LAW OF VELOCITY DISTRIBUTION FOR TURBULENT BOUNDARY LAYER WITH ADVERSE PRESSURE GRADIENT OVER ROUGH SURFACE

Dr. Samir F. Hanna
Faculty of Engineering
EL-Mansoura university

Eng. B. O. Djebedjian
Assistant Lecturer
Faculty Of Engineering

(Received Aug. 12, 1987, accepted Dec. 1987)

قانون توزيع السرعه للطبقه الجدارية المضطربة مع تدرج ضغط عكسى على الاسطح الخشنة

خلاصه: تعتبر دراسة الطبقه الحديه غير المنضغطه على السطوح الخشنه غير سلسه لما يعترضها من عقبات نتيجة لصعوبة الحصول على صوره واضحه لميكانيكية الحركة المضطربه بالاضافة الى الاشكال والاحجام المختلفه لعناصر الخشونه وطريقة توزيعها على السطح ومن اهداف هذا البحث ايجاد قانون لتوزيع السرعه داخل الطبقه الحدية المضطربة فى وجود خشونه مختلفه للسطح وخاصة مع وجود تدرج عكسى فى الضغط. ويعتمد هذا البحث على نتائج معمليه لبعض الباحثين وكذلك على قوانين توزيع السرعة داخل الطبقة الجدارية المضطربه على الاسطح الخشنه.

وقد تم عمل برنامج حاسب الى لحل قانون السرعه مع باقى المعادلات الأخرى المستنتجه وعلى ضوء ذلك تم رسم العلاقات بين المتغيرات المختلفه وبناءا عليها يمكن دراسة اى طبقه حديه حدد لها توزيع الضغط ،ونسبة ارتفاع الخشونة الى سمك كمية الحركه K/δ^{**} وذلك بالنسبه الى نوع الخشونة المستخدمه فى التجارب السابقه. وقد اوضحت الدراسه ايضا ان ثابت دالة الخشونه C_r يتغير بتغير التدرج العكسى وهذا يتعارض مع الفروض السابقه بثبوته.

ABSTRACT - The aim of this investigation is to obtain a law for the velocity distribution in turbulent boundary layer in the presence of different surface roughness and adverse pressure gradient. This work has been developed on the basis of experimental results of different investigators[1:4] and on the velocity law in turbulent flow over rough surface [5].

In order to obtain the numerical results for the mathematical solution outlined in this work, a computer program was constructed to perform all the necessary calculations. These results are presented in charts and can be utilised for the geometric shape of roughness. On these basis it is possible to study turbulent boundary layer for which the pressure distribution, and the ratio of the roughness height to the momentum thickness k/δ^{**} are defined. Moreover, this work shows also that the constant of surface roughness function, C_r, varies with pressure gradient in form of Euler number for different ratios k/δ^{**}. This contradicts suggestions of previous investigations.

NOMENCLATURE

Symbols

A	velocity profile parameter	—
B	constant	—
c_∞	free stream velocity	(m/s)
$\bar{c}$	velocity at the outer edge of the boundary layer	(m/s)
c_τ	friction velocity	(m/s)

$c_\tau k/\nu$	dimensionless roughness height	—
$c_\tau y/\nu$	dimensionless distance normal to the wall	—
c_f	local skin friction coefficient, $\tau_w / 1/2\rho\bar{c}^2$	—
c_x	velocity of the fluid inside the boundary layer in x-direction	(m/s)
c_y	velocity component inside the boundary layer in y-direction	(m/s)
$C(\frac{c_\tau k}{\nu})$	surface roughness function	—
C_r	constant of surface roughness function	—
H_{12}	boundary layer form parameter, δ^* / δ^{**}	
I	boundary layer shape parameter, $\int_0^\infty (\frac{\bar{c}-c_x}{c_\tau})^2 d(\frac{y c_\tau}{\delta^* \bar{c}})$	
k	roughness height	(m)
M	Mach number	—
$Re_{\delta^{**}}$	momentum thickness Reynolds number, $\frac{\bar{c}\cdot\delta^{**}}{\nu}$	—
x	coordinate in the direction of the wall	(m)
y	coordinate normal to the direction of the wall	(m)
δ	boundary layer thickness	(m)
δ^*	boundary layer displacement thickness, $\int_0^\infty (1 - \frac{c_x}{\bar{c}}) dy$	(m)
δ^{**}	boundary layer momentum thickness, $\int_0^\infty \frac{c_x}{\bar{c}} (1 - \frac{c_x}{\bar{c}}) dy$	(m)
$\varkappa$	von Karman's universal constant	
Λ	Euler number, $-\frac{1}{\bar{c}} \cdot \frac{d\bar{c}}{dx} \cdot \delta^{**}$	—
λ	roughness density	—
ν	kinematic viscosity of fluid	(m²/s)
Π	pressure gradient parameter, $-\frac{1}{\bar{c}} \cdot \frac{d\bar{c}}{dx} \cdot \frac{\delta^*}{\tau_w/\rho\bar{c}^2}$	
ρ	density of fluid	(kg/m³)
τ_w	wall shear stress	(N/m²)

1- INTRODUCTION

In recent years, there have been increasing attempts to obtain better understanding of the behaviour of the turbulent boundary layer of incompressible flow in the presence of surface roughness. Measurements have been made in many experimental configurations and many prediction methods have been developed to get better understanding of the exact mechanism of turbulent motion, and to introduce a complete theoretical solution for that mechanism. These experimental and prediction methods included not only the effect of roughness but also the height of roughness elements, the size distribution, the shape of roughness elements, and their density distribution over the wall surface.

The framework of rough-wall flow was established by Nikuradse [6] who investigated flow in sand-roughened pipes, and found that with increasing Reynolds number the flow behaviour deviated from the turbulent smooth wall law and depended on k_s/d (k_s is sand grain roughness height and d is pipe diameter) as well as on Reynolds number, this was termed transition flow. At higher Reynolds number the flow becomes independent of viscosity and is a function of k_s/d alone and the latter flow was termed fully rough.

Prandtl and Schlichting assumed that the pipe and flat plate velocity profiles were nearly identical; [7]. Using Nikuradse's [6] results they calculated a skin-friction relation as a function of plate Reynolds number.

