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AN ENHANCED NUMERICAL ONE-DIMENSIONAL TRANSIENT FUEL ROD CODE FOR LIGHT WATER REACTORS

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نموذج عددي محسن احادي الابعاد لدرجات الحراره الغير مستقره
في قضبان الوقود النووي لمفاعلات الماء الخفيف

تهدف هذه الورقه الى بحث التطور الزمني لدرجات الحراره في قضبان الوقود النووي لمفاعلات الماء الخفيف وذلك في الحاله الغير مستقره ، وذلك بتطوير نموذج رياضي عددي احادي الابعاد ، ويعتمد هذا النموذج على تحويل المعادله التفاضليه لتوصيل الحراره الغير مستقر مع وجود مصدر طاقه داخلي الى معادله جبريه ، وبتقسيم قضيب الوقود في الاتجاه القطري الى رقائق اسطوانيه وتطبيق المعادله الجبريه على كل رقيقه نحصل على مجموعه من المعادلات الانيه التي يمكن حلها مع اعتبار الشروط الحدوديه . وتتميز هذه الطريقه بطول الفتره الزمنيه الازمه كشرط للحصول على حل عددي مستقر وذلك باستخدام الطريقه الصريحه والتي تتميز ببساطتها وسرعة الحل . كما تم في هذا العمل تعديل معادله تحليليه رياضيه للتوصيل الحراري الغير مستقر في قضيب وقود غير مغلف وذلك لتطبيقها على قضيب مغلف. وتدل النتائج على ان الحل العددي المقترح يعطي نتائج تتفق مع الحل التحليلي المعقد والذي يحتاج ايضا الى برمجه ، كما تدل النتائج على ان درجه حرارة سطح الغلاف تصل الى الحاله المستقره اسرع من درجة حرارة مركز الوقود عند اي معدل لانتاج الطاقه الداخليه .

ABSTRACT

In this paper a new enhanced simple numerical fuel rod code is developed, which is used to calculate the transient one-dimensional fuel temperature behavior in Light Water Reactors. The proposed code is then applied to determine the transient temperature behavior of fuel due to sudden rise in heat generation rate. The obtained results are then compared with that obtained from an analytical transient solution. The steady state values obtained from both solutions are then compared with that obtained from the corresponding closed form solution of this problem. It is proved that the numerical code is more powerful and accurate.

INTRODUCTION

Heat conduction with internal heat generation is an important problem in the fields of nuclear and electrical engineering. Solid reactor and radioisotopic fuel rods and electrical resistance heaters are elements in which heat is both generated and conducted. Temperature development in both steady and transient operation is important in evaluating reactor core thermal performance. The amount of power generation in a given reactor is limited by thermal rather than by nuclear considerations [1,2]. The reactor core must be operated such that with adequate heat removal system, the temperature of the fuel and cladding anywhere in the core must not exceed safe limits. The behavior of the fuel-coolant combination in response to reactivity insertions, loss of coolant, or other transient effects is of vital importance. In case of loss of coolant accident, the time after which the fuel or cladding meltdown temperatures is reached is very important, therefore a *fuel rod code* is used in conjunction with a nuclear code, core code, and loop code to determine the temperature behavior of cladding surface [3].

Exact analytical solutions of this problem in only a few cases for different geometries may exist. Exact solution to the transient, one-dimensional form of the heat equation have been developed for an infinite bare cylinder with internal heat generation [2,4]. The solution with surface condition which is characterized by convective heat transfer coefficient h_{eff} is as follows [2]:

$$T^* = \sum_{n=1}^{\infty} (C_n/\zeta_n^2).[1.0 - EXP(-\zeta_n^2 Fo)].J_o(\zeta_n r^*) \qquad (1)$$

where $T^* = (T-T_a)/T_o$, $\qquad T_o = (q''' R_o^2)/k$

$C_n = (2/\zeta_n)[J_1(\zeta_n)/(J_o^2(\zeta_n)+J_1^2(\zeta_n))$

$r^* = r/R_o$, $\qquad Fo = k\theta/(R_o^2 \rho C)$, $\qquad Bi = h_{eff} R_o/k$

and the discrete values (eigenvalues) ζ_n are positive roots of the transcendental equation

$$\zeta_n \frac{J_1(\zeta_n)}{J_o(\zeta_n)} = Bi$$

The quantities J_1 and J_o are Bessel functions of the first kind.

