W.T. "Numerical Recipes" New York, (1989).

16-Rossby, H.T. "A study of Benard convection with and without rotation" J. Fluid Mech. vol. 36 part 2, pp. 309-335, (1969).

17-Swinnery. H.L. and Gollub, J.P. (1981) "Hydrodynamic Instabilities and the Transition to Turbulence", part 45, Berlin, Heidelberg, New York.

18-Veronis, G. "Large-amplitude Benard convection in a rotating fluid" J. Fluid Mech. vol. 31, pp. 113-139, (1968).

19-Wagener, J. "Principles of Programming" New York, (1989).

20-Warner, W.H. "Poloidal and toroidal potentials for solenoidal fields" J. Appl. Math. Phys. (ZAMP) vol. 23, pp. 221 (1972).

21-Wolfram, S. "Mathematica, A system for Doing Mathematic by Computer" Bonn, New York, (1990).

22-Zhang K.K. & Busse, F.H. "On the onset of convection in rotating spherical shells" Geophys, Astrophys. Fluid Dynamic, vol. 39, pp. 119-147, (1986).

23-Zhang K.K. & Busse, F.H. "Generation of magnetic fields by convection in a rotating spherical fluid shell of infinite prandtl number" Phys. Earth. & Plan. Int. vol. 59, pp. 208-222, (1990).

Fig. 1

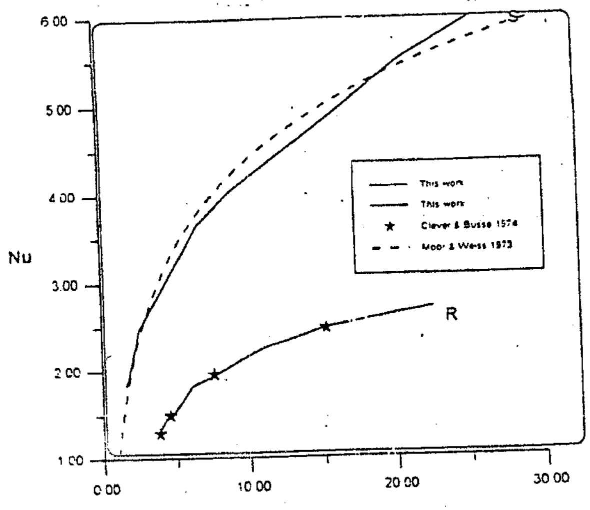


Fig. 2

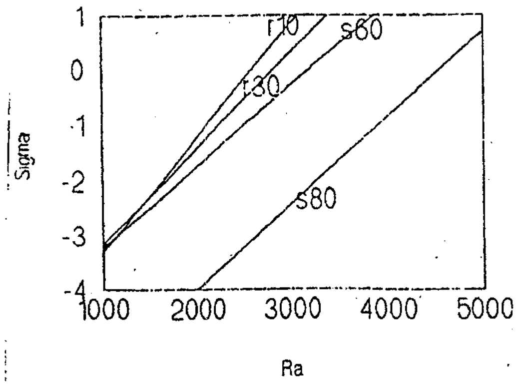
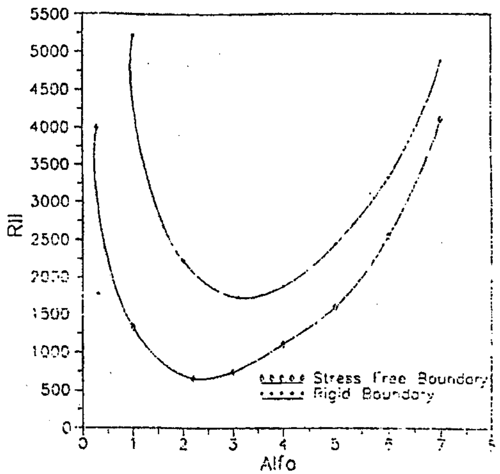


Fig. 3



التأفق فى الإنتقال الحرارى بالحمل فى وجود مجالات خارجية

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الملخص العربى

فى هذا البحث تم دراسة الإنتقال الحرارى بالحمل فى وسط مائع محدود بسطحين بينهما مسافة d وإستخدمنا لوصف الظاهرة معادلة كمية الحركة و الإتصال و الطاقة والحث المغناطيسى وأثرنا على المائع اثناء الإنتقال الحرارى بمجالات خارجية (المغناطيسية و الدوران) . إستخدمنا التقريبات العددية (تقريب*بواسنسك) لحل المعادلات التفاضلية الجزئية الغير خطية عدديا بفرض الحل بطريقة طيفية (طريقة جالركن) وتم إيجاد الحل بطريقة نيوتن ، ودراسة إستقرارة ، وإستخراج القيم الذاتية وتم تمثيل النتائج هندسيا . وتطبيق ذلك فى مسألتين 1-السطوح الصلبة 2- السطوح خالية من الإجهادات فى كلا من الحالات الفلكية و الجيوفيزيقا.