Clauser;[8]; gives the form of the logarithmic velocity distribution for flow over rough walls with zero pressure gradients as:

$$\frac{c_x}{c_\tau} = \frac{1}{\varkappa} \ln \frac{c_\tau \cdot y}{\nu} + B - C\left(\frac{c_\tau \cdot k}{\nu}\right), \qquad (1.1)$$

where c_τ is the friction velocity, $C(c_\tau .k/\nu)$ is the roughness function which is zero for smooth walls and $\varkappa$, B are universal constants. The Clauser form of the roughness function for fully rough flow is:

$$C\left(\frac{c_\tau . k}{\nu}\right) = \frac{1}{\varkappa} \ln\left(\frac{c_\tau . k}{\nu}\right) + C_1 , \qquad (1.2)$$

where C_1 is a constant.

Hama; [9]; showed from the results of an extensive experimental programme that equations (1.1) and (1.2) are both universal for a given roughness geometry in pipe, channel and zero pressure gradient boundary layer flow.

Clauser and Hama;[9]; have determined the value of C $(c_\tau k/\nu)$ for quite different types of roughness, with the constant C_1 in equation (1.2), dependent on the type of roughness.

Perry and Joubert;[8];showed that the universality of equations(1.1) and (1.2) extends to boundary-layer flow in adverse pressure gradients.

Perry, Schofield and Joubert [8] distinguished between two types of roughness, the "k" and "d" types.

The 'k' type roughness follows the Nikuradse - Clauser scheme in which eddies with a length scale proportional to 'k' are assumed to be shed into the flow above the crests of the elements. This type of roughness has a roughness function depending on Reynolds number based on the friction velocity and on a length associated with the size of the roughness.

The other type of roughness, d-type, is characterised by a smooth surface with a series of depressions or narrow leteral grooves within which the outer flow generates stable vortices. It has been tested in pipes by many researchers, and the investigators in [9] indicated from the results obtained, that the corresponding roughness function does not depend on roughness scale but depends instead on the pipe diameter.

A simplified solution for the boundary layer development on rough surface using Coles law of velocity distribution has been obtained [10], and indicated that the existance of roughness increases the shape parameter, I, while maintaining a constant pressure gradient parameter, π.

Two additional variables were introduced in the study of boundary layers on rough walls, the error in origin, ε, and the roughness density, λ. The roughness density, λ, is the ratio of total surface area to roughness area, $\lambda = L/S$, where L is spacing between roughness elements and S is length of roughness element. Its effect is introduced in the constant C_1 of equation (1.2).

Einstein and EL'Samni [10] observed that the origin of logarithmic velocity distribution was situated below the top of the roughness elements; consisting of hemispherical caps; by a distance of $\varepsilon = 0.2k$.

The error in origin can be considered as a measure of the interaction between the mean flow and roughness. Its value depends on the type of roughness [12], and [13].

Perry, Schofield and Joubert [8] stated equation (1.1) for k-type roughness as:

$$\frac{c_x}{c_\tau} = \frac{1}{\varkappa} \ln \frac{c_\tau (y_T + \varepsilon)}{\nu} + B - C\left(\frac{c_\tau . \varepsilon}{\nu}\right) , \qquad (1.3)$$

where y_T the distance measured from the top of the roughness elements for fully rough flow, the roughness function, C $(c_\tau . \varepsilon/\nu)$, takes the form:

$$C\left(\frac{c_\tau . \varepsilon}{\nu}\right) = \frac{1}{\varkappa} \ln \frac{c_\tau . \varepsilon}{\nu} + \text{constant} . \qquad (1.4)$$

Bettermann;[9]; correlated his measurements on two-dimensional roughness elements (rod) of varying spacing, L, in terms of equation (1.2) with the constant, C, being a function of the roughness density, λ, and observed that for a certain spacing of rods, the value of

$C\,(c_\tau .k/\nu)$ was maximum, and as the spacing was increased, $C\,(c_\tau .k/\nu)$ decreased.

Comparison between the results obtained from the different resources of velocity distribution on rough surfaces indicates that there is no acceptable form for this distribution.

The main objective of this investigation is to obtain a law for the velocity distribution in turbulent boundary layer in the presence of different surface roughness and adverse pressure gradient. This work has been developed on the basis of theoritical and experimental results of different investigators [1 : 4] and the velocity law in turbulent flow over rough surface [5].

2- ANALYSIS OF VELOCITY PROFILES

2.1. Velocity Distribution in Turbulent Flow over Rough Surfaces

The prediction of the development of turbulent boundary layers over smooth surfaces can be achieved by many methods but the corresponding published methods for rough surfaces are few and all are restricted to the prediction of the momentum thickness and the skin-friction coefficients in the fully rough regime in zero-pressure gradient.

The calculation concerning boundary layers with non-zero pressure gradients are more difficult than those concerning flat plates, owing to the large number of independary variables. However, the most evaluated values of the different boundary layer parameters depend on the used form of the velocity distribution. Therefore the general acceptable form for this distribution is given as:

$$\frac{c_x}{c_\tau} = \underbrace{\frac{1}{\varkappa}\,.\ln\frac{c_\tau .y}{\nu} + B}_{\text{Law of the wall}} + \underbrace{\frac{2A}{\varkappa}\,.\frac{y}{\delta}}_{\text{Correction Function}} + \underbrace{C\left(\frac{c_\tau .k}{\nu}\right)}_{\text{Surface Roughness Function}} \quad \ldots .\ 0 \le y \le \delta\ , \qquad (2.1)$$

where $\varkappa$, B are two emperical constants of 0.4 and 5.2 respectively, and the $C\,(c_\tau .k/\nu)$ equals zero for smooth surfaces.

The velocity distribution; equation (2.1) is similar to that suggested by Rotta [13] and independently by Ross and Robertson [15] with exception that it does not contain the constant B taking $C\,(c_\tau .k/\nu) = 5.2$ for smooth surface. Equation (2.1) has a more generalization in its use than that given by [14].