According to [2] it can be shown that for values of $Fo \geq 0.2$, the infinite series solution can be approximated by the first term of the series.

In light water reactors, the coolant in the core is considered as a boundary condition which supplies the fuel rods with a sink temperature and a heat transfer coefficient. Conduction in the radial direction is usually taken into account [5,6] and some codes consider conduction in the axial conduction also. With nonuniform cooling, conduction in the azimuthal direction must be considered [7]. Thermal radiation from rod-to-rod is negligible when the core is filled with liquid. There are many fuel rod codes to predict the steady and transient temperature behavior of nuclear fuel rods. The problem is to improve the time and space of computation. For this purpose, the present fuel rod code is developed, in which one dimensional transient heat conduction is taken into account.

MATHEMATICAL PROCEDURE

The general heat conduction equation is given by

$$\nabla^2 T + \frac{q'''}{k} = \frac{\rho C}{k} \frac{\partial T}{\partial \theta} \qquad (2)$$

To transform the above equation into algebraic equation, the time variable θ and space variable r are broken into discrete intervals $\Delta\theta$ and Δr as shown in Fig.(1).

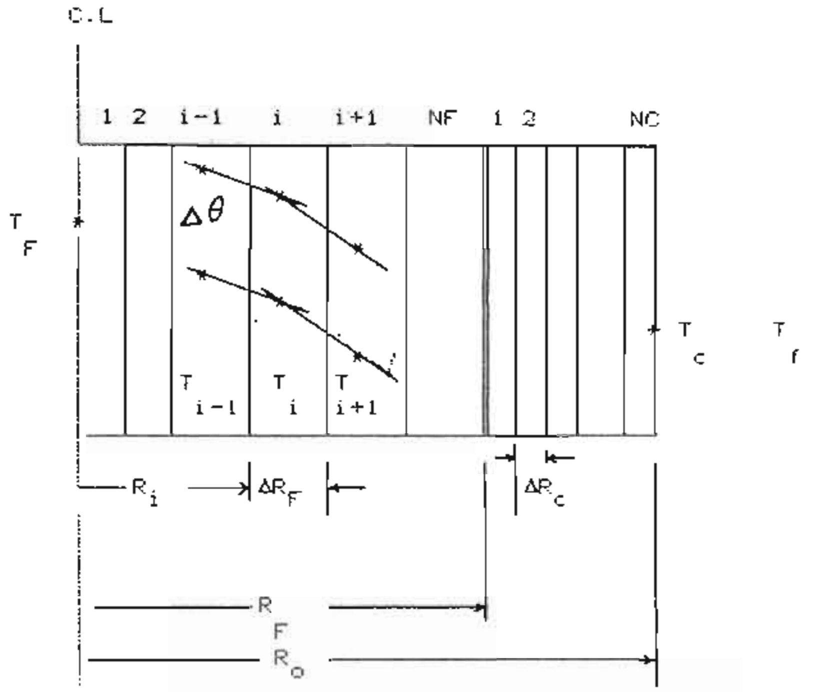

Fig. 1 Nodes in one-dimensional cylindrical system

Applying Eq.(2) to the nodal point i for one dimensional Laplacian in cylindrical coordinates and rearranging, one get the following linear equation :

$$T_i^{\theta+\Delta\theta} = (1-F_1-F_2)T_i^{\theta} + F_1 T_{i-1}^{\theta} + F_2 T_{i+1}^{\theta} + FO\ \Delta T_g \qquad (3)$$

where $F_1 = \dfrac{2FO\ R_i \Delta R}{A}$, $F_2 = \dfrac{2FO\ R_{i+1} \Delta R}{A}$

$A = R_{i+1}^2 - R_i^2$, $FO = \dfrac{k\ \Delta\theta}{\rho c\ \Delta R^2}$

and $\Delta T_g = \dfrac{q''' \Delta R^2}{k}$

The following boundary conditions are applicable for the considered case :

* For the innermost layer i = 1, we have

$$(\partial T/\partial r)_{r=0} = 0.0$$

To satisfy this condition , the coefficient F_1 must equal zero.

