Menofia University
Faculty of Engineering Shebien El-kom
Basic Engineering Sci. Department.
First semester Examination, 2017-2018
Date of Exam : 13 /1/2018

Subject : Complex Analysis .
Code: BES 505
Year : Diploma
Time Allowed : 3 hrs
Total Marks: 100 Marks

Answer the following questions

Question 1 (30 MARKS)

(A) Suppose we choose the principal branch of $sin^{-1} z$ to be that one for which $sin^{-1} 0 = 0$. Prove that $sin^{-1} z = \frac{1}{i} \ln(iz + \sqrt{1-z^2})$ (5 Marks)

(B) Let w = f(z) = z^2. Find the values of w that correspond to

(a) z = -2 + i and (b) z = 1 - 3i,

and show how the correspondence can be represented graphically. (5 Marks)

(C) Solve the partial differential equation $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = x^2 - y^2$ using complex analysis (5 Marks)

(D) (i) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic. (15 Marks)

(ii) Find v such that $f(z) = u + iv$ is analytic.

(iii) Find f(z)

(iv) Find the orthogonal trajectories of the family of curves in the xy plane which are defined by $e^{-x}(x \sin y - y \cos y) = \alpha$ where α is a real constant.

Question 2 (40 MARKS)

(A) Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)\, dy$ along the parabola

$x = 2t, y = t^2 + 3;$ (5 Marks)

(B) Evaluate $\int_c \bar{z}\, dz$ from z = 0 to z = 4 + 2i along the curve C given by: (5 Marks)

the line from $z = 0$ to $z = 2i$ and then the line from $z = 2i$ to $z = 4 + 2i$.

(C) Verify Green's theorem in the plane for $\oint_c (2xy - x^2)dx + (x + y^2)dy$

Where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$

(10 Marks)

(D) . Evaluate:

$(a) \oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz, \quad (b) \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 3$.

(10 Marks)

(E) Find the residues of

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$$

at all its poles in the finite plane. (10 Marks)

Question 3 (30 MARKS)

(A) Prove that (i) $\int_0^\infty \frac{\ln(x^2+1)}{x^2+1}\, dx = \pi \ln 2$ (ii) $\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$.

(iii) $\sum_{n=-\infty}^{\infty} \frac{1}{n^2+a^2} = \frac{\pi}{a} \coth \pi a$ where $a > 0$.

(iv) $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \cdots = \frac{\pi^3}{32}$. (15 Marks)

(B) (i) Determine the region of the w plane into which each of the following is mapped by the transformation $w = z^2$. (a) First quadrant of the z plane. (b) Region bounded by $x = 1, y = 1, and\ x + y = 1$.

(ii) Find a bilinear transformation that maps points $z = 0, -i, -1$ into $w = i, 1, 0$, respectively.

(iii) Find a transformation that maps the real axis in the w plane onto the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ in the z plane. (15 Marks)

This exam measures the following ILOs											
Question Number	Q1-a	Q2-a	Q3-b	Q2-e	Q2-b	Q3-b	Q2-d		Q1-b	Q3-a	Q1-d
Skills			b-ii			b-i					
	Knowledge &understanding skills				Intellectual Skills				Professional Skills		

With my best wishes

Associate Prof. Dr. Islam M. Eldesoky