At the outer edge of the boundary layer, at $y = \delta$, $c_x = \bar{c}$. Substituting these values in equation (2.1), the following relation is obtained:

$$\frac{\bar{c}}{c_\tau} = \frac{1}{\varkappa}\ln\frac{c_\tau .\delta}{\nu} + B + \frac{2A}{\varkappa} + C\left(\frac{c_\tau .k}{\nu}\right), \qquad (2.2)$$

which gives an expression for the local skin friction coefficient, $c_f/2 = (c_\tau/\bar{c})^2$.

From equation (2.1) and (2.2) the velocity distribution in the boundary layer is obtained as :

$$\frac{c_x}{\bar{c}} = 1 + \sqrt{\frac{\tau_w}{\rho .\bar{c}^2}}\ .\left[\frac{1}{\varkappa}\ \ln\frac{y}{\delta} - \frac{2A}{\varkappa}\left(1 - \frac{y}{\delta}\right)\right], \qquad (2.3)$$

It may be noticed from equation (2.3) that the surface roughness (through $C\,(c_\tau .k/\nu)$) has no effect on the velocity distrbution.

The two boundary layer parameters defined in the nomenclature; the dimensionless displacement thickness δ^*/δ and the dimensionless momentum thickness δ^{**}/δ, are obtained by substituting the value of $c_x/\bar{c}$ in their formulae so that:

$$\frac{\delta^*}{\delta} = \frac{1}{\varkappa}\ .\sqrt{\frac{\tau_w}{\rho\bar{c}^2}}\ .\ (1 + A)\ ; \qquad (2.4)$$

$$\frac{\delta^{**}}{\delta} = \frac{1}{\varkappa} \cdot \sqrt{\frac{\tau_w}{\rho \bar{c}^2}} \cdot (1 + A) - \frac{1}{\varkappa^2} \cdot \frac{\tau_w}{\rho \bar{c}^2} \cdot (2 + 3A + \frac{4}{3} A^2) \cdot \qquad (2.5)$$

The remaining boundary layer parameters such as the form parameter H_{12}, shape parameter I, and the pressure gradient parameter Π may all be represented by separate formulas. They are :

Form parameter : $$H_{12} = \frac{\delta^*/\delta}{\delta^{**}/\delta} = \frac{1}{1 - \frac{1}{\varkappa}\sqrt{\frac{\tau_w}{\rho \bar{c}^2}} \cdot \frac{(2 + 3A + \frac{4}{3}A^2)}{(1 + A)}} \qquad (2.6)$$

Shape parameter : $$I = (1 - \frac{1}{H_{12}}) \cdot \frac{1}{\sqrt{\tau_w/\rho \bar{c}^2}} = \frac{2 + 3A + \frac{4}{3}A^2}{(1 + A)} \qquad (2.7)$$

Solving the previous equations for A ; which has two roots; the suitable form is :

$$A = \frac{1}{8} [3.\varkappa . I - 9 + \sqrt{(3.\varkappa . I - 5)(3\varkappa . I + 3)}] ; \qquad (2.8)$$

Pressure gradient parameter $$\Pi = \Lambda \cdot \frac{H_{12}}{\tau_w / \rho \bar{c}^2} \qquad (2.9)$$

For flow past a flat plate at zero incidence, $\Pi = 0$, also $\Lambda = 0$, while for boundary layers with adverse pressure gradient $\Pi > 0$ and $\Lambda > 0$.

2.2. Surface Roughness Function

The contribution of surface roughness effect to the velocity distribution c_x/c_τ is represented by the surface roughness function $C(c_\tau . k/\nu)$. For completly rough surface, it is given by the relation :

$$C(\frac{c_\tau k}{\nu}) = C_r - \frac{1}{\varkappa} . \ln \frac{c_\tau k}{\nu} , \qquad (2.10)$$

with C_r the constant of surface roughness function, which can be estimated after substituting equation (2.10) into equation (2.1) which yields :

$$\frac{c_x}{c_\tau} = - \frac{1}{\varkappa} (\ln \frac{k}{y} - 2A \frac{y}{\delta}) + B + C_r . \qquad (2.11a)$$

At the outer edge of the boundary layer, i.e., y = 0, $c_x = \bar{c}$, equation (2.11a) becomes :

$$\frac{\bar{c}}{c_\tau} = - \frac{1}{\varkappa} (\ln \frac{k}{\delta} - 2A) + B + C_r . \qquad (2.11b)$$

To get the ratio of the roughness height to the momentum thickness, k/δ^{**}, this will be obtained by substituting the ratios δ^*/δ and δ^{**}/δ from (2.4) and (2.6) in (2.11b), thus

$$\frac{\bar{c}}{c_\tau} = - \frac{1}{\varkappa} [\ln \frac{k}{\delta^{**}} + \ln \frac{1}{H_{12}} + \ln (\sqrt{\frac{\tau_w}{\rho \bar{c}^2}} . \frac{1 + A}{\varkappa}) - 2A] + B + C_r .$$

Then, the expression for C_r is given by :

$$C_r = \sqrt{\frac{2}{c_f}} - B + \frac{1}{\varkappa} \ln \frac{1 + A}{\varkappa} - \frac{2A}{\varkappa} + \frac{1}{\varkappa} \ln (\frac{k}{\delta^{**}} . \frac{1}{H_{12}} . \sqrt{\frac{c_f}{2}}) \; .... \; (2.12)$$

Equation (2.12) gives a method, which is used in the computer programme;

to estimate the constant of surface roughness, C_r , based on knowing the velocity profile parameter, A, the ratio k/δ^{**}, the form parameter, H_{12}, and the dimensionless wall shear stress, $c_f/2$.