* For the outermost cladding layer i = N, we have

$$(\partial T/\partial r)_{r=R_o} = -(\alpha/k_c).(T_N - T_F)$$

To satisfy the above condition, the coefficients F_1, F_2 are given by:

$$F_1 = \frac{2(FO)_c R_N \ \Delta R_c}{(R_o^2 - R_N^2)} \ , \text{ and}$$

$$F_2 = \frac{2(FO)_c \ h \ R_o \ \Delta R_c^2}{k_c (R_o^2 - R_N^2)} = \frac{2(FO)_c \ (Bi)_c \Delta R_c^2}{(R_o^2 - R_N^2)} \qquad (4)$$

To account for the effect of the helium gas gap at the interface between the fuel surface and the inner cladding surface, the coefficients F_1 and F_2 for both the last fuel layer NF and the first cladding layer NF+1 are given by :

* For the layer i=NF

$$F_1 = \frac{2(FO)_F \ R_{NF} \ \Delta R_F}{(R_{NF+1}^2 - R_{NF}^2)}$$

$$F_2 = \frac{2(FO)_F \ R_{NF+1} \ \Delta R_F^2}{k_F (R_{NF+1}^2 - R_{NF}^2)[(\Delta R_F/2k_F)+(1/h_g)+(\Delta R_c/2k_c)]} \qquad (5)$$

* For the layer i= NF + 1

$$F_1 = \frac{2(FO)_c \ R_{NF+1} \ \Delta R_c^2}{k_c (R_{NF+2}^2 - R_{NF+1}^2)[(\Delta R_F/2k_F)+(1/h_g)+(\Delta R_c/2k_c)]}$$

$$F_2 = \frac{2(FO)_c \ R_{NF+2} \ \Delta R_c}{(R_{NF+2}^2 - R_{NF+1}^2)}$$

Each body of the fuel and cladding is divided into a number of concentric cylindrical layers of equal thickness ΔR_F and ΔR_c ,Fig.(1). Applying Eq.(3) to each layer one gets a system of finite difference equations. There are several methods for the solution of a system of simultaneous linear equations [2,4,5,6,8]. The forward difference technique (explicit scheme) is however simpler but is not unconditionally stable, while the backward technique (implicit scheme) is unconditionally stabel but more complex [4,6,8]. The forward differences applied in the present code result in errors that are proportional to $\Delta r^2 \Delta \theta$ [2,8]. A KWU 1300 MWe pressurized water reactor is chosen as an illustration example [9]. The following data are required :

Average volumetric heat generation rate $q'''_{av} = 3.\times10^8$ W/m^3

Inlet temperature 291 ^oC, Fuel rod pitch = 12.7 mm,

Fuel is UO_2, Cladding is Zircaloy-4,

Pellet radius R_F = 4.025 mm. Cladding thickness = 0.64 mm,

Fuel rod outside radius R_o = 4.75 mm,

Fuel rod active height H = 3.9 m,

$h = 4.0\times10^4$ W/m^{2}deg [5-7], h_g = 4500 W/m^{2}deg [5-7]

k_F = 2.50 W/m.deg., ρ_F = 10200 Kg/m^3, c_F = 296 J/Kg.deg.
k_c = 15.13 W/m.deg., ρ_c = 630 Kg/m^3, c_c = 319 J/Kg.deg.

For simplicity, material properties are considered temperature independent and the heat transfer coefficient h is assumed constant along the length of the fuel rod. First, the maximum fuel temperature and its corresponding coolant and cladding temperatures are calculated analytically using separate subroutine. The developed code is then applied at this section. The volumetric heat generation rate q''' obeys the relation :

$$q''' = q_c''' \, \mathrm{Cos}(\pi z/H_e) = (\pi/2) \, q_{av}''' \, \mathrm{Cos}(\pi z/H_e)$$

where q''' and q_c''' are the volumetric heat generation rate at any point z and the center of the fuel element, and H_e is the extrapolated fuel element height (He ≈ H).

RESULTS AND DISCUSSION

To examine the validity of the proposed code, the time behavior of temperature in a cladded fuel rod is estimated using the proposed code and the results are compared with that obtained from the infinite series solution given by Eq.(1), which is also computerized using the approximate series of Bessel functions [10].