The effect of momentum thickness Reynolds number, $Re_{\delta^{**}}$, on the surface roughness function, $C(c_\tau . k/\nu)$, can be found by performing $Re_{\delta^{**}}$ to have the following form :

$$Re_{\delta^{**}} = \frac{c . \delta^{**}}{\nu} = \frac{\bar{c}}{c_\tau} . \frac{c_\tau . \delta}{\nu} . \frac{\delta^{*}}{\delta} . \frac{\delta^{**}}{\delta^{*}}$$

$$= \frac{1+A}{\varkappa} . \frac{1}{H_{12}} . \exp\left[\varkappa\left(\sqrt{\frac{2}{c_f}} - B - C\left(\frac{c_\tau . k}{\nu}\right) - 2A\right]\right. , \qquad (2.13)$$

Equation (2.12) give the relation between $Re_{\delta^{**}}$ and $C(c_\tau . k/\nu)$. For smooth surfaces, i.e., $C(c_\tau . k/\nu) = 0$ eqation (2.13) yields :

$$Re_{\delta^{**}})_{rough} = \frac{1+A}{\varkappa} . \frac{1}{H_{12}} . \exp\left[\varkappa\left(\sqrt{\frac{2}{c_f}} - B\right) - 2A\right] , \qquad (2.14)$$

where $Re_{\delta^{**}})_{rough}$ is the admissible minimum Reynolds number for rough surface.

When the actual Reynolds number, $Re_{\delta^{**}}$ is lower than $Re_{\delta^{**}})_{rough}$ the surface is to be considered as being hydraulically smooth.

The surface roughness function can be estimated from equation (2.13), this gives:

$$C\left(\frac{c_\tau k}{\nu}\right) = \sqrt{\frac{2}{c_f}} - B - \frac{1}{\varkappa}\left[\ln Re_{\delta^{**}} - \ln\left(\frac{1+A}{\varkappa} . \frac{1}{H_{12}}\right) + 2A\right] \qquad (2.15)$$

Equation (2.15) was used in the computer programme for the evaluation of $C(c_\tau . k/\nu)$ for different values of $Re_{\delta^{**}}$.

3 – PRESENTATION AND DISCUSSION OF RESULTS

The velocity profiles at ratios of roughness height to momentum thickness $k/\delta^{**} = 0.05$ and 0.3 for Euler numbers, $A = 0$, $1*10^{-3}$, $2*10^{-3}$, and $3*10^{-3}$ are plotted in Fig. (la & lb) .

Comparing figures (la) & (lb), shows that for constant Euler number A, the velocity distribution decreases slightly with the increase of the ratio k/δ^{**} .

The velocity profiles for constant velocity profile parameter, are represented in Fig. (2). This figure shows turbulent boundary layers at velocity profile parameters A = 1.0, Fig. (2a), and A = 4.0, Fig. (2b), with the ratio k/δ^{**} as a parameter taking the values 0.05 and 0.3. For the same velocity profile parameter, the velocity ratio $c_x/\bar{c}$ decreases with the increase of the ratio k/δ^{**}, due to the decrease of the dimensionless wall shear stress, $\tau_w / \rho \bar{c}^2$.

Figure (3) represents the variation in the velocity profile parameter, A, with Euler number, Λ, for different values of the ratio k/δ^{**}. The figure consists of four curves for values of k/δ^{**} equal to 0.05, 0.08, 0.1 and 0.3 .

For boundary layers at the same value of k/δ^{**}, the velocity profile parameter, A, increases as Euler number, Λ, increases. But the increment in the value of the parameter A for values of Euler number $\Lambda > 1.5*10^{-3}$ is greater than that for Euler numbers below that value.

For the same value of the velocity profile parameter, A = constant, Euler number Λ increases as the ratio k/δ^{**}, increases.

Figure (4) presents the change in the velocity profile parameter, A with the pressure gradient parameter, π , for flows over rough and smooth surfaces. This relation for smooth surface is adopted from [16] and its intersection with the vertical axis ($\pi = 0$) referring to the turbulent boundary layers on flat plate with zero pressure gradient occurs at $A \simeq 0.64$

corresponding to shape parameter $I \approx 6.8$. For flows over rough surface all points for different ratios of k/δ^{**} lie on one curve.

Figure (5) illustrates the relation between I and Π for ratios of $k/\delta^{**} = 0.03, 0.05, 0.1$ and 0.3. also presents this relation for flow over smooth surfaces investigated theoretically by Mellor and Gibson [15] ; adopted from Felsh [3]; and [16]. The results show good agreement to Mellor-Gibson curve.

Figure (6) illustrates the variation of the constant of surface roughness function C_r with the ratio k/δ^{**} and Euler number Λ as parameter. It has the functional form $C_r = C_r(k/\delta^{**}, \Lambda)$ which results from the simplification of the form $C_r = C_r(A, k/\delta^{**}, c_f/2)$; equation (2.12); by making use of the relations $A = A(\Lambda, k/\delta^{**})$; Fig. (3).

Figure (6) shows the previous relation of C_r for Euler numbers $\Lambda = 0, 1*10^{-3}, 2*10^{-3}$ and $3*10^{-3}$. For constant Euler number, Λ = const., each curve can be divided into two parts, the first part for values of the ratio $k/\delta^{**} < 0.1$, at which the constant C_r decreases as the ratio k/δ^{**} increases. In the second part, $k/\delta^{**} > 0.1$, the constant C_r increases with the increase of the ratio k/δ^{**}.

At the same value of the ratio k/δ^{**}, the constant C_r increases as Euler number, Λ, increases. Also, the velocity profile parameter, A, increases; Fig. (3).

Another observation from the plot at ratios $k/\delta^{**} < 0.1$, is that the decreasing rate in the constant C_r increases with an increment in Euler number, Λ.

The results obtained for the value of C_r contradicts with the suggestion of Rotta [13] taking the constant C_r ; $C_r + B = 8.4$; for sand grain type of roughness to remain unaffected by the existence of pressure gradient.

The influence of pressure gradient in form of Euler number, Λ, on the surface roughness function $C(\frac{c_\tau k}{\nu})$ is illustrated in Fig. (7) for values of the ratio $k/\delta^{**} = 0.05$ & 0.30 with the momentum thickness Reynolds number, $Re_{\delta^{**}}$, as a parameter. At constant k/δ^{**} & Λ the function $C(\frac{c_\tau k}{\nu})$ increases as $Re_{\delta^{**}}$ decreases. Comparing Fig. 3a with 3b indicates that at the same $Re_{\delta^{**}}$ the function $C(\frac{c_\tau k}{\nu})$ decreases as the ratio k/δ^{**} increases.

The variation of the surface roughness function, $C(\frac{c_\tau k}{\nu})$, with the dimensionless wall shear stress, $\tau_w / \rho \bar{c}^2$, and the momentum thickness Reynolds number, $Re_{\delta^{**}}$, as a pparameter are represented in figure 8 . This figure is drawn for values of the ratio k/δ^{**} = 0.05 and 0.3 . Each figure contains five curves for constant Reynolds number $Re_{\delta^{**}} = 4*10$, $8*10^4$, $2*10^5$, $4*10^5$ and 10^6 .

For constant ratio of k/δ^{**} and constant dimensionless wall shear stress, $\tau_w/\rho\bar{c}^2$, the function $C(\frac{c_\tau k}{\nu})$ increases as the momentum thickness Reynolds number, $Re_{\delta^{**}}$, decreases. For a constant $Re_{\delta^{**}}$, the surface roughness function $C(\frac{c_\tau k}{\nu})$ increases as the dimensionless wall shear stress, $\tau_w/\rho\bar{c}^2$, decreases.

For the same value of Reynolds number $Re_{\delta^{**}}$ the surface roughness function, $C(\frac{c_\tau k}{\nu})$, decreases as the ratio k/δ^{**} increases, knowing that as the roughness height k increases, the surface roughness function $C(\frac{c_\tau k}{\nu})$ decreases, Eq. (2.10).

Figure 9 illustrate the variation of surface roughness function, $C(\frac{c_\tau k}{\nu})$ with the velocity profile parameter, A. The figure drawn for constant ratios of k/δ^{**}.

For constant ratio of k/δ^{**} and constant parameter A, the roughness function $C(\frac{c_\tau k}{\nu})$ increases as Reynolds number $Re_{\delta^{**}}$ decreases, this is similar to that for the variation with Euler number, Λ. For a constant Reynolds number $Re_{\delta^{**}}$, the surface roughness function C ($\frac{c_\tau k}{\nu}$) increases as the velocity profile parameter, A, increases. Observing that, for this last case, from figure(8) the dimensionless wall shear stress, $\tau_w/\rho \bar{c}^2$, decreases, i.e, the velocity profile $\bar{c}/c_\tau$ becomes fuller.

For constant momentum thickness Reynolds number, $Re_{\delta^{**}}$, the surface roughness function C ($\frac{c_\tau k}{\nu}$) decreases as the ratio k/δ^{**} increases.

Figure (10) indicates the relation between the admissible minimum Reynolds number for rough surface, $Re_{\delta^{**}})_{rough}$, and Euler number, Λ, with the ratio k/δ^{**} as a parameter. Equation (3.22) is used for calculating these values, which gives the case when the surface roughness function C ($\frac{c_\tau k}{\nu}$) = 0.

A surface is to be considered as being hydraulically smooth when the actual momentum thickness Reynolds number, $Re_{\delta^{**}}$, is lower than $Re_{\delta^{**}})_{rough}$ given by Fig.(10). The figure contains four curves for values of the ratio k/δ^{**} = 0.05, 0.08, 0.1 and 0.3.

For constant Euler number, Λ, the admissible minimum Reynolds number for rough surface $Re_{\delta^{**}})_{rough}$ increases with the increase of Euler number, Λ. Each curve can be divided into two parts. First part, the slope of the curve is positive and approximately constant, this means the increase of $Re_{\delta^{**}})_{rough}$ with the increase of Euler number, Λ. Second part, the slope of the curve increases which also increases with the decreasing of the ratio k/δ^{**}, that means large increase in the value of $Re_{\delta^{**}})_{rough}$ for small increase in Euler number, Λ. For example, at the ratio k/δ^{**} = 0.05, $Re_{\delta^{**}})_{rough}$ increases from 1023.8 at $\Lambda = 2.5*10^{-3}$ to 2058 at $\Lambda = 3*10^{-3}$. As can be detected from the figure, as the ratio k/δ^{**} tends to zero; i.e. smooth surface; the value of $Re_{\delta^{**}})_{rough}$ goes to infinity .

CONCLUSIONS

As a result of the present investigation the following conclusions concerning specific aspects about the velocity profile on turbulent boundary layers over rough surfaces are obtained.

For boundary layers at the same ratio of the roughness height to momentum thickness, k/δ^{**}, the velocity profile decreases with the increase of Euler number, Λ.

For boundary layers at the same velocity profile parameter. A, the velocity profile decreases with the increase of the ratio, k/δ^{**}. Moreover, the velocity parameter, A, increases with the increase of Euler number, Λ, and the decrease of the ratio, k/δ^{**}, it also increases with the increase of the pressure gradient parameter, .

The variation of the shape parameter, f, with the pressure gradient parameter, π, for different ratios of, k/δ^{**}, is in good agreement with the investigation of Mellor - Gibson [15] .

The constant of surface roughness function, C_r , varies with the pressure gradient in form of Euler number for different ratios of, k/δ^{**}. This contradicts the suggestion of Rotta [13] and others that it remains constant , C_r ,i. e., unaffected by the existance of pressure gradient. Also, for certain ratio of, k/δ^{**}; k/δ^{**} = 0.1; the constant, C_r , is a minimum for all Euler numbers and as the ratio is increased or decreased, C_r , increases. A similar observation was made by Betermann; [2]; and Liu et al; for the variation of C_r , with the roughness density, .

The surface roughness function, C ($\frac{c_\tau k}{\nu}$); for boundary layers at the same ratio of, k/δ^{**}, and the momentum thickness Reynolds number, $Re_{\delta^{**}}$, increase with the increase of Euler number, A. Furthermore the admissible Reynolds number, $Re_{\delta^{**}})_{rough}$, increases with the increase of the ratio, k/δ^{**} .

REFERENCES

[1] Dvorak, F. A. "Calculation of Turbulent Boundary layer on Rough Surfaces in Pressure Gradient" . AIAA Journal, Vol. 7, No. 9, pp 1752-1759, Sept. 1969.

[2] Perry, A. E Schofield, W. H. Joubert, P. N. "Rough Wall Turbulent Boundary Layers". Journal of Fluid Mechanichs, Vol. 37, Part 2, pp. 38-413, 1969 .

[3] Felsh, K. O. "Eine neue Methode zur Berechnung Turbulenter Grenzschichten" Wissenschaftliche Zeitschrift der Technischen Universität Dresden. 16 (1967) Heft 4.