To make sure that the solution converges, the coefficients of the finite difference equation (Eq. (3)) must be positive. Applying this principle to the outer most cladding layer where the term F is the largest term because it includes the heat transfer coefficient h.

$$(1 - F1 - F2) \geq 0.0$$

Substituting for F1 and F2 from Eqs.(4) , one obtains,

$$(FO)_c \leq \frac{1}{2\left[\dfrac{R_N \, \Delta R_c}{R_o^2 - R_N^2} + \dfrac{(Bi)_c \, \Delta R_c^2}{R_o^2 - R_N^2}\right]} \qquad (6)$$

and the time step $\Delta\theta$ is then given by:

$$\Delta\theta = (\rho C)_c \, \Delta R_c^2 \, (FO)_c / k_c \qquad (7)$$

The above stability criteria can be approximately reduced to:

$$\Delta\theta \leq \frac{(\rho C)_c \, \Delta R_c^2}{k_c + \Delta R_c h} \qquad (8)$$

which about twice the time interval used in [5].

It is concluded that the new condition improves the economics of computations due the higher time interval $\Delta\theta$.

The analytical calculations (Eq.(1)) are conducted considering the effective heat transfer coefficient given by:

$$h_{eff} = \frac{1}{\left[\dfrac{1}{h_g} + \dfrac{R_F}{k_c}\ln(R_I/R_F) + \dfrac{R_F}{R_o h}\right]} \qquad (9)$$

The effective heat transfer coefficient of the considered case

takes the convection through the gas gap, the conduction through the cladding material, and the convection through the coolant into consideration. Using the corresponding values h_{eff} is then alculated and has the value of 3481 $W/m^2.K$.
Using the proposed code for fluid temperature of 311 °C one gets the numerical solution as shown in figure (2). Fig.(2) illustrates the time behavior of fuel temperature for a normal cooling channel ($q'''=3.0\times10^8$ W/m^3). According to the figure it is clear the fuel centerline temperature reaches the steady value 970 °C in approximately 27 second. Comparing this value of the centerline temperature with the analytically obtained corresponding value (948 °C) a well agreement is found. In addition, the numerically obtained values lie above the analytical values, which means more conservative solution is obtained. Another important fact is that the steady state values obtained numerically is closer to that obtained from the closed form analytical steady state closed form solution which is given by:

$$T_F - T_f = \frac{q'''R_F^2}{4\,k_F} + \frac{q'''R_F^2}{2}\left[\frac{1}{k_c}\ln\frac{R_o}{R_F} + \frac{1}{h\,R_o}\right] + \frac{q'''R_F}{2\,h_g} \qquad (10)$$

According to the relation (10), the steady state fuel centerline temperature for the considered case is 974 °C, while the analytical transient solution (Eq.(1)) gives the value of 948 and the value obtained numerically is 970 °C.
Figures (3) and (4) illustrate the temperature behavior for different volumetric heat generation rates q'''. It is clear that raising the neutron flux from zero to the value which produces 6.0×10^8 W/m^3 in a hot channel leads to raising the fuel temperature from initial value of 311 °C to a steady value of 1629 °C in approximately 30 seconds, while the cladding temperature reaches the value of 337 °C.. This steady state fuel temperature is less than the melting point of the UO2 fuel material which is 2749 °C [2]. In addition , the obtained values of fuel and cladding temperature are comparable to the values obtained from the closed form solution given by Eq. (10) (1636 and 337 °C), while the value obtained from the analytical transient solution (Eq.(1)) is 1584 °C. Investigating the time behavior of fuel and cladding temperatures as shown on Figs.(2) and (4), it can be deduced that the surface of the cladding reaches its stable temperature faster than the fuel cnterline temperature. This is because the thermal diffusivity of the cladding material (7.5×10^{-6} m^2/s) is higher than that of the fuel material (8.3×10^{-7} m^2/s).

CONCLUSIONS

From the above discussion it is concluded that the proposed code is structurally simple and requires small storage capacity. Consequently, the speed and accuracy of the calculations are greatly enhanced. In addition, even though exact analytical solutions may be available a numerical technique might prove economical and convenient.