[4] Zancow, S. "Integration der Impulsgleichung mit Berücksichigung der Wandrauhigkeit und des Druckgradient der Aussenströmung". Dissertation TU Dresden, 1974.

[5] Coles, D. "The law of the wake in the turbulent boundary layer". Journal of Fluid Mechanics, Vol. 1, pp. 191-226, 1956.

[6] Nikuradse, J., "Strömungsgesetze in rauhen Rohren". V. D. Forshungsheft, No. 361, 1933.

[7] Hinze, J. O., "Turbulence". 2 nd Edition, McGraw-Hill Book Company, New York, 1975.

[8] Perry, A. E. Schofield, W. H. Joubert, P. N. "Rough Wall Turbulent Boundary Layers". Journal of Fluid Mechanics, Vol. 37, Part 2, pp. 383-413, 1969 .

[9] Abou - Rayan, S. M. , "Study of Boundary Layer with Roughness Effect" . M. Sc. Thesis, El-Mansoura University, 1984.

[10] Einstein, H. A. EL-Samni, S. A. , "Hydrodynamic Forces on a Rough Wall". Review Modern Physics, Vol. 21, pp. 520-524, 1949.

[11] Blinco, P. H. Partheniades, E. , "Turbulence Characteristics In Free Surface Flows over smooth and Rough Boundaries". Journal of Hydraulic Research, Vol. 9, No. 1, 1971.

[12] Jackson, P. S. , "On the displacement height in the logarithmic Velocity Profile" . Journal of Fluid Mechanics, Vol. 111, pp. 15-25, 1981.

[13] Rotta , J. C. , "Turbulent Boundary Layers in Incompressible Flows" . Progress in Aeronautical Sciences, Vol. 2, pp. 1-220, Pergamon Press, 1962 .

[14] Ross, D. Robertson, J. M. , "A superposition analysis of the turbulent boundary layer in an adverse pressure gradient". Journal of Applied Mechanics, Vol. 18, pp. 95-100, 1951.

[15] Mellor, G. L. , Gibson, D. M. , "Equilibrium turbulent boundary layers". Journal of Fluid Mechanics, Vol. 24, Part 2, pp. 225-253, 1966 .

[16] Saad, M. S. , "An Investigation of Some Turbulent Boundary layer parameters". M. Sc. Thesis, Mansoura University, 1980.

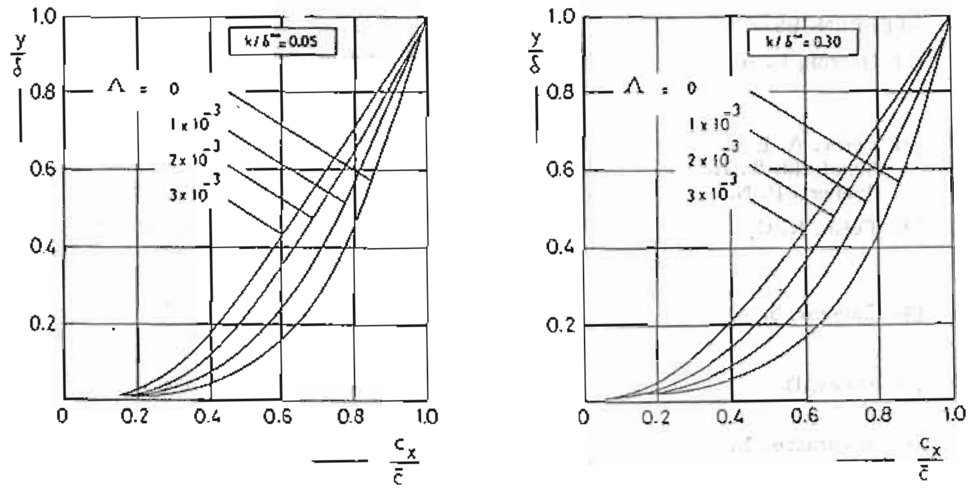

Fig.(1) Turbulent boundary layer velocity profiles at constant ratios of roughness height to momentum thickness k/δ^{**}

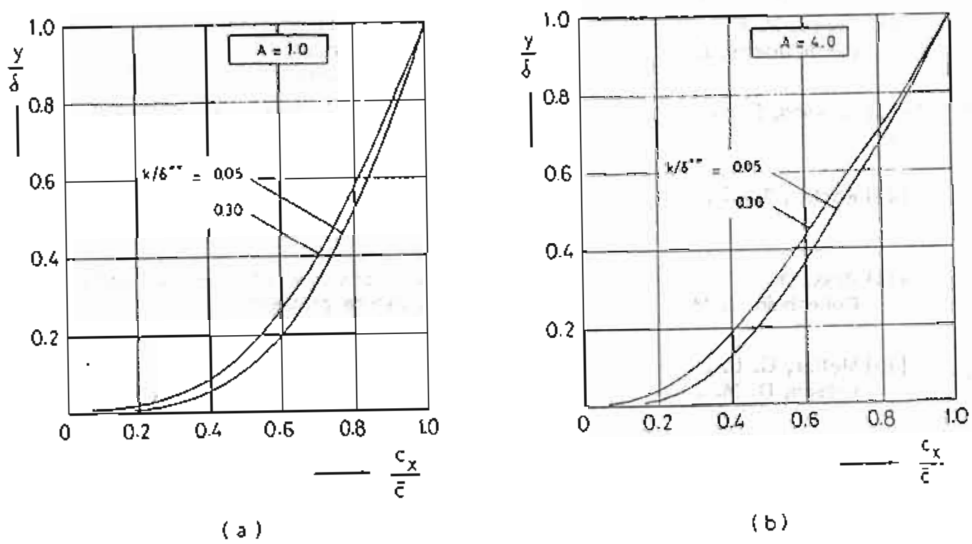

Fig (2) Turbulent boundary layer velocity profiles at constant velocity profile parameters A

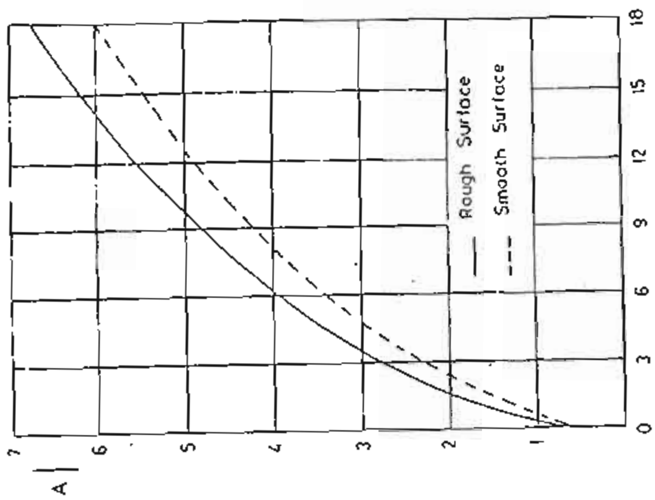

Fig.(4) Velocity profile parameter versus pressure gradient parameter

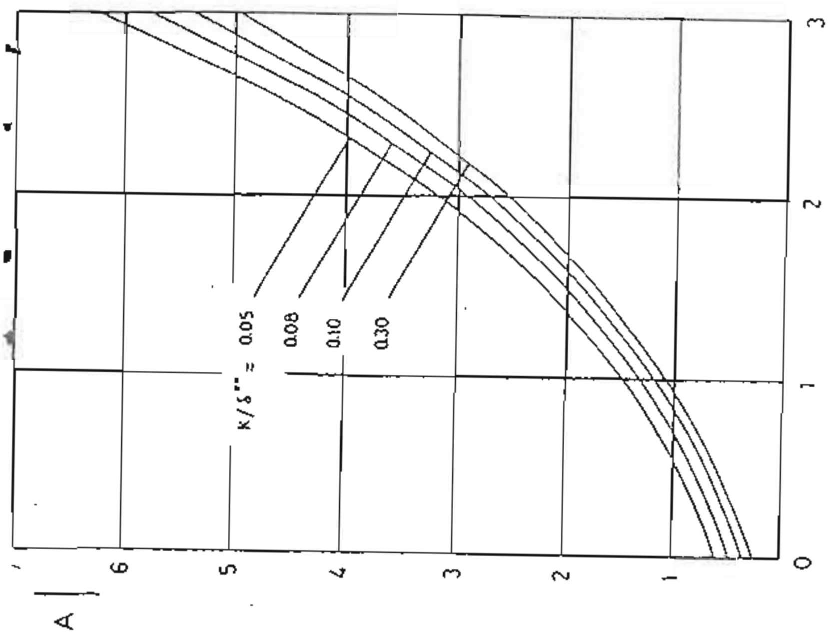

Fig.(3) Velocity profile parameter versus Euler number with the ratio k/δ** as parameter

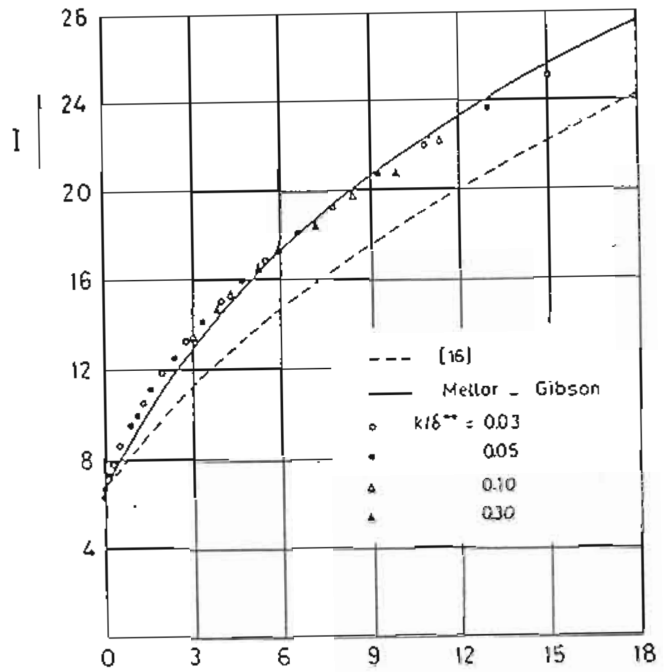

Fig.(5) Shape parameter versus pressure gradient parameter

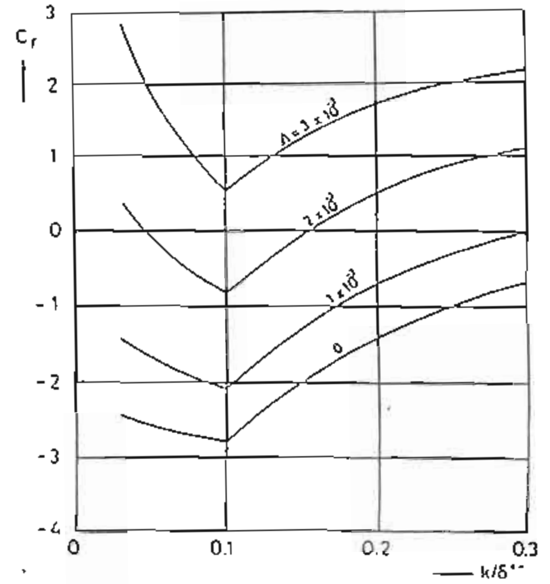

Fig.(6) Constant of surface roughness function versus the ratio k/δ** with Euler number as parameter