NOMENCLATURE

Bi Biot Number = $h\ R_o/k_c$
C Specific heat (J/Kg.deg.)
F Coefficients (dimensionless)
Fo Fouriers modulus = $k\ \Delta\theta/(\rho c \Delta R^2)$
h Cladding to fluid heat transfer coefficient (W/m^2deg)
h_g Gas heat transfer coefficient (W/m^2deg)
h_{eff} Effective heat transfer coefficient (Eq.(9))
H Fuel element active height (m)
k Thermal conductivity (W/m.deg.)
q''' Volumetric heat generation rate (W/m^3)
NF Number of fuel layers
NC Number of cladding layers
r Radial distance (m)
R_o Cladding outer radius (m)
R_F Fuel radius (m)
T Temperature (°K)
z Vertical distance measured from fuel rod center
Z Vertical distance measured from the bottom of fuel rod
ρ Density (Kg/m^3)
θ Time (seconds)
ΔR Thickness of a cylindrical layer (m)

SUBSCRIPTS

F Fuel
f fluid (coolant)
c Cladding / center

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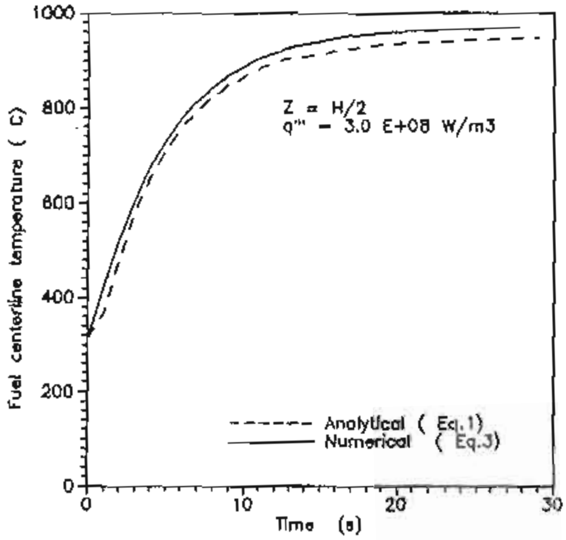

Fig. (2) Comparison between analytical and numerical solutions

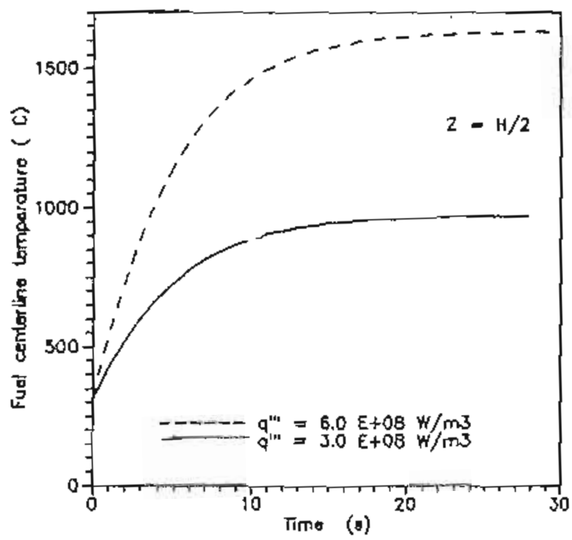

Fig. (3) Fuel temperature behavior for different heat generation rates

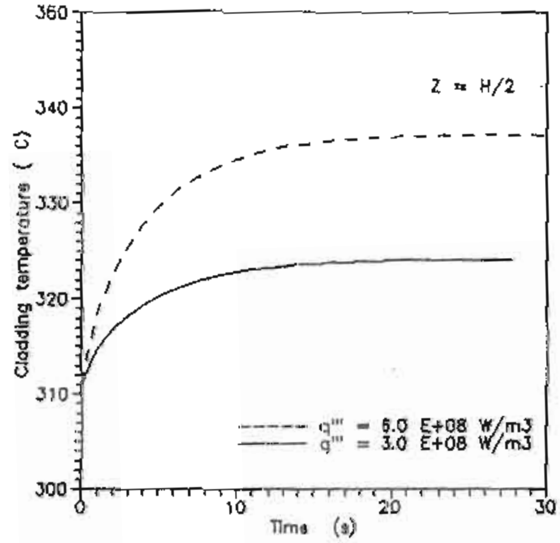

Fig. (4) Cladding temperature behavior for different heat generation rates