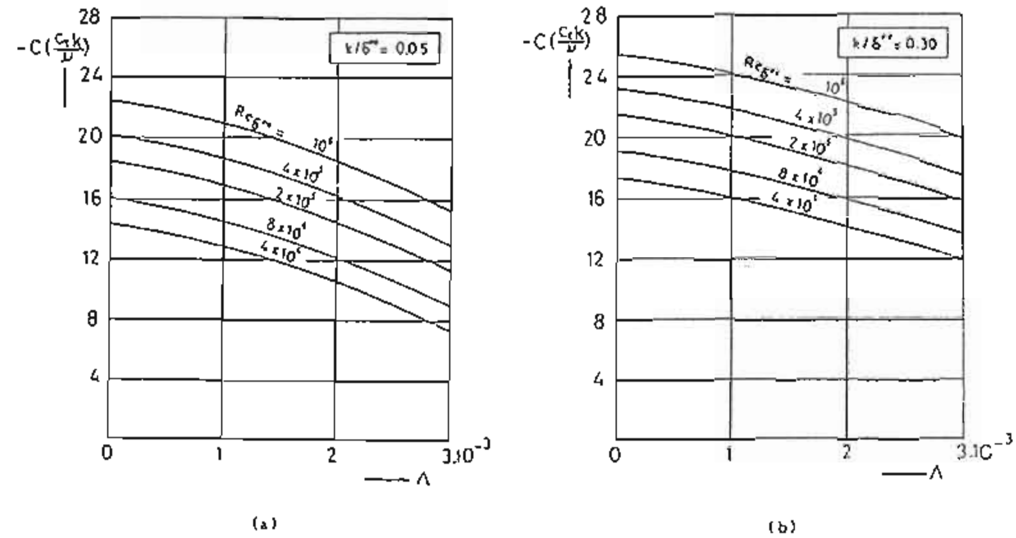

Fig.(7) Surface roughness function versus Euler number with momentum thickness Reynolds number as parameter, at constant ratios of k/δ^{**}

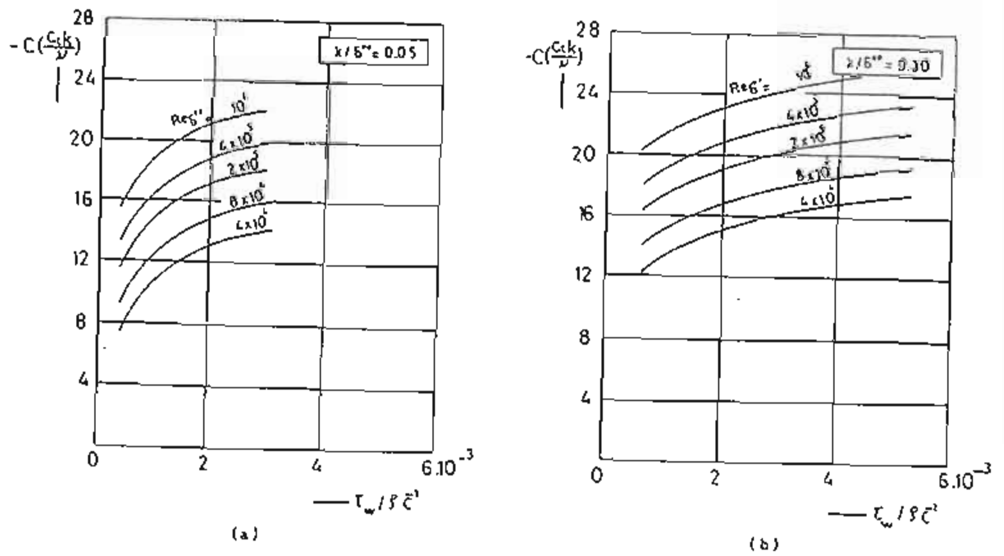

Fig.(8) Surface roughness function versus dimensionless wall shear stress with momentum thickness Reynolds number as parameter, at const. ratios of k/δ^{**}

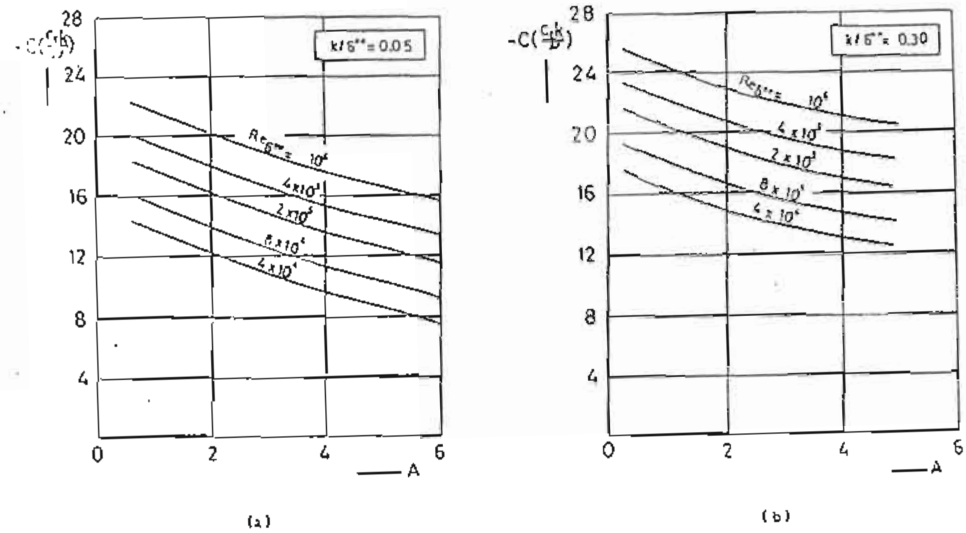

Fig.(9) Surface roughness function versus velocity profile parameter with momentum thickness Reynolds number as parameter, at const. ratios of k/δ**

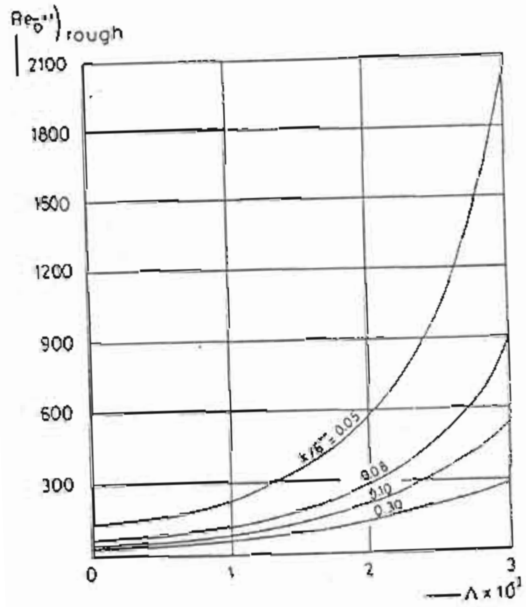

Fig. (10) Admissible minimum Reynolds number for rough surface versus Euler number with the ratio k/δ** as